Computer algebra independent integration tests

Summer 2022 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/165-6.2.1-c+d-x-^m-a+b-cosh-^n

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [183]. This is test number [165].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (183)	0.00 (0)
Mathematica	98.91 (181)	1.09 (2)
Fricas	81.97 (150)	18.03 (33)
Maxima	78.14 (143)	21.86 (40)
Maple	60.66 (111)	39.34 (72)
Giac	56.28 (103)	43.72 (80)
Mupad	38.25 (70)	61.75 (113)
Sympy	33.88 (62)	66.12 (121)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

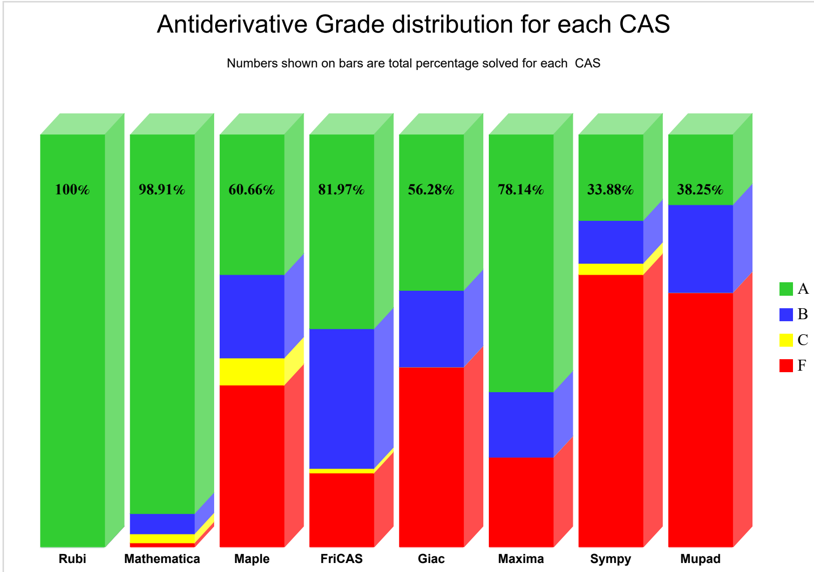
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

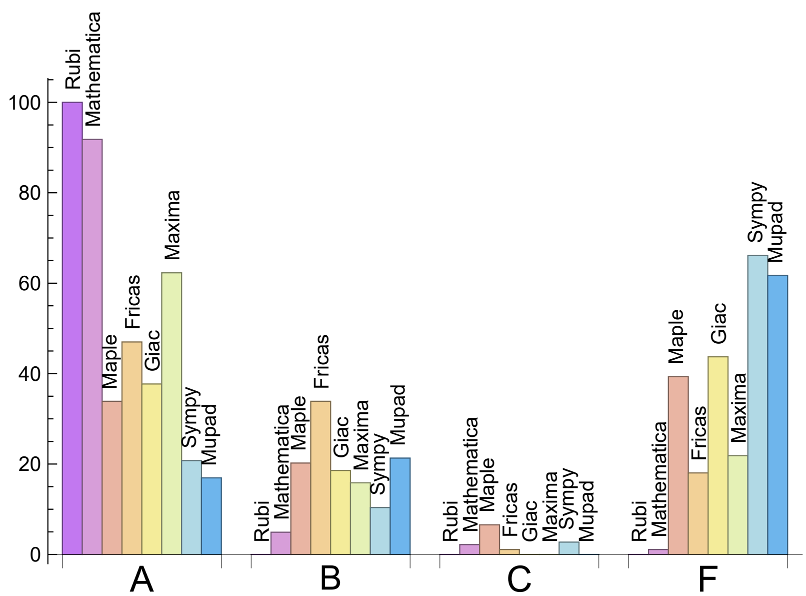
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	91.80	4.92	2.19	1.09
Maxima	62.30	15.85	0.00	21.86
Fricas	46.99	33.88	1.09	18.03
Giac	37.70	18.58	0.00	43.72
Maple	33.88	20.22	6.56	39.34
Sympy	20.77	10.38	2.73	66.12
Mupad	N/A	21.31	0.00	61.75

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	2	0.00 %	100.00 %	0.00 %
Maple	72	100.00 %	0.00 %	0.00 %
Fricas	33	18.18 %	0.00 %	81.82 %
Giac	80	100.00 %	0.00 %	0.00 %
Maxima	40	80.00 %	0.00 %	20.00 %
Sympy	121	76.03 %	10.74 %	13.22 %
Mupad	113	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

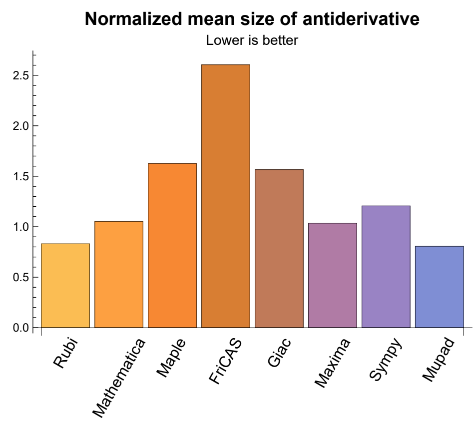
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

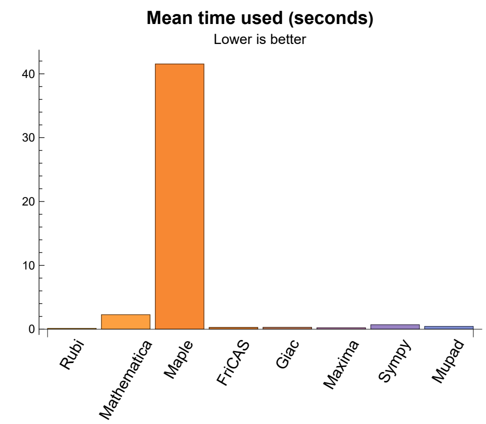
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.13	121.48	0.83	89.00	1.00
Mathematica	2.27	230.86	1.05	79.00	0.85
Maple	41.54	198.80	1.63	103.00	1.37
Maxima	0.23	134.94	1.04	102.00	0.93
Fricas	0.28	539.15	2.60	174.00	1.66
Sympy	0.70	146.45	1.21	68.00	1.11
Giac	0.30	187.26	1.57	107.00	1.15
Mupad	0.43	84.53	0.81	45.50	0.94

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
2.5
2.0
1.5
1.0
0.5
0.0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
40
30
20
10
0
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi $\{\}$

Mathematica $\{71, 73, 112, 117\}$

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

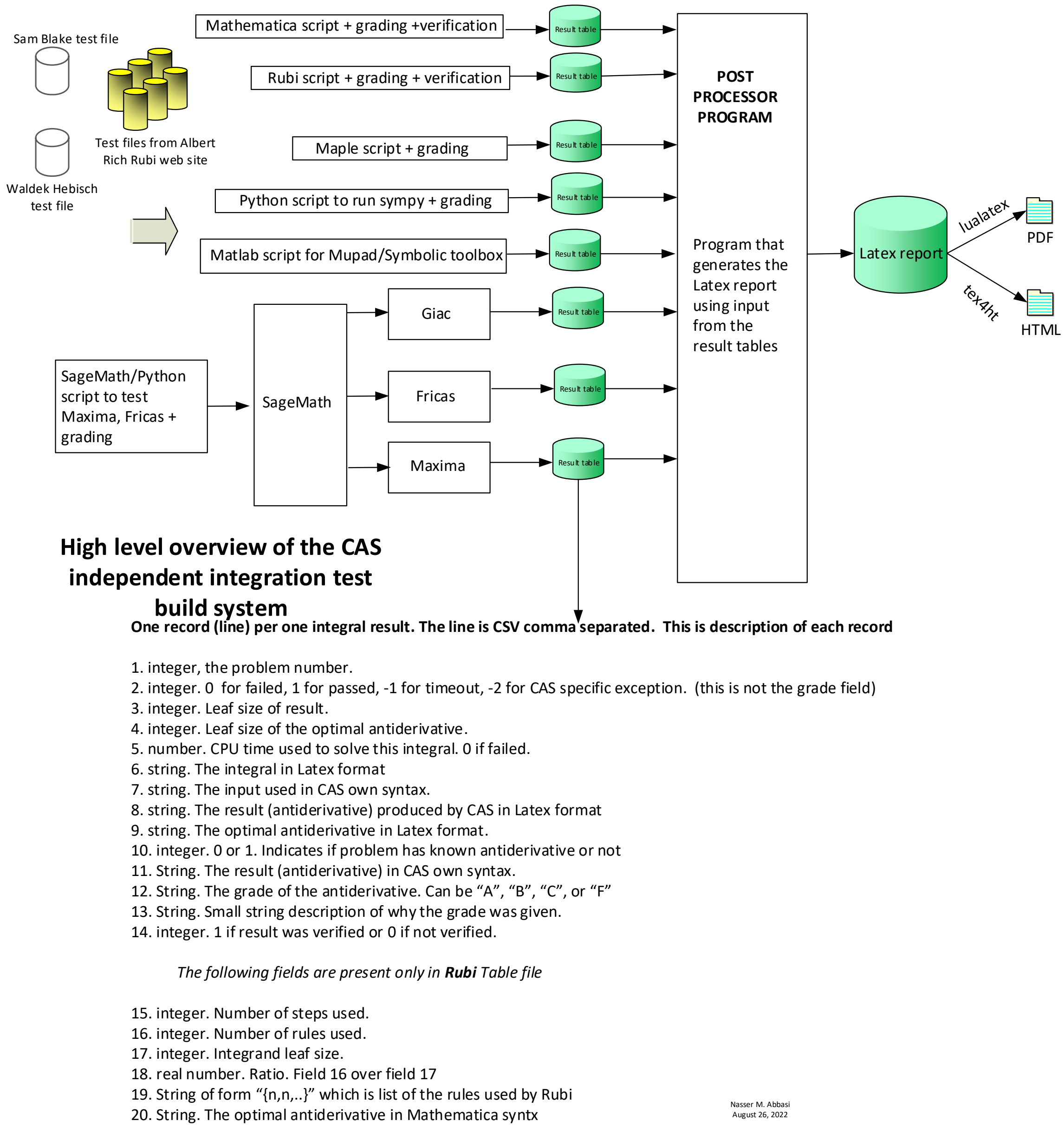
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 33, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 53, 55, 56, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 175, 176, 177, 178, 180, 181, 182, 183 }

B grade: { 28, 52, 54, 60, 62, 71, 173, 174, 179 }

C grade: { 32, 74, 112, 117 }

F grade: { 39, 40 }

2.1.3 Maple

A grade: { 4, 5, 6, 12, 13, 19, 20, 21, 24, 25, 28, 29, 30, 33, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 102, 103, 107, 108, 109, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 127, 128, 129, 142, 143, 147, 148, 149, 150, 154, 155, 159, 160, 164, 165, 166, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 7, 8, 9, 10, 11, 14, 15, 16, 17, 18, 22, 23, 31, 32, 38, 99, 100, 101, 104, 105, 106, 110, 111, 112, 116, 156, 157, 158, 161, 162, 163, 167, 170, 175 }

C grade: { 63, 64, 65, 66, 67, 81, 82, 83, 84, 85, 86, 87 }

F grade: { 26, 27, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 76, 77, 78, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 173, 174, 179, 180, 181 }

2.1.4 Maxima

A grade: { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 29, 30, 34, 35, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 68, 69, 70, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 107, 108, 109, 110, 113, 114, 115, 119, 120, 121, 122, 123, 127, 128, 129, 133, 134, 135, 142, 143, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 163, 164, 165, 166, 167, 171, 172, 176, 177, 178, 179, 180, 181, 182, 183 }

B grade: { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 31, 33, 41, 42, 43, 44, 63, 64, 65, 99, 100, 105, 106, 111, 116, 118, 156, 157, 162 }

C grade: { }

F grade: { 26, 27, 28, 32, 36, 37, 38, 71, 72, 73, 74, 93, 94, 95, 96, 97, 98, 112, 117, 124, 125, 126, 130, 131, 132, 136, 137, 138, 139, 140, 141, 144, 145, 146, 168, 169, 170, 173, 174, 175 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 44, 51, 59, 65, 68, 69, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 106, 107, 108, 114, 115, 119, 120, 142, 143, 147, 148, 150, 151, 154, 155, 156, 157, 158, 159, 160, 162, 163, 164, 165, 171, 172, 176, 177, 178, 179, 180, 182, 183 }

B grade: { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 26, 27, 28, 33, 36, 37, 38, 41, 42, 43, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 72, 104, 109, 110, 111, 112, 113, 116, 117, 118, 152, 153, 161, 166, 167, 168, 169, 170, 173, 174, 175, 181 }

C grade: { 31, 32 }

F grade: { 70, 71, 73, 74, 95, 96, 97, 98, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 149 }

2.1.6 Sympy

A grade: { 4, 19, 23, 24, 25, 29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 101, 107, 114, 115, 118, 119, 120, 142, 143, 147, 148, 149, 154, 155, 158, 164, 171, 172, 176, 182, 183 }

B grade: { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 99, 100, 105, 106, 113, 156, 157, 162, 163 }

C grade: { 63, 64, 65, 66, 67 }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 33, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 150, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 177, 178, 179, 180, 181 }

2.1.7 Giac

A grade: { 4, 5, 10, 11, 12, 19, 20, 23, 24, 25, 29, 30, 34, 35, 39, 40, 41, 42, 43, 44, 51, 63, 64, 65, 68, 69, 70, 75, 79, 80, 101, 102, 107, 108, 113, 114, 115, 119, 120, 121, 122, 123, 124, 125, 126, 133, 134, 135, 136, 137, 142, 143, 147, 148, 149, 150, 154, 155, 158, 159, 164, 165, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 33, 99, 100, 103, 104, 105, 106, 109, 110, 118, 138, 156, 157, 160, 161, 162, 163, 166, 167 }

C grade: { }

F grade: { 26, 27, 28, 31, 32, 36, 37, 38, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 116, 117, 127, 128, 129, 130, 131, 132, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

2.1.8 Mupad

A grade: { 29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183 }

B grade: { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 33, 71, 72, 73, 99, 100, 101, 105, 106, 107, 113, 118, 121, 122, 123, 127, 128, 129, 156, 157, 158, 162, 163, 164 }

C grade: { }

F grade: { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	76	547	326	171	311	324	215
N.S.	1	1.00	0.84	6.01	3.58	1.88	3.42	3.56	2.36
time (sec)	N/A	0.094	0.178	0.652	0.268	0.374	0.357	0.419	0.185

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	61	308	222	111	202	204	143
N.S.	1	1.00	0.87	4.40	3.17	1.59	2.89	2.91	2.04
time (sec)	N/A	0.061	0.114	0.688	0.266	0.413	0.225	0.407	0.935

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	44	147	135	64	112	112	82
N.S.	1	1.00	0.90	3.00	2.76	1.31	2.29	2.29	1.67
time (sec)	N/A	0.034	0.083	0.673	0.265	0.356	0.135	0.424	0.905

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	53	68	30	46	46	35
N.S.	1	1.00	0.96	1.89	2.43	1.07	1.64	1.64	1.25
time (sec)	N/A	0.014	0.044	0.732	0.264	0.389	0.083	0.412	0.073

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	57	94	0	56	-1
N.S.	1	1.00	0.96	1.61	1.12	1.84	0.00	1.10	-0.02
time (sec)	N/A	0.074	0.057	0.757	0.307	0.346	0.000	0.399	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	133	81	150	0	615	-1
N.S.	1	1.00	0.92	1.87	1.14	2.11	0.00	8.66	-0.01
time (sec)	N/A	0.097	0.175	0.783	0.310	0.374	0.000	0.446	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	88	277	95	254	0	298	-1
N.S.	1	1.00	0.85	2.66	0.91	2.44	0.00	2.87	-0.01
time (sec)	N/A	0.120	0.401	0.796	0.299	0.369	0.000	0.431	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	132	910	382	312	660	372	332
N.S.	1	1.00	0.81	5.62	2.36	1.93	4.07	2.30	2.05
time (sec)	N/A	0.074	0.471	0.793	0.284	0.363	0.581	0.420	1.502

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	104	523	263	209	456	243	229
N.S.	1	1.00	0.78	3.90	1.96	1.56	3.40	1.81	1.71
time (sec)	N/A	0.057	0.425	0.771	0.296	0.421	0.404	0.419	1.178

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	75	262	165	123	264	136	127
N.S.	1	1.00	0.79	2.76	1.74	1.29	2.78	1.43	1.34
time (sec)	N/A	0.035	0.288	0.779	0.274	0.365	0.259	0.407	0.983

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	103	88	66	126	63	58
N.S.	1	1.00	0.93	1.87	1.60	1.20	2.29	1.15	1.05
time (sec)	N/A	0.018	0.210	0.832	0.256	0.393	0.144	0.400	0.099

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	97	72	104	0	68	-1
N.S.	1	1.00	0.82	1.24	0.92	1.33	0.00	0.87	-0.01
time (sec)	N/A	0.116	0.084	3.581	0.296	0.369	0.000	0.415	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	152	88	164	0	574	-1
N.S.	1	1.00	0.93	1.88	1.09	2.02	0.00	7.09	-0.01
time (sec)	N/A	0.108	0.300	3.431	0.298	0.365	0.000	0.443	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	102	299	99	278	0	330	-1
N.S.	1	1.00	0.91	2.67	0.88	2.48	0.00	2.95	-0.01
time (sec)	N/A	0.132	0.974	3.560	0.306	0.450	0.000	0.423	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	121	555	110	409	0	537	-1
N.S.	1	1.00	0.75	3.43	0.68	2.52	0.00	3.31	-0.01
time (sec)	N/A	0.126	0.642	3.498	0.305	0.369	0.000	0.403	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	385	1263	644	528	772	654	532
N.S.	1	1.00	1.71	5.61	2.86	2.35	3.43	2.91	2.36
time (sec)	N/A	0.196	0.604	1.368	0.304	0.382	0.819	0.425	1.350

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	122	709	439	343	495	414	364
N.S.	1	1.00	0.70	4.05	2.51	1.96	2.83	2.37	2.08
time (sec)	N/A	0.123	0.636	1.275	0.288	0.388	0.544	0.417	1.140

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	93	340	272	199	284	230	183
N.S.	1	1.00	0.76	2.76	2.21	1.62	2.31	1.87	1.49
time (sec)	N/A	0.076	0.371	1.260	0.282	0.352	0.344	0.423	1.218

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	124	143	95	126	98	77
N.S.	1	1.00	0.69	1.65	1.91	1.27	1.68	1.31	1.03
time (sec)	N/A	0.031	0.175	1.343	0.281	0.386	0.211	0.412	0.217

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	117	186	0	112	-1
N.S.	1	1.00	0.84	1.37	0.97	1.54	0.00	0.93	-0.01
time (sec)	N/A	0.178	0.169	2.919	0.319	0.382	0.000	0.428	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	196	271	145	305	0	1075	-1
N.S.	1	1.00	1.35	1.87	1.00	2.10	0.00	7.41	-0.01
time (sec)	N/A	0.181	0.357	2.950	0.319	0.348	0.000	0.497	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	218	562	145	527	0	602	-1
N.S.	1	1.00	1.18	3.05	0.79	2.86	0.00	3.27	-0.01
time (sec)	N/A	0.244	0.611	2.918	0.341	0.383	0.000	0.439	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	100	367	176	195	253	150	129
N.S.	1	1.00	0.58	2.13	1.02	1.13	1.47	0.87	0.75
time (sec)	N/A	0.101	0.303	1.311	0.269	0.349	0.662	0.411	0.368

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	90	213	132	147	209	118	94
N.S.	1	1.00	0.67	1.59	0.99	1.10	1.56	0.88	0.70
time (sec)	N/A	0.074	0.120	1.261	0.270	0.339	0.445	0.410	0.249

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	53	101	96	114	138	86	68
N.S.	1	1.00	0.66	1.26	1.20	1.42	1.72	1.08	0.85
time (sec)	N/A	0.030	0.180	1.222	0.285	0.345	0.292	0.421	0.146

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	343	0	0	497	0	0	-1
N.S.	1	1.00	1.92	0.00	0.00	2.78	0.00	0.00	-0.01
time (sec)	N/A	0.093	2.155	180.000	0.000	0.414	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	199	0	0	305	0	0	-1
N.S.	1	1.00	1.67	0.00	0.00	2.56	0.00	0.00	-0.01
time (sec)	N/A	0.058	1.103	180.000	0.000	0.389	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	127	101	0	157	0	0	-1
N.S.	1	1.00	2.08	1.66	0.00	2.57	0.00	0.00	-0.02
time (sec)	N/A	0.027	0.116	0.960	0.000	0.387	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.016	2.990	180.000	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.014	6.204	180.000	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	135	298	238	1332	0	0	-1
N.S.	1	1.00	1.31	2.89	2.31	12.93	0.00	0.00	-0.01
time (sec)	N/A	0.143	1.365	1.517	0.387	0.428	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	186	159	0	715	0	0	-1
N.S.	1	1.00	2.55	2.18	0.00	9.79	0.00	0.00	-0.01
time (sec)	N/A	0.091	5.844	1.362	0.000	0.466	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	57	72	161	0	78	50
N.S.	1	1.00	1.76	1.97	2.48	5.55	0.00	2.69	1.72
time (sec)	N/A	0.021	0.080	0.835	0.286	0.414	0.000	0.412	0.089

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	16.083	180.000	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.025	16.318	180.000	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	455	0	0	4785	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	16.17	0.00	0.00	-0.00
time (sec)	N/A	0.153	20.952	180.000	0.000	0.455	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	270	0	0	2651	0	0	-1
N.S.	1	1.00	1.54	0.00	0.00	15.15	0.00	0.00	-0.01
time (sec)	N/A	0.110	3.990	180.000	0.000	0.434	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	178	216	0	1267	0	0	-1
N.S.	1	1.00	1.75	2.12	0.00	12.42	0.00	0.00	-0.01
time (sec)	N/A	0.052	2.409	1.190	0.000	0.394	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.028	180.018	180.000	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	F(-1)	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.026	180.009	180.000	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	107	0	308	523	0	232	-1
N.S.	1	1.00	0.63	0.00	1.80	3.06	0.00	1.36	-0.01
time (sec)	N/A	0.257	0.043	180.000	0.272	0.368	0.000	0.471	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	107	0	268	387	0	202	-1
N.S.	1	1.00	0.73	0.00	1.84	2.65	0.00	1.38	-0.01
time (sec)	N/A	0.169	0.075	180.000	0.288	0.366	0.000	0.450	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	105	0	230	302	0	169	-1
N.S.	1	1.00	0.85	0.00	1.87	2.46	0.00	1.37	-0.01
time (sec)	N/A	0.125	0.092	180.000	0.263	0.372	0.000	0.445	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	0	180	123	0	89	-1
N.S.	1	1.00	1.01	0.00	1.73	1.18	0.00	0.86	-0.01
time (sec)	N/A	0.090	0.029	180.000	0.280	0.361	0.000	0.436	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	118	0	104	338	0	0	-1
N.S.	1	1.00	0.99	0.00	0.87	2.84	0.00	0.00	-0.01
time (sec)	N/A	0.131	0.257	180.000	0.288	0.351	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	150	0	115	534	0	0	-1
N.S.	1	1.00	1.01	0.00	0.77	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.514	180.000	0.332	0.365	0.000	0.000	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	191	0	115	853	0	0	-1
N.S.	1	1.00	1.10	0.00	0.66	4.90	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.269	180.000	0.323	0.414	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	189	0	281	1001	0	0	-1
N.S.	1	1.00	0.79	0.00	1.18	4.19	0.00	0.00	-0.00
time (sec)	N/A	0.287	1.185	180.000	0.480	0.389	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	163	0	239	755	0	0	-1
N.S.	1	1.00	0.77	0.00	1.13	3.58	0.00	0.00	-0.00
time (sec)	N/A	0.227	0.544	180.000	0.474	0.445	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	-1
N.S.	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	-0.01
time (sec)	N/A	0.208	0.326	180.000	0.494	0.388	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	141	0	107	155	0	115	-1
N.S.	1	1.00	1.02	0.00	0.78	1.12	0.00	0.83	-0.01
time (sec)	N/A	0.159	0.087	180.000	0.475	0.388	0.000	0.414	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	570	0	116	569	0	0	-1
N.S.	1	1.00	4.01	0.00	0.82	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.158	2.482	180.000	0.331	0.424	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	156	0	118	861	0	0	-1
N.S.	1	1.00	0.90	0.00	0.68	4.95	0.00	0.00	-0.01
time (sec)	N/A	0.231	0.947	180.000	0.335	0.391	0.000	0.000	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	825	0	116	1350	0	0	-1
N.S.	1	1.00	3.75	0.00	0.53	6.14	0.00	0.00	-0.00
time (sec)	N/A	0.236	2.513	180.000	0.336	0.428	0.000	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	222	0	116	1825	0	0	-1
N.S.	1	1.00	0.88	0.00	0.46	7.27	0.00	0.00	-0.00
time (sec)	N/A	0.278	0.547	180.000	0.321	0.405	0.000	0.000	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	243	0	513	2092	0	0	-1
N.S.	1	1.00	0.64	0.00	1.35	5.49	0.00	0.00	-0.00
time (sec)	N/A	0.690	3.222	180.000	0.478	0.429	0.000	0.000	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	243	0	429	1545	0	0	-1
N.S.	1	1.00	0.75	0.00	1.32	4.74	0.00	0.00	-0.00
time (sec)	N/A	0.525	1.379	180.000	0.498	0.379	0.000	0.000	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	210	0	334	1217	0	0	-1
N.S.	1	1.00	0.76	0.00	1.21	4.43	0.00	0.00	-0.00
time (sec)	N/A	0.343	0.208	180.000	0.489	0.373	0.000	0.000	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	192	0	177	253	0	0	-1
N.S.	1	1.00	0.84	0.00	0.78	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.273	0.144	180.000	0.485	0.380	0.000	0.000	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	717	0	196	1344	0	0	-1
N.S.	1	1.00	2.91	0.00	0.80	5.46	0.00	0.00	-0.00
time (sec)	N/A	0.302	2.388	180.000	0.366	0.471	0.000	0.000	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	253	0	194	2058	0	0	-1
N.S.	1	1.00	0.91	0.00	0.70	7.43	0.00	0.00	-0.00
time (sec)	N/A	0.425	2.307	180.000	0.371	0.396	0.000	0.000	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	3211	0	196	3280	0	0	-1
N.S.	1	1.00	9.70	0.00	0.59	9.91	0.00	0.00	-0.00
time (sec)	N/A	0.489	6.292	180.000	0.361	0.431	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	51	133	174	191	131	145	-1
N.S.	1	1.00	0.46	1.20	1.57	1.72	1.18	1.31	-0.01
time (sec)	N/A	0.109	0.011	0.717	0.268	0.370	14.469	0.420	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	B	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	48	121	148	138	100	108	-1
N.S.	1	1.00	0.52	1.32	1.61	1.50	1.09	1.17	-0.01
time (sec)	N/A	0.074	0.010	0.722	0.276	0.382	0.861	0.427	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	B	A	C	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	48	72	117	59	66	60	-1
N.S.	1	1.00	0.62	0.94	1.52	0.77	0.86	0.78	-0.01
time (sec)	N/A	0.054	0.007	0.713	0.269	0.425	0.566	0.400	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	67	115	76	136	99	0	-1
N.S.	1	1.00	0.76	1.31	0.86	1.55	1.12	0.00	-0.01
time (sec)	N/A	0.076	0.026	0.709	0.265	0.371	1.982	0.000	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	B	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	78	126	58	179	124	0	-1
N.S.	1	1.00	0.68	1.11	0.51	1.57	1.09	0.00	-0.01
time (sec)	N/A	0.102	0.065	0.714	0.303	0.416	16.030	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.020	8.879	180.000	0.000	0.000	0.000	0.000	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.022	8.182	180.000	0.000	0.000	0.000	0.000	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	63	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	2.696	0.207	0.000	0.000	0.000	0.000	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	39
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	1.95
time (sec)	N/A	0.036	0.273	1.199	0.000	0.000	0.000	0.000	0.968

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	42
N.S.	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.75
time (sec)	N/A	0.035	0.048	1.257	0.000	0.377	0.000	0.000	0.936

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	110
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	2.34
time (sec)	N/A	0.051	0.450	1.609	0.000	0.000	0.000	0.000	1.082

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	-1
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.064	0.709	1.187	0.000	0.000	0.000	0.000	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.031	1.994	180.000	0.000	0.000	0.000	0.000	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	0	161	340	0	0	-1
N.S.	1	1.00	0.86	0.00	0.68	1.43	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.137	180.000	0.094	0.117	0.000	0.000	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	102	241	0	0	-1
N.S.	1	1.00	0.92	0.00	0.71	1.67	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.152	180.000	0.061	0.109	0.000	0.000	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	79	168	0	0	-1
N.S.	1	1.00	0.93	0.00	0.72	1.53	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.040	180.000	0.062	0.105	0.000	0.000	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.016	4.128	180.000	0.000	0.000	0.000	0.000	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	19	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.027	2.428	180.000	0.000	0.000	0.000	0.000	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.055	0.030	0.323	0.071	0.096	0.000	0.000	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.052	0.016	0.356	0.075	0.088	0.000	0.000	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.048	0.024	0.347	0.078	0.095	0.000	0.000	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	78	0	0	-1
N.S.	1	1.00	0.92	1.24	0.93	1.32	0.00	0.00	-0.02
time (sec)	N/A	0.044	0.014	0.339	0.089	0.088	0.000	0.000	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	43	78	0	0	-1
N.S.	1	1.00	1.00	1.37	0.88	1.59	0.00	0.00	-0.02
time (sec)	N/A	0.045	0.016	0.389	0.083	0.113	0.000	0.000	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	67	55	86	0	0	-1
N.S.	1	1.00	0.95	1.22	1.00	1.56	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.015	0.413	0.092	0.098	0.000	0.000	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	71	55	86	0	0	-1
N.S.	1	1.00	0.93	1.20	0.93	1.46	0.00	0.00	-0.02
time (sec)	N/A	0.049	0.017	0.332	0.087	0.111	0.000	0.000	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.078	0.660	0.075	0.102	0.000	0.000	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.072	0.507	0.077	0.082	0.000	0.000	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	-1
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.076	0.551	0.083	0.160	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	-1
N.S.	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.065	0.538	0.073	0.101	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	55	117	0	0	-1
N.S.	1	1.00	0.89	0.00	0.76	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.043	0.648	0.084	0.090	0.000	0.000	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	136	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	1.64	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.074	0.664	0.000	0.149	0.000	0.000	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	136	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.101	0.074	0.645	0.000	0.141	0.000	0.000	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.061	0.052	1.116	0.000	0.000	0.000	0.000	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.059	0.092	1.082	0.000	0.000	0.000	0.000	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	-1
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.071	0.089	1.113	0.000	0.000	0.000	0.000	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	55	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.116	0.063	1.128	0.000	0.000	0.000	0.000	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	122	482	250	174	264	258	187
N.S.	1	1.00	1.37	5.42	2.81	1.96	2.97	2.90	2.10
time (sec)	N/A	0.097	0.309	0.828	0.282	0.453	0.258	0.416	1.019

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	240	149	108	151	146	112
N.S.	1	1.00	1.19	3.58	2.22	1.61	2.25	2.18	1.67
time (sec)	N/A	0.066	0.205	0.837	0.283	0.551	0.158	0.408	0.135

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	91	70	57	68	64	53
N.S.	1	1.00	1.16	2.02	1.56	1.27	1.51	1.42	1.18
time (sec)	N/A	0.033	0.135	0.898	0.314	0.454	0.093	0.411	0.079

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	94	72	122	0	67	-1
N.S.	1	1.00	0.84	1.47	1.12	1.91	0.00	1.05	-0.02
time (sec)	N/A	0.108	0.079	1.138	0.291	0.400	0.000	0.420	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	68	149	89	178	0	631	-1
N.S.	1	1.00	0.78	1.71	1.02	2.05	0.00	7.25	-0.01
time (sec)	N/A	0.136	0.240	1.057	0.306	0.454	0.000	0.427	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	296	100	293	0	316	-1
N.S.	1	1.00	0.73	2.41	0.81	2.38	0.00	2.57	-0.01
time (sec)	N/A	0.159	0.330	1.053	0.311	0.373	0.000	0.423	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	217	1071	554	410	779	577	452
N.S.	1	1.00	0.92	4.52	2.34	1.73	3.29	2.43	1.91
time (sec)	N/A	0.187	0.989	0.921	0.286	0.483	0.470	0.429	2.252

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	192	541	344	242	456	329	257
N.S.	1	1.00	1.14	3.22	2.05	1.44	2.71	1.96	1.53
time (sec)	N/A	0.138	0.343	0.911	0.286	0.547	0.302	0.426	1.288

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	81	211	176	128	219	151	123
N.S.	1	1.00	0.69	1.79	1.49	1.08	1.86	1.28	1.04
time (sec)	N/A	0.073	0.322	0.946	0.268	0.501	0.169	0.432	0.135

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	113	191	153	251	0	135	-1
N.S.	1	1.00	0.78	1.32	1.06	1.73	0.00	0.93	-0.01
time (sec)	N/A	0.251	0.146	4.393	0.329	0.442	0.000	0.424	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	207	308	186	544	0	1134	-1
N.S.	1	1.00	1.32	1.96	1.18	3.46	0.00	7.22	-0.01
time (sec)	N/A	0.237	0.466	3.498	0.325	0.396	0.000	0.470	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	353	618	206	840	0	682	-1
N.S.	1	1.00	1.71	2.99	1.00	4.06	0.00	3.29	-0.00
time (sec)	N/A	0.351	0.750	3.437	0.336	0.375	0.000	0.434	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	158	325	239	672	0	0	-1
N.S.	1	1.00	1.35	2.78	2.04	5.74	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.580	1.682	0.376	0.421	0.000	0.000	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	295	174	0	359	0	0	-1
N.S.	1	1.00	3.35	1.98	0.00	4.08	0.00	0.00	-0.01
time (sec)	N/A	0.143	4.435	1.508	0.000	0.502	0.000	0.000	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	70	63	76	116	76	66	53
N.S.	1	1.00	1.43	1.29	1.55	2.37	1.55	1.35	1.08
time (sec)	N/A	0.049	0.163	1.228	0.263	0.411	0.328	0.408	0.903

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	6.383	180.000	0.000	0.000	0.000	0.000	0.000

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	6.493	180.000	0.000	0.000	0.000	0.000	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	492	600	633	2683	0	0	-1
N.S.	1	1.00	1.93	2.35	2.48	10.52	0.00	0.00	-0.00
time (sec)	N/A	0.253	2.665	2.041	0.417	0.454	0.000	0.000	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	637	313	0	1347	0	0	-1
N.S.	1	1.00	3.18	1.56	0.00	6.74	0.00	0.00	-0.00
time (sec)	N/A	0.180	6.270	1.605	0.000	0.447	0.000	0.000	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	114	108	255	472	156	192	138
N.S.	1	1.00	0.93	0.88	2.07	3.84	1.27	1.56	1.12
time (sec)	N/A	0.066	0.290	1.571	0.294	0.383	0.532	0.402	0.893

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	21.273	180.000	0.000	0.000	0.000	0.000	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.035	21.729	180.000	0.000	0.000	0.000	0.000	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	53	108	120	0	0	147	117
N.S.	1	1.00	0.48	0.98	1.09	0.00	0.00	1.34	1.06
time (sec)	N/A	0.109	0.152	0.805	0.484	0.000	0.000	0.398	0.207

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	44	86	90	0	0	107	95
N.S.	1	1.00	0.50	0.98	1.02	0.00	0.00	1.22	1.08
time (sec)	N/A	0.086	0.113	0.477	0.479	0.000	0.000	0.410	0.934

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	34	64	60	0	0	67	56
N.S.	1	1.00	0.64	1.21	1.13	0.00	0.00	1.26	1.06
time (sec)	N/A	0.047	0.084	0.477	0.477	0.000	0.000	0.426	0.907

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	54	0	0	0	0	32	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.39	-0.01
time (sec)	N/A	0.115	0.060	0.766	0.000	0.000	0.000	0.421	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	75	0	0	0	0	68	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.62	-0.01
time (sec)	N/A	0.104	0.105	0.444	0.000	0.000	0.000	0.419	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	97	0	0	0	0	107	-1
N.S.	1	1.00	0.64	0.00	0.00	0.00	0.00	0.71	-0.01
time (sec)	N/A	0.123	0.184	0.444	0.000	0.000	0.000	0.411	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	33	62	88	0	0	0	63
N.S.	1	1.00	0.49	0.91	1.29	0.00	0.00	0.00	0.93
time (sec)	N/A	0.094	0.044	0.474	0.474	0.000	0.000	0.000	0.911

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	31	50	66	0	0	0	51
N.S.	1	1.00	0.58	0.94	1.25	0.00	0.00	0.00	0.96
time (sec)	N/A	0.069	0.034	0.408	0.476	0.000	0.000	0.000	0.071

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	22	38	44	0	0	0	39
N.S.	1	1.00	0.69	1.19	1.38	0.00	0.00	0.00	1.22
time (sec)	N/A	0.038	0.019	0.405	0.476	0.000	0.000	0.000	0.880

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	0	0	0	-1
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.068	0.007	0.408	0.000	0.000	0.000	0.000	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	33	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.068	0.042	0.369	0.000	0.000	0.000	0.000	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	44	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.056	0.381	0.000	0.000	0.000	0.000	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	70	0	180	0	0	192	-1
N.S.	1	1.00	0.38	0.00	0.97	0.00	0.00	1.04	-0.01
time (sec)	N/A	0.146	0.212	0.329	0.478	0.000	0.000	0.411	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	54	0	136	0	0	144	-1
N.S.	1	1.00	0.37	0.00	0.94	0.00	0.00	0.99	-0.01
time (sec)	N/A	0.106	0.184	0.324	0.490	0.000	0.000	0.431	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	56	0	92	0	0	96	-1
N.S.	1	1.00	0.63	0.00	1.03	0.00	0.00	1.08	-0.01
time (sec)	N/A	0.062	0.074	0.322	0.478	0.000	0.000	0.417	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	36	0	0	0	0	40	-1
N.S.	1	1.00	0.65	0.00	0.00	0.00	0.00	0.73	-0.02
time (sec)	N/A	0.092	0.016	0.323	0.000	0.000	0.000	0.406	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	53	0	0	0	0	112	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	1.42	-0.01
time (sec)	N/A	0.094	0.065	0.316	0.000	0.000	0.000	0.413	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	69	0	0	0	0	170	-1
N.S.	1	1.00	0.63	0.00	0.00	0.00	0.00	1.56	-0.01
time (sec)	N/A	0.123	0.049	0.319	0.000	0.000	0.000	0.416	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	383	383	213	0	0	0	0	0	-1
N.S.	1	1.00	0.56	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.150	1.107	0.457	0.000	0.000	0.000	0.000	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	163	0	0	0	0	0	-1
N.S.	1	1.00	0.61	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.120	1.135	0.437	0.000	0.000	0.000	0.000	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	117	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.059	0.544	0.432	0.000	0.000	0.000	0.000	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.049	1.957	0.432	0.000	0.000	0.000	0.000	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.067	1.067	0.436	0.000	0.000	0.000	0.000	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	716	0	0	0	0	0	-1
N.S.	1	1.00	1.78	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.198	1.925	0.326	0.000	0.000	0.000	0.000	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	214	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.128	0.692	0.332	0.000	0.000	0.000	0.000	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	0	0	0	0	0	-1
N.S.	1	1.00	0.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.083	0.325	0.000	0.000	0.000	0.000	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.050	6.464	0.332	0.000	0.000	0.000	0.000	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.051	7.878	0.326	0.000	0.000	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-2)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.05
time (sec)	N/A	0.039	1.955	0.382	0.000	0.000	0.000	0.000	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.031	4.348	180.000	0.000	0.000	0.000	0.000	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	429	0	381	783	0	0	-1
N.S.	1	1.00	1.07	0.00	0.95	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.407	1.640	180.000	0.136	0.136	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	302	0	213	539	0	0	-1
N.S.	1	1.00	1.15	0.00	0.81	2.05	0.00	0.00	-0.00
time (sec)	N/A	0.240	0.775	180.000	0.084	0.098	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	189	0	102	271	0	0	-1
N.S.	1	1.00	1.44	0.00	0.78	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.231	180.000	0.057	0.101	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	2.942	180.000	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	6.077	180.000	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	482	250	174	264	258	187
N.S.	1	1.00	1.38	5.42	2.81	1.96	2.97	2.90	2.10
time (sec)	N/A	0.095	0.290	0.836	0.279	0.382	0.281	0.410	0.988

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	240	149	108	151	146	110
N.S.	1	1.00	1.24	3.58	2.22	1.61	2.25	2.18	1.64
time (sec)	N/A	0.064	0.209	0.768	0.268	0.370	0.175	0.403	0.946

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	91	70	57	68	64	49
N.S.	1	1.00	1.02	2.02	1.56	1.27	1.51	1.42	1.09
time (sec)	N/A	0.031	0.087	0.788	0.263	0.360	0.091	0.416	0.084

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	72	122	0	67	-1
N.S.	1	1.00	0.89	1.47	1.12	1.91	0.00	1.05	-0.02
time (sec)	N/A	0.093	0.087	0.872	0.309	0.385	0.000	0.403	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	89	178	0	631	-1
N.S.	1	1.00	0.82	1.71	1.02	2.05	0.00	7.25	-0.01
time (sec)	N/A	0.121	0.251	0.885	0.295	0.346	0.000	0.432	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	100	293	0	316	-1
N.S.	1	1.00	0.77	2.41	0.81	2.38	0.00	2.57	-0.01
time (sec)	N/A	0.146	0.367	0.899	0.305	0.349	0.000	0.406	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	232	1061	550	424	779	599	481
N.S.	1	1.00	0.93	4.24	2.20	1.70	3.12	2.40	1.92
time (sec)	N/A	0.200	0.811	0.950	0.301	0.362	0.507	0.423	2.636

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	252	535	341	255	456	345	281
N.S.	1	1.00	1.38	2.94	1.87	1.40	2.51	1.90	1.54
time (sec)	N/A	0.133	0.634	0.931	0.288	0.366	0.323	0.419	1.335

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	96	208	174	137	219	160	135
N.S.	1	1.00	0.83	1.79	1.50	1.18	1.89	1.38	1.16
time (sec)	N/A	0.076	0.511	0.972	0.275	0.429	0.163	0.412	0.149

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	133	202	152	253	0	144	-1
N.S.	1	1.00	0.85	1.29	0.97	1.62	0.00	0.92	-0.01
time (sec)	N/A	0.231	0.174	4.142	0.316	0.381	0.000	0.418	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	233	319	185	540	0	1135	-1
N.S.	1	1.00	1.27	1.74	1.01	2.95	0.00	6.20	-0.01
time (sec)	N/A	0.258	0.497	4.213	0.316	0.381	0.000	0.502	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	394	626	205	830	0	678	-1
N.S.	1	1.00	1.63	2.59	0.85	3.43	0.00	2.80	-0.00
time (sec)	N/A	0.325	0.812	4.085	0.348	0.380	0.000	0.429	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	436	436	384	0	0	1420	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.570	1.034	180.000	0.000	0.376	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	247	0	0	938	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	2.93	0.00	0.00	-0.00
time (sec)	N/A	0.467	0.667	180.000	0.000	0.458	0.000	0.000	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	152	437	0	559	0	0	-1
N.S.	1	1.00	0.75	2.15	0.00	2.75	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.631	1.461	0.000	0.375	0.000	0.000	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	0.673	180.000	0.000	0.000	0.000	0.000	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	0.787	180.000	0.000	0.000	0.000	0.000	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	823	823	11178	0	0	11778	0	0	-1
N.S.	1	1.00	13.58	0.00	0.00	14.31	0.00	0.00	-0.00
time (sec)	N/A	0.975	23.005	0.833	0.000	0.574	0.000	0.000	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	593	593	2854	0	0	6143	0	0	-1
N.S.	1	1.00	4.81	0.00	0.00	10.36	0.00	0.00	-0.00
time (sec)	N/A	0.726	18.618	0.847	0.000	0.683	0.000	0.000	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	509	585	0	2297	0	0	-1
N.S.	1	1.00	1.86	2.14	0.00	8.38	0.00	0.00	-0.00
time (sec)	N/A	0.325	3.308	2.035	0.000	0.410	0.000	0.000	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.039	33.691	180.000	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	36.258	180.000	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.034	3.020	180.000	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	2639	0	383	883	0	0	-1
N.S.	1	1.00	4.86	0.00	0.71	1.63	0.00	0.00	-0.00
time (sec)	N/A	0.539	9.709	180.000	0.132	0.126	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	241	0	212	555	0	0	-1
N.S.	1	1.00	0.85	0.00	0.75	1.97	0.00	0.00	-0.00
time (sec)	N/A	0.281	5.831	180.000	0.080	0.147	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	202	0	102	271	0	0	-1
N.S.	1	1.00	1.54	0.00	0.78	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.967	180.000	0.058	0.087	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.038	0.883	180.000	0.000	0.000	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.036	3.942	180.000	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [98] had the largest ratio of [24]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	2	1.00	14	0.143
2	A	4	2	1.00	14	0.143
3	A	3	2	1.00	14	0.143
4	A	2	2	1.00	12	0.167
5	A	3	3	1.00	14	0.214
6	A	4	4	1.00	14	0.286
7	A	5	4	1.00	14	0.286
8	A	6	4	1.00	16	0.250
9	A	4	3	1.00	16	0.188
10	A	4	4	1.00	16	0.250
11	A	2	1	1.00	14	0.071
12	A	5	4	1.00	16	0.250
13	A	5	5	1.00	16	0.312
14	A	7	6	1.00	16	0.375
15	A	7	7	1.00	16	0.438
16	A	12	4	1.00	16	0.250
17	A	8	4	1.00	16	0.250
18	A	6	4	1.00	16	0.250
19	A	3	3	1.00	14	0.214
20	A	8	4	1.00	16	0.250
21	A	8	4	1.00	16	0.250
22	A	12	5	1.00	16	0.312
23	A	8	3	1.00	12	0.250
24	A	8	4	1.00	12	0.333
25	A	3	2	1.00	10	0.200

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	9	5	1.00	14	0.357
27	A	7	4	1.00	14	0.286
28	A	5	3	1.00	12	0.250
29	A	0	0	0.00	0	0.000
30	A	0	0	0.00	0	0.000
31	A	6	6	1.00	16	0.375
32	A	5	5	1.00	16	0.312
33	A	2	2	1.00	14	0.143
34	A	0	0	0.00	0	0.000
35	A	0	0	0.00	0	0.000
36	A	15	8	1.00	16	0.500
37	A	9	6	1.00	16	0.375
38	A	6	4	1.00	14	0.286
39	A	0	0	0.00	0	0.000
40	A	0	0	0.00	0	0.000
41	A	8	5	1.00	16	0.312
42	A	7	5	1.00	16	0.312
43	A	6	5	1.00	16	0.312
44	A	5	4	1.00	16	0.250
45	A	6	5	1.00	16	0.312
46	A	7	5	1.00	16	0.312
47	A	8	5	1.00	16	0.312
48	A	10	8	1.00	18	0.444
49	A	9	7	1.00	18	0.389
50	A	8	6	1.00	18	0.333
51	A	7	5	1.00	18	0.278
52	A	7	6	1.00	18	0.333
53	A	9	7	1.00	18	0.389
54	A	9	8	1.00	18	0.444
55	A	11	7	1.00	18	0.389
56	A	23	7	1.00	18	0.389
57	A	20	7	1.00	18	0.389
58	A	14	6	1.00	18	0.333
59	A	12	5	1.00	18	0.278
60	A	12	5	1.00	18	0.278

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	18	6	1.00	18	0.333
62	A	19	7	1.00	18	0.389
63	A	7	5	1.00	12	0.417
64	A	6	5	1.00	12	0.417
65	A	5	4	1.00	12	0.333
66	A	6	5	1.00	12	0.417
67	A	7	5	1.00	12	0.417
68	A	0	0	0.00	0	0.000
69	A	0	0	0.00	0	0.000
70	A	0	0	0.00	0	0.000
71	A	2	1	1.00	17	0.059
72	A	2	1	1.00	20	0.050
73	A	3	1	1.00	20	0.050
74	A	3	2	1.00	21	0.095
75	A	0	0	0.00	0	0.000
76	A	8	3	1.00	16	0.188
77	A	5	3	1.00	16	0.188
78	A	3	2	1.00	14	0.143
79	A	0	0	0.00	0	0.000
80	A	0	0	0.00	0	0.000
81	A	3	2	1.00	12	0.167
82	A	3	2	1.00	12	0.167
83	A	3	2	1.00	12	0.167
84	A	3	2	1.00	10	0.200
85	A	3	2	1.00	12	0.167
86	A	3	2	1.00	12	0.167
87	A	3	2	1.00	12	0.167
88	A	5	3	1.00	14	0.214
89	A	5	3	1.00	14	0.214
90	A	5	3	1.00	14	0.214
91	A	5	3	1.00	12	0.250
92	A	5	3	1.00	14	0.214
93	A	5	3	1.00	14	0.214
94	A	5	3	1.00	14	0.214
95	A	4	2	1.00	20	0.100

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	4	2	1.00	20	0.100
97	A	5	2	1.00	20	0.100
98	A	7	5	1.00	24	0.208
99	A	6	3	1.00	18	0.167
100	A	5	3	1.00	18	0.167
101	A	4	3	1.00	16	0.188
102	A	5	4	1.00	18	0.222
103	A	6	5	1.00	18	0.278
104	A	7	5	1.00	18	0.278
105	A	10	6	1.00	20	0.300
106	A	9	7	1.00	20	0.350
107	A	6	4	1.00	18	0.222
108	A	9	5	1.00	20	0.250
109	A	9	5	1.00	20	0.250
110	A	15	6	1.00	20	0.300
111	A	7	7	1.00	20	0.350
112	A	6	6	1.00	20	0.300
113	A	3	3	1.00	18	0.167
114	A	0	0	0.00	0	0.000
115	A	0	0	0.00	0	0.000
116	A	10	9	1.00	20	0.450
117	A	9	9	1.00	20	0.450
118	A	4	4	1.00	18	0.222
119	A	0	0	0.00	0	0.000
120	A	0	0	0.00	0	0.000
121	A	5	3	1.00	18	0.167
122	A	4	3	1.00	18	0.167
123	A	3	3	1.00	16	0.188
124	A	4	4	1.00	18	0.222
125	A	5	5	1.00	18	0.278
126	A	6	5	1.00	18	0.278
127	A	5	3	1.00	14	0.214
128	A	4	3	1.00	14	0.214
129	A	3	3	1.00	12	0.250
130	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	3	3	1.00	14	0.214
132	A	4	3	1.00	14	0.214
133	A	9	5	1.00	14	0.357
134	A	7	5	1.00	14	0.357
135	A	4	4	1.00	12	0.333
136	A	5	3	1.00	14	0.214
137	A	5	3	1.00	14	0.214
138	A	7	4	1.00	14	0.286
139	A	10	6	1.00	18	0.333
140	A	8	5	1.00	18	0.278
141	A	6	4	1.00	16	0.250
142	A	0	0	0.00	0	0.000
143	A	0	0	0.00	0	0.000
144	A	16	9	1.00	14	0.643
145	A	10	7	1.00	14	0.500
146	A	7	5	1.00	12	0.417
147	A	0	0	0.00	0	0.000
148	A	0	0	0.00	0	0.000
149	A	0	0	0.00	0	0.000
150	A	0	0	0.00	0	0.000
151	A	12	4	1.00	20	0.200
152	A	9	4	1.00	20	0.200
153	A	5	3	1.00	18	0.167
154	A	0	0	0.00	0	0.000
155	A	0	0	0.00	0	0.000
156	A	6	3	1.00	18	0.167
157	A	5	3	1.00	18	0.167
158	A	4	3	1.00	16	0.188
159	A	5	4	1.00	18	0.222
160	A	6	5	1.00	18	0.278
161	A	7	5	1.00	18	0.278
162	A	10	6	1.00	20	0.300
163	A	9	7	1.00	20	0.350
164	A	6	4	1.00	18	0.222
165	A	10	5	1.00	20	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	11	7	1.00	20	0.350
167	A	14	8	1.00	20	0.400
168	A	12	7	1.00	20	0.350
169	A	10	6	1.00	20	0.300
170	A	8	5	1.00	18	0.278
171	A	0	0	0.00	0	0.000
172	A	0	0	0.00	0	0.000
173	A	22	9	1.00	20	0.450
174	A	18	10	1.00	20	0.500
175	A	11	8	1.00	18	0.444
176	A	0	0	0.00	0	0.000
177	A	0	0	0.00	0	0.000
178	A	0	0	0.00	0	0.000
179	A	18	4	1.00	20	0.200
180	A	10	4	1.00	20	0.200
181	A	5	3	1.00	18	0.167
182	A	0	0	0.00	0	0.000
183	A	0	0	0.00	0	0.000

Chapter 3

Listing of integrals

Local contents

3.1 $\int (c+dx)^4 \cosh(a+bx)\,dx$

Optimal. Leaf size=91

$$-\frac{24d^3(c+dx)\cosh(a+bx)}{b^4}-\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{24d^4\sinh(a+bx)}{b^5}+\frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3}+\frac{(}{}$$

```
[Out] -24*d^3*(d*x+c)*cosh(b*x+a)/b^4-4*d*(d*x+c)^3*cosh(b*x+a)/b^2+24*d^4*sinh(b
*x+a)/b^5+12*d^2*(d*x+c)^2*sinh(b*x+a)/b^3+(d*x+c)^4*sinh(b*x+a)/b
```

Rubi [A]

time = 0.09, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3377, 2717}

$$\frac{24d^4\sinh(a+bx)}{b^5}-\frac{24d^3(c+dx)\cosh(a+bx)}{b^4}+\frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3}-\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x],x]
```

```
[Out] (-24*d^3*(c + d*x)*Cosh[a + b*x])/b^4 - (4*d*(c + d*x)^3*Cosh[a + b*x])/b^2
 + (24*d^4*Sinh[a + b*x])/b^5 + (12*d^2*(c + d*x)^2*Sinh[a + b*x])/b^3 + ((
c + d*x)^4*Sinh[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^4\cosh(a+bx)\,dx &= \frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{(4d)\int (c+dx)^3\sinh(a+bx)\,dx}{b}\\
&= -\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{(12d^2)\int (c+dx)^2\,\mathrm{co}}{b^2}\\
&= -\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^4\sinh(a}{b}\\
&= -\frac{24d^3(c+dx)\cosh(a+bx)}{b^4}-\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{12d^2(c+dx)^2\,\mathrm{sin}}{b^3}\\
&= -\frac{24d^3(c+dx)\cosh(a+bx)}{b^4}-\frac{4d(c+dx)^3\cosh(a+bx)}{b^2}+\frac{24d^4\sinh(a+bx}{b^5}
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 76, normalized size = 0.84

$$\frac{-4bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)+\left(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4\right)\sinh(a+bx)}{b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x],x]
```

```
[Out] (-4*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (24*d^4 + 12*b^
2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sinh[a + b*x])/b^5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(91) = 182.
time = 0.65, size = 547, normalized size = 6.01

method	result
risch	$\frac{(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2-4b^3d^4x^3+4b^4c^3dx-12b^3c\,d^3x^2+b^4c^4-12b^3c^2d^2x+12b^2d^4x^2-4b^3c^3d+24b^2c\,d^3x+12b^2c}{2b^5}$
derivativedivides	$\frac{\frac{d^4\left((bx+a)^4\sinh(bx+a)-4(bx+a)^3\cosh(bx+a)+12(bx+a)^2\sinh(bx+a)-24(bx+a)\cosh(bx+a)+24\sinh(bx+a)\right)}{b^4}+\frac{d^4a^4\sinh(bx+a)}{b^4}+12}{}$
default	$\frac{\frac{d^4\left((bx+a)^4\sinh(bx+a)-4(bx+a)^3\cosh(bx+a)+12(bx+a)^2\sinh(bx+a)-24(bx+a)\cosh(bx+a)+24\sinh(bx+a)\right)}{b^4}+\frac{d^4a^4\sinh(bx+a)}{b^4}+12}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^4/b^4*((b*x+a)^4*sinh(b*x+a)-4*(b*x+a)^3*cosh(b*x+a)+12*(b*x+a)^2*si
nh(b*x+a)-24*(b*x+a)*cosh(b*x+a)+24*sinh(b*x+a))+d^4/b^4*a^4*sinh(b*x+a)+12
*d^3/b^3*a^2*c*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-12*d^2/b^2*a*c^2*((b*x+a)*
sinh(b*x+a)-cosh(b*x+a))-12*d^3/b^3*a*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*co
sh(b*x+a)+2*sinh(b*x+a))-4*d/b*a*c^3*sinh(b*x+a)-4*d^3/b^3*a^3*c*sinh(b*x+a
)+6*d^2/b^2*a^2*c^2*sinh(b*x+a)-4*d^4/b^4*a^3*((b*x+a)*sinh(b*x+a)-cosh(b*x
+a))+4*d/b*c^3*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+6*d^2/b^2*c^2*((b*x+a)^2*s
inh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))+4*d^3/b^3*c*((b*x+a)^3*sinh
(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*sinh(b*x+a)-6*cosh(b*x+a))+6*d^4/
b^4*a^2*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-4*d^4/b
^4*a*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*sinh(b*x+a)-6
*cosh(b*x+a))+c^4*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 326 vs. 2(91) = 182.
time = 0.27, size = 326, normalized size = 3.58

$$\frac{c^4e^{(bx+a)}}{2\,b}+\frac{2\,(bxe^a-e^a)c^3de^{(bx)}}{b^2}-\frac{c^4e^{(-bx-a)}}{2\,b}-\frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2}+\frac{3\,(b^2x^2e^a-2\,bxe^a+2\,e^a)c^2d^2e^{(bx)}}{b^3}-\frac{3\,(b^2x^2+2\,bx+2)c^2d^2e^{(-bx-a)}}{b^3}+\frac{2\,(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)cd^3e^{(bx)}}{b^4}-\frac{2\,(b^3x^3+3\,b^2x^2+6\,bx+6)cd^3e^{(-bx-a)}}{b^4}+\frac{(b^4x^4e^a-4\,b^3x^3e^a+12\,b^2x^2e^a-24\,bxe^a+24\,e^a)d^4e^{(bx)}}{2\,b^5}-\frac{(b^4x^4+4\,b^3x^3+12\,b^2x^2+24\,bx+24)d^4e^{(-bx-a)}}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 - 1/2*c^4*e^(-b
*x - a)/b - 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e^a
 + 2*e^a)*c^2*d^2*e^(b*x)/b^3 - 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x - a
)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*x)/b
^4 - 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*(b^4*
x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e^(b*x)
/b^5 - 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-b*x - a
)/b^5
```

Fricas [A]
time = 0.37, size = 171, normalized size = 1.88

$$-\frac{4\,(b^3d^4x^3+3\,b^3cd^3x^2+b^3c^3d+6\,bcd^3+3\,(b^3c^2d^2+2\,bd^4)x)\cosh\left(bx+a\right)-(b^4d^4x^4+4\,b^4cd^3x^3+b^4c^4+12\,b^2c^2d^2+24\,d^4+6\,(b^4c^2d^2+2\,b^2d^4)x^2+4\,(b^4c^3d+6\,b^2cd^3)x)\sinh\left(bx+a\right)}{b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2
 + 2*b*d^4)*x)*cosh(b*x + a) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 + 1
2*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d + 6
*b^2*c*d^3)*x)*sinh(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(92) = 184$.
time = 0.36, size = 311, normalized size = 3.42

$$\begin{cases} \frac{c^4\sinh(a+bx)}{b}+\frac{4c^3dx\sinh(a+bx)}{b}+\frac{6c^2d^2x^2\sinh(a+bx)}{b}+\frac{4cd^3x^3\sinh(a+bx)}{b}+\frac{d^4x^4\sinh(a+bx)}{b}-\frac{4c^3d\cosh(a+bx)}{b^2}-\frac{12c^2d^2x\cosh(a+bx)}{b^2}-\frac{12cd^3x^2\cosh(a+bx)}{b^2}-\frac{4d^4x^3\cosh(a+bx)}{b^2}+\frac{12c^2d^2\sinh(a+bx)}{b^3}+\frac{24cd^3x\sinh(a+bx)}{b^3}+\frac{12d^4x^2\sinh(a+bx)}{b^3}-\frac{24cd^3\cosh(a+bx)}{b^4}-\frac{24d^4x\cosh(a+bx)}{b^4}+\frac{24d^4\sinh(a+bx)}{b^5} & \text{for}\ b\neq 0 \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**4*sinh(a + b*x)/b + 4*c**3*d*x*sinh(a + b*x)/b + 6*c**2*d**2*
x**2*sinh(a + b*x)/b + 4*c*d**3*x**3*sinh(a + b*x)/b + d**4*x**4*sinh(a + b
*x)/b - 4*c**3*d*cosh(a + b*x)/b**2 - 12*c**2*d**2*x*cosh(a + b*x)/b**2 - 1
2*c*d**3*x**2*cosh(a + b*x)/b**2 - 4*d**4*x**3*cosh(a + b*x)/b**2 + 12*c**2
*d**2*sinh(a + b*x)/b**3 + 24*c*d**3*x*sinh(a + b*x)/b**3 + 12*d**4*x**2*si
nh(a + b*x)/b**3 - 24*c*d**3*cosh(a + b*x)/b**4 - 24*d**4*x*cosh(a + b*x)/b
**4 + 24*d**4*sinh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c
**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cosh(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(91) = 182$.
time = 0.42, size = 324, normalized size = 3.56

$$\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2-4\,b^3d^4x^3+4\,b^4c^3dx-12\,b^3cd^3x^2+b^4c^4-12\,b^3c^2d^2x+12\,b^2d^4x^2-4\,b^3c^3d+24\,b^2cd^3x+12\,b^2c^2d^2-24\,bd^4x-24\,bcd^3+24\,d^4)e^{(bx+a)}}{2\,b^5}-\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^3d^4x^3+4\,b^4c^3dx+12\,b^3cd^3x^2+b^4c^4+12\,b^3c^2d^2x+12\,b^2d^4x^2+4\,b^3c^3d+24\,b^2cd^3x+12\,b^2c^2d^2+24\,bd^4x+24\,bcd^3+24\,d^4)e^{(-bx-a)}}{2\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*
b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^
2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3
 + 24*d^4)*e^(b*x + a)/b^5 - 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2
*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*
b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*
d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

Mupad [B]
time = 0.19, size = 215, normalized size = 2.36

$$\frac{\sinh(a+b\,x)\,\left(b^4\,c^4+12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5}-\frac{4\cosh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^4}-\frac{4\,d^4\,x^3\cosh(a+b\,x)}{b^2}-\frac{12\,x\cosh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^4}+\frac{d^4\,x^4\sinh(a+b\,x)}{b}+\frac{4\,x\sinh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^3}+\frac{6\,x^2\sinh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^3}-\frac{12\,c\,d^3\,x^2\cosh(a+b\,x)}{b^2}+\frac{4\,c\,d^3\,x^3\sinh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^4,x)
```

```
[Out] (sinh(a + b*x)*(24*d^4 + b^4*c^4 + 12*b^2*c^2*d^2))/b^5 - (4*cosh(a + b*x)*
(6*c*d^3 + b^2*c^3*d))/b^4 - (4*d^4*x^3*cosh(a + b*x))/b^2 - (12*x*cosh(a +
 b*x)*(2*d^4 + b^2*c^2*d^2))/b^4 + (d^4*x^4*sinh(a + b*x))/b + (4*x*sinh(a
+ b*x)*(6*c*d^3 + b^2*c^3*d))/b^3 + (6*x^2*sinh(a + b*x)*(2*d^4 + b^2*c^2*d
^2))/b^3 - (12*c*d^3*x^2*cosh(a + b*x))/b^2 + (4*c*d^3*x^3*sinh(a + b*x))/b
```

3.2 $\int (c+dx)^3 \cosh(a+bx)\,dx$

Optimal. Leaf size=70

$$-\frac{6d^3\cosh(a+bx)}{b^4}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}+\frac{(c+dx)^3\sinh(a+bx)}{b}$$

```
[Out] -6*d^3*cosh(b*x+a)/b^4-3*d*(d*x+c)^2*cosh(b*x+a)/b^2+6*d^2*(d*x+c)*sinh(b*x
+a)/b^3+(d*x+c)^3*sinh(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2718}

$$-\frac{6d^3\cosh(a+bx)}{b^4}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{(c+dx)^3\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x],x]
```

```
[Out] (-6*d^3*Cosh[a + b*x])/b^4 - (3*d*(c + d*x)^2*Cosh[a + b*x])/b^2 + (6*d^2*(
c + d*x)*Sinh[a + b*x])/b^3 + ((c + d*x)^3*Sinh[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\cosh(a+bx)\,dx = \frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\sinh(a+bx)\,dx}{b}$$

$$= -\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{(6d^2)\int (c+dx)\cosh}{b^2}$$

$$= -\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}+\frac{(c+dx)^3\sinh(a+}{b}$$

$$= -\frac{6d^3\cosh(a+bx)}{b^4}-\frac{3d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{6d^2(c+dx)\sinh(a+bx)}{b^3}$$

Mathematica [A]
time = 0.11, size = 61, normalized size = 0.87

$$\frac{-3d(2d^2+b^2(c+dx)^2)\cosh(a+bx)+b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x],x]
```

```
[Out] (-3*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + b*(c + d*x)*(6*d^2 + b^2*(c
 + d*x)^2)*Sinh[a + b*x])/b^4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 307 vs. 2(70) = 140.
time = 0.69, size = 308, normalized size = 4.40

method	result
risch	$\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx-3b^2d^3x^2+b^3c^3-6b^2c\,d^2x-3b^2c^2d+6b\,d^3x+6bc\,d^2-6d^3\right)\mathrm{e}^{bx+a}}{2b^4}-\frac{\left(d^3x^3b^3+3b^3c\,d^2x^2+3b\right.}{}$
derivativedivides	$\frac{\frac{d^3\left((bx+a)^3\sinh(bx+a)-3(bx+a)^2\cosh(bx+a)+6(bx+a)\sinh(bx+a)-6\cosh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)\right.}{b^3}}{}$
default	$\frac{\frac{d^3\left((bx+a)^3\sinh(bx+a)-3(bx+a)^2\cosh(bx+a)+6(bx+a)\sinh(bx+a)-6\cosh(bx+a)\right)}{b^3}-\frac{3d^3a\left((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)\right.}{b^3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^3/b^3*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*sinh(
b*x+a)-6*cosh(b*x+a))-3*d^3/b^3*a*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x
+a)+2*sinh(b*x+a))+3*d^2/b^2*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)
+2*sinh(b*x+a))+3*d^3/b^3*a^2*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-6*d^2/b^2*a
*c*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+3*d/b*c^2*((b*x+a)*sinh(b*x+a)-cosh(b*
x+a))-d^3/b^3*a^3*sinh(b*x+a)+3*d^2/b^2*a^2*c*sinh(b*x+a)-3*d/b*a*c^2*sinh(
b*x+a)+c^3*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(70) = 140.
time = 0.27, size = 222, normalized size = 3.17

$$\frac{c^3e^{(bx+a)}}{2\,b}+\frac{3\,(bxe^a-e^a)c^2de^{(bx)}}{2\,b^2}-\frac{c^3e^{(-bx-a)}}{2\,b}-\frac{3\,(bx+1)c^2de^{(-bx-a)}}{2\,b^2}+\frac{3\,(b^2x^2e^a-2\,bxe^a+2\,e^a)cd^2e^{(bx)}}{2\,b^3}-\frac{3\,(b^2x^2+2\,bx+2)cd^2e^{(-bx-a)}}{2\,b^3}+\frac{(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)d^3e^{(bx)}}{2\,b^4}-\frac{(b^3x^3+3\,b^2x^2+6\,bx+6)d^3e^{(-bx-a)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 - 1/2*c^3*e^(
-b*x - a)/b - 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2*b
```

```
*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 - 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-b*x
 - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e^(b*
x)/b^4 - 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Fricas [A]
time = 0.41, size = 111, normalized size = 1.59

$$-\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+2\,d^3\right)\cosh\left(bx+a\right)-\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3+6\,bcd^2+3\left(b^3c^2d+2\,bd^3\right)x\right)\sinh\left(bx+a\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - (b^3*
d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*d^3)*x
)*sinh(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(70) = 140.
time = 0.22, size = 202, normalized size = 2.89

$$\begin{cases}\frac{c^3\sinh\left(a+bx\right)}{b}+\frac{3c^2dx\sinh\left(a+bx\right)}{b}+\frac{3cd^2x^2\sinh\left(a+bx\right)}{b}+\frac{d^3x^3\sinh\left(a+bx\right)}{b}-\frac{3c^2d\cosh\left(a+bx\right)}{b^2}-\frac{6cd^2x\cosh\left(a+bx\right)}{b^2}-\frac{3d^3x^2\cosh\left(a+bx\right)}{b^2}+\frac{6cd^2\sinh\left(a+bx\right)}{b^3}+\frac{6d^3x\sinh\left(a+bx\right)}{b^3}-\frac{6d^3\cosh\left(a+bx\right)}{b^4} & \text{for}\ b\neq 0\\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cosh\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**3*sinh(a + b*x)/b + 3*c**2*d*x*sinh(a + b*x)/b + 3*c*d**2*x**
2*sinh(a + b*x)/b + d**3*x**3*sinh(a + b*x)/b - 3*c**2*d*cosh(a + b*x)/b**2
 - 6*c*d**2*x*cosh(a + b*x)/b**2 - 3*d**3*x**2*cosh(a + b*x)/b**2 + 6*c*d**
2*sinh(a + b*x)/b**3 + 6*d**3*x*sinh(a + b*x)/b**3 - 6*d**3*cosh(a + b*x)/b
**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cos
h(a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 204 vs. 2(70) = 140.
time = 0.41, size = 204, normalized size = 2.91

$$\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx-3\,b^2d^3x^2+b^3c^3-6\,b^2cd^2x-3\,b^2c^2d+6\,bd^3x+6\,bcd^2-6\,d^3\right)e^{(bx+a)}}{2\,b^4}-\frac{\left(b^3d^3x^3+3\,b^3cd^2x^2+3\,b^3c^2dx+3\,b^2d^3x^2+b^3c^3+6\,b^2cd^2x+3\,b^2c^2d+6\,bd^3x+6\,bcd^2+6\,d^3\right)e^{(-bx-a)}}{2\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c^
3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x + a
)/b^4 - 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x^2
+ b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3)*e^
(-b*x - a)/b^4
```

Mupad [B]
time = 0.94, size = 143, normalized size = 2.04

$$\frac{\sinh(a+b\,x)\,\left(b^2\,c^3+6\,c\,d^2\right)}{b^3}-\frac{3\cosh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^4}-\frac{3\,d^3\,x^2\cosh(a+b\,x)}{b^2}+\frac{d^3\,x^3\sinh(a+b\,x)}{b}+\frac{3\,x\sinh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^3}-\frac{6\,c\,d^2\,x\cosh(a+b\,x)}{b^2}+\frac{3\,c\,d^2\,x^2\sinh(a+b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^3,x)
```

```
[Out] (sinh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*cosh(a + b*x)*(2*d^3 + b^2*c^2
*d))/b^4 - (3*d^3*x^2*cosh(a + b*x))/b^2 + (d^3*x^3*sinh(a + b*x))/b + (3*x
*sinh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*cosh(a + b*x))/b^2 + (
3*c*d^2*x^2*sinh(a + b*x))/b
```

3.3 $\int (c+dx)^2 \cosh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^2\sinh(a+bx)}{b}$$

```
[Out] -2*d*(d*x+c)*cosh(b*x+a)/b^2+2*d^2*sinh(b*x+a)/b^3+(d*x+c)^2*sinh(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3377, 2717}

$$\frac{2d^2\sinh(a+bx)}{b^3}-\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{(c+dx)^2\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x],x]
```

```
[Out] (-2*d*(c + d*x)*Cosh[a + b*x])/b^2 + (2*d^2*Sinh[a + b*x])/b^3 + ((c + d*x)
^2*Sinh[a + b*x])/b
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cosh(a+bx)\,dx &= \frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{(2d)\int (c+dx)\sinh(a+bx)\,dx}{b}\\ &= -\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{(2d^2)\int \cosh(a+bx)\,dx}{b^2}\\ &= -\frac{2d(c+dx)\cosh(a+bx)}{b^2}+\frac{2d^2\sinh(a+bx)}{b^3}+\frac{(c+dx)^2\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 44, normalized size = 0.90

$$\frac{-2bd(c+dx)\cosh(a+bx)+(2d^2+b^2(c+dx)^2)\sinh(a+bx)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x],x]
```

```
[Out] (-2*b*d*(c + d*x)*Cosh[a + b*x] + (2*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])/
b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(49) = 98$.
time = 0.67, size = 147, normalized size = 3.00

method	result
risch	$\frac{(b^2d^2x^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)\mathrm{e}^{bx+a}}{2b^3}-\frac{(b^2d^2x^2+2b^2cdx+b^2c^2+2b\,d^2x+2bcd+2d^2)\mathrm{e}^{-bx-a}}{2b^3}$
derivativedivides	$\frac{\frac{d^2\left((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b^2}+\frac{2dc((bx+a)\sinh(bx+a)-\cosh(\cdots}{b}}{b}$
default	$\frac{\frac{d^2\left((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh(bx+a)\right)}{b^2}-\frac{2d^2a((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b^2}+\frac{2dc((bx+a)\sinh(bx+a)-\cosh(\cdots}{b}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^2/b^2*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-2*
d^2/b^2*a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+2*d/b*c*((b*x+a)*sinh(b*x+a)-co
sh(b*x+a))+d^2/b^2*a^2*sinh(b*x+a)-2*d/b*a*c*sinh(b*x+a)+sinh(b*x+a)*c^2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.
time = 0.27, size = 135, normalized size = 2.76

$$\frac{c^2e^{(bx+a)}}{2\,b}+\frac{(bxe^a-e^a)cde^{(bx)}}{b^2}-\frac{c^2e^{(-bx-a)}}{2\,b}-\frac{(bx+1)cde^{(-bx-a)}}{b^2}+\frac{(b^2x^2e^a-2\,bxe^a+2\,e^a)d^2e^{(bx)}}{2\,b^3}-\frac{(b^2x^2+2\,bx+2)d^2e^{(-bx-a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 - 1/2*c^2*e^(-b*x -
 a)/b - (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2*e
^a)*d^2*e^(b*x)/b^3 - 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Fricas [A]
time = 0.36, size = 64, normalized size = 1.31

$$-\frac{2\,(bd^2x+bcd)\cosh\left(bx+a\right)-(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,d^2)\sinh\left(bx+a\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(2*(b*d^2*x + b*c*d)*cosh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
+ 2*d^2)*sinh(b*x + a))/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.
time = 0.14, size = 112, normalized size = 2.29

$$\begin{cases} \frac{c^2 \sinh{(a+bx)}}{b} + \frac{2cdx \sinh{(a+bx)}}{b} + \frac{d^2x^2 \sinh{(a+bx)}}{b} - \frac{2cd \cosh{(a+bx)}}{b^2} - \frac{2d^2x \cosh{(a+bx)}}{b^2} + \frac{2d^2 \sinh{(a+bx)}}{b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \cosh{(a)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a),x)
```

```
[Out] Piecewise((c**2*sinh(a + b*x)/b + 2*c*d*x*sinh(a + b*x)/b + d**2*x**2*sinh(
a + b*x)/b - 2*c*d*cosh(a + b*x)/b**2 - 2*d**2*x*cosh(a + b*x)/b**2 + 2*d**
2*sinh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)
, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(49) = 98$.
time = 0.42, size = 112, normalized size = 2.29

$$\frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2)e^{(bx+a)}}{2\,b^3} - \frac{(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{2\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^(
b*x + a)/b^3 - 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b*c
*d + 2*d^2)*e^(-b*x - a)/b^3
```

Mupad [B]
time = 0.90, size = 82, normalized size = 1.67

$$\frac{\sinh(a + b\,x)\,\left(b^2\,c^2 + 2\,d^2\right)}{b^3} + \frac{d^2\,x^2\,\sinh(a + b\,x)}{b} - \frac{2\,c\,d\cosh(a + b\,x)}{b^2} - \frac{2\,d^2\,x\cosh(a + b\,x)}{b^2} + \frac{2\,c\,d\,x\sinh(a + b\,x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^2,x)
```

```
[Out] (sinh(a + b*x)*(2*d^2 + b^2*c^2))/b^3 + (d^2*x^2*sinh(a + b*x))/b - (2*c*d*
cosh(a + b*x))/b^2 - (2*d^2*x*cosh(a + b*x))/b^2 + (2*c*d*x*sinh(a + b*x))/
b
```

3.4 $\int (c+dx)\cosh(a+bx)\,dx$

Optimal. Leaf size=28

$$-\frac{d\cosh(a+bx)}{b^2}+\frac{(c+dx)\sinh(a+bx)}{b}$$

```
[Out] -d*cosh(b*x+a)/b^2+(d*x+c)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.01, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3377, 2718}

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] -((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cosh(a+bx)\,dx &= \frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)\,dx}{b}\\ &= -\frac{d\cosh(a+bx)}{b^2}+\frac{(c+dx)\sinh(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 27, normalized size = 0.96

$$\frac{-d\cosh(a+bx)+b(c+dx)\sinh(a+bx)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x],x]
```

```
[Out] (-(d*Cosh[a + b*x]) + b*(c + d*x)*Sinh[a + b*x])/b^2
```

Maple [A]
time = 0.73, size = 53, normalized size = 1.89

method	result
risch	$\frac{(bdx+bc-d)\,\mathrm{e}^{bx+a}}{2b^2}-\frac{(bdx+bc+d)\,\mathrm{e}^{-bx-a}}{2b^2}$
derivativedivides	$\frac{\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\sinh(bx+a)}{b}$
default	$\frac{\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\sinh(bx+a)}{b}$
meijerg	$-\frac{2d\cosh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(bx)}{2\sqrt{\pi}}-\frac{bx\sinh(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{d\sinh(a)(\cosh(bx)bx-\sinh(bx))}{b^2}+\frac{c\cosh(a)\sinh(bx)}{b}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-d/b*a*sinh(b*x+a)+c*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.
time = 0.26, size = 68, normalized size = 2.43

$$\frac{ce^{(bx+a)}}{2\,b}+\frac{(bxe^a-e^a)de^{(bx)}}{2\,b^2}-\frac{ce^{(-bx-a)}}{2\,b}-\frac{(bx+1)de^{(-bx-a)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 - 1/2*c*e^(-b*x - a
)/b - 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Fricas [A]
time = 0.39, size = 30, normalized size = 1.07

$$-\frac{d\cosh\left(bx+a\right)-\left(bdx+bc\right)\sinh\left(bx+a\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -(d*cosh(b*x + a) - (b*d*x + b*c)*sinh(b*x + a))/b^2
```

Sympy [A]
time = 0.08, size = 46, normalized size = 1.64

$$\begin{cases} \frac{c\sinh(a+bx)}{b} + \frac{dx\sinh(a+bx)}{b} - \frac{d\cosh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\cosh(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x)
```

```
[Out] Piecewise((c*sinh(a + b*x)/b + d*x*sinh(a + b*x)/b - d*cosh(a + b*x)/b**2,
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a), True))
```

Giac [A]
time = 0.41, size = 46, normalized size = 1.64

$$\frac{(bdx + bc - d)e^{(bx+a)}}{2\,b^2} - \frac{(bdx + bc + d)e^{(-bx-a)}}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] 1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 - 1/2*(b*d*x + b*c + d)*e^(-b*x - a)/
b^2
```

Mupad [B]
time = 0.07, size = 35, normalized size = 1.25

$$\frac{c\sinh(a + b\,x) + d\,x\sinh(a + b\,x)}{b} - \frac{d\cosh(a + b\,x)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x),x)
```

```
[Out] (c*sinh(a + b*x) + d*x*sinh(a + b*x))/b - (d*cosh(a + b*x))/b^2
```

3.5 $\int \frac{\cosh(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=51

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

```
[Out] Chi(b*c/d+b*x)*cosh(a-b*c/d)/d+Shi(b*c/d+b*x)*sinh(a-b*c/d)/d
```

Rubi [A]
time = 0.07, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3384, 3379, 3382}

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x),x]
```

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*Sinh
Integral[(b*c)/d + b*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx + \sinh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx$$
$$= \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.96

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)+\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x),x]
```

```
[Out] (Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x] + Sinh[a - (b*c)/d]*SinhInte
gral[(b*c)/d + b*x])/d
```

Maple [A]
time = 0.76, size = 82, normalized size = 1.61

method	result	size
risch	$-\frac{e^{-\frac{ad-bc}{d}}\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{2d}-\frac{e^{\frac{ad-bc}{d}}\text{expIntegral}\left(1,-bx-a-\frac{-ad+bc}{d}\right)}{2d}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.31, size = 57, normalized size = 1.12

$$-\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2\,d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)*
exp_integral_e(1, -(d*x + c)*b/d)/d
```

Fricas [A]
time = 0.35, size = 94, normalized size = 1.84

$$\frac{\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (E
i((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.40, size = 56, normalized size = 1.10

$$\frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d
))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{\cosh(a+b\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x), x)
```

3.6 $\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=71

$$-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

```
[Out] -cosh(b*x+a)/d/(d*x+c)+b*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^2+b*Chi(b*c/d+b*x)*
sinh(a-b*c/d)/d^2
```

Rubi [A]
time = 0.10, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d^2}+\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}-\frac{\cosh(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] -(Cosh[a + b*x]/(d*(c + d*x))) + (b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b
*c)/d])/d^2 + (b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(a+bx)}{c+dx}\,dx}{d}$$

$$= -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,d}{d}$$

$$= -\frac{\cosh(a+bx)}{d(c+dx)} + \frac{b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^2} + \frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

Mathematica [A]
time = 0.17, size = 65, normalized size = 0.92

$$\frac{-\frac{d\cosh(a+bx)}{c+dx} + b\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right) + b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^2,x]
```

```
[Out] (-((d*Cosh[a + b*x])/(c + d*x)) + b*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*c
)/d] + b*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Maple [A]
time = 0.78, size = 133, normalized size = 1.87

method	result	siz
risch	$-\frac{b\,e^{-bx-a}}{2d(bdx+bc)} + \frac{b\,e^{-\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{2d^2} - \frac{b\,e^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)} - \frac{b\,e^{\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,-bx-a-\frac{-ad+bc}{d}\right)}{2d^2}$	13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*
d-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(1,
-b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.31, size = 81, normalized size = 1.14

$$\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d}-\frac{\cosh\left(bx+a\right)}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - e^(a - b*c/d)*ex
p_integral_e(1, -(d*x + c)*b/d)/d)/d - cosh(b*x + a)/((d*x + c)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(71) = 142.
time = 0.37, size = 150, normalized size = 2.11

$$-\frac{2\,d\cosh\left(bx+a\right)-\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*d*cosh(b*x + a) - ((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*c
)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b*d*x + b*c)*Ei((b*d*x + b
*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*x +
 c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(71) = 142.
time = 0.45, size = 615, normalized size = 8.66

$$-\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}-ab^2d\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^2de^{\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}+\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-ab^2d\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-b^2de^{\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}\right)d^2}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c*E
i(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c -
 a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
 - a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c) + a*d
/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b
*c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2
*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c
 - a*d)/d) + b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c
- a*d)/d)*e^(-(b*c - a*d)/d) - a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a
*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - b^2*d*e^((d*x + c)*(b -
b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d
/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + b\,x)}{(c + d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^2,x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x)^2, x)
```

3.7 $\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=104

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}+\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

```
[Out] 1/2*b^2*Chi(b*c/d+b*x)*cosh(a-b*c/d)/d^3-1/2*cosh(b*x+a)/d/(d*x+c)^2+1/2*b^
2*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-1/2*b*sinh(b*x+a)/d^2/(d*x+c)
```

Rubi [A]
time = 0.12, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3378, 3384, 3379, 3382}

$$\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}+\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}-\frac{\cosh(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] -1/2*Cosh[a + b*x]/(d*(c + d*x)^2) + (b^2*Cosh[a - (b*c)/d]*CoshIntegral[(b
*c)/d + b*x]])/(2*d^3) - (b*Sinh[a + b*x])/(2*d^2*(c + d*x)) + (b^2*Sinh[a -
 (b*c)/d]*SinhIntegral[(b*c)/d + b*x]])/(2*d^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx &= -\frac{\cosh(a+bx)}{2d(c+dx)^2}+\frac{b\int \frac{\sinh(a+bx)}{(c+dx)^2}\,dx}{2d}\\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}+\frac{b^2\int \frac{\cosh(a+bx)}{c+dx}\,dx}{2d^2}\\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}+\frac{\left(b^2\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\,dx}{2d^2}+\frac{\left(b^2\sinh\left(a-\right.\right.}{}\\ &= -\frac{\cosh(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3}-\frac{b\sinh(a+bx)}{2d^2(c+dx)}+\frac{b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{S}}{2d^3}\end{aligned}$$

Mathematica [A]
time = 0.40, size = 88, normalized size = 0.85

$$\frac{b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(b\left(\frac{c}{d}+x\right)\right)-\frac{d(d\cosh(a+bx)+b(c+dx)\sinh(a+bx))}{(c+dx)^2}+b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^3,x]
```

```
[Out] (b^2*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*(d*Cosh[a + b*x] + b*
(c + d*x)*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Sinh[a - (b*c)/d]*SinhIntegral[
b*(c/d + x)])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. 2(96) = 192.
time = 0.80, size = 277, normalized size = 2.66

method	result
risch	$\frac{b^3 e^{-bx-a} x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^3 e^{-bx-a} c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 e^{-bx-a}}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 e^{-\frac{ad-bc}{d}}\,\mathrm{expIntegral}\left(1,bx+a-\frac{ad-}{d}\right.}{4d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+1/4*b^3*exp(-b*x-
a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/4*b^2*exp(-b*x-a)/d/(b^2*d^2*x
^2+2*b^2*c*d*x+b^2*c^2)-1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/
d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-
1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Maxima [A]
time = 0.30, size = 95, normalized size = 0.91

$$\frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d}-\frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4\,d}-\frac{\cosh\left(bx+a\right)}{2\left(dx+c\right)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - e^(a
 - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*cosh(b*x
 + a)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(96) = 192.
time = 0.37, size = 254, normalized size = 2.44

$$-\frac{2\,d^2\cosh\left(bx+a\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+2\left(bd^2x+bcd\right)\sinh\left(bx+a\right)-\left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*d^2*cosh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*
x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))*c
osh(-(b*c - a*d)/d) + 2*(b*d^2*x + b*c*d)*sinh(b*x + a) - ((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^
2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c
^2*d^3)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(96) = 192$.
time = 0.43, size = 298, normalized size = 2.87

$$\frac{b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+2\,b^2cdx\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+b^2c^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+b^2c^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}-bd^2xe^{(bx+a)}+bd^2xe^{(-bx-a)}-bcde^{(bx+a)}+bcde^{(-bx-a)}-d^2e^{(bx+a)}-d^2e^{(-bx-a)}}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + b^2*d^2*x^2*Ei(-(b*d*x
 + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) +
 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b*c)
/d)*e^(a - b*c/d) + b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2*x*e
^(b*x + a) + b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) + b*c*d*e^(-b*x - a)
- d^2*e^(b*x + a) - d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)}{{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^3,x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x)^3, x)
```

3.8 $\int (c+dx)^4 \cosh^2(a+bx)\, dx$

Optimal. Leaf size=162

$$\frac{3d^4x}{4b^4}+\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^4\cosh(a+bx)\sinh(}{4b^5}$$

```
[Out] 3/4*d^4*x/b^4+1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/2*d^3*(d*x+c)*cosh(b*x
+a)^2/b^4-d*(d*x+c)^3*cosh(b*x+a)^2/b^2+3/4*d^4*cosh(b*x+a)*sinh(b*x+a)/b^5
+3/2*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^4*cosh(b*x+a)*si
nh(b*x+a)/b
```

Rubi [A]
time = 0.07, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{3d^4\sinh(a+bx)\cosh(a+bx)}{4b^5}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}+\frac{3d^2(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b^3}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{3d^4x}{4b^4}+\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x]^2,x]
```

```
[Out] (3*d^4*x)/(4*b^4) + (d*(c + d*x)^3)/(2*b^2) + (c + d*x)^5/(10*d) - (3*d^3*(
c + d*x)*Cosh[a + b*x]^2)/(2*b^4) - (d*(c + d*x)^3*Cosh[a + b*x]^2)/b^2 + (
3*d^4*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^5) + (3*d^2*(c + d*x)^2*Cosh[a + b*
x]*Sinh[a + b*x])/(2*b^3) + ((c + d*x)^4*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cosh^2(a+bx)\,dx &= -\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c\\ &= \frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^2(c\\ &= \frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a\\ &= \frac{3d^4x}{4b^4}+\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3}{}\end{aligned}$$

Mathematica [A]
time = 0.47, size = 132, normalized size = 0.81

$$\frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+10(3d^4+6b^2d^2(c+dx)^2+2b^4(c+dx)^4)\sinh(2(a+bx))}{80b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x]^2,x]
```

```
[Out] (8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 20
*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4 +
6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 909 vs. $2(148) = 296$.
time = 0.79, size = 910, normalized size = 5.62

method	result
risch	$\frac{d^4x^5}{10}+\frac{d^3cx^4}{2}+d^2c^2x^3+dc^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}+\frac{(2d^4x^4b^4+8b^4cd^3x^3+12b^4c^2d^2x^2-4b^3d^4x^3+8b^4c^3dx-1}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(-12*d^3/b^3*a*c*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1
/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+12*d^3/
b^3*a^2*c*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a
)^2)-12*d^2/b^2*a*c^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/
4*cosh(b*x+a)^2)-4*d^3/b^3*a^3*c*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a
)+6*d^2/b^2*a^2*c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)-4*d/b*a*c^3
*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+6*d^2/b^2*c^2*(1/2*(b*x+a)^2*c
osh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x
+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-4*d^4/b^4*a^3*(1/2*(b*x+a)*cosh(b*x+a)*sinh(
b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+4*d/b*c^3*(1/2*(b*x+a)*cosh(b*x+a)*
sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+4*d^3/b^3*c*(1/2*(b*x+a)^3*cos
h(b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*
cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+6*d^4/b^4*a^2*(1/2
*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)*cosh(b*x+a)^2+
1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-4*d^4/b^4*a*(1/2*(b*x+a)^3*cosh(
b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^4-3/4*(b*x+a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*co
sh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*cosh(b*x+a)^2)+d^4/b^4*(1/2*(b*x+a)
^4*cosh(b*x+a)*sinh(b*x+a)+1/10*(b*x+a)^5-(b*x+a)^3*cosh(b*x+a)^2+3/2*(b*x+
a)^2*cosh(b*x+a)*sinh(b*x+a)+1/2*(b*x+a)^3-3/2*(b*x+a)*cosh(b*x+a)^2+3/4*co
sh(b*x+a)*sinh(b*x+a)+3/4*b*x+3/4*a)+d^4/b^4*a^4*(1/2*cosh(b*x+a)*sinh(b*x+
a)+1/2*b*x+1/2*a)+c^4*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(148) = 296$.
time = 0.28, size = 382, normalized size = 2.36

$$\frac{1}{4}\left(4x^2 + \frac{(2bxe^{(2a)} - e^{(2a)})e^{(2bx)}}{b^2} - \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^3d + \frac{1}{8}\left(8x^3 + \frac{3(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)})e^{(2bx)}}{b^3} - \frac{3(2b^2x^2 + 2bx + 1)e^{(-2bx-2a)}}{b^3}\right)c^2d^2 + \frac{1}{8}\left(4x^4 + \frac{(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bxe^{(2a)} - 3e^{(2a)})e^{(2bx)}}{b^4} - \frac{(4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx-2a)}}{b^4}\right)cd^3 + \frac{1}{80}\left(8x^5 + \frac{5(2b^4x^4e^{(2a)} - 4b^3x^3e^{(2a)} + 6b^2x^2e^{(2a)} - 6bxe^{(2a)} + 3e^{(2a)})e^{(2bx)}}{b^5} - \frac{5(2b^4x^4 + 4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx-2a)}}{b^5}\right)d^4 + \frac{1}{8}c^4\left(4x + \frac{e^{(2bx+2a)}}{b} - \frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b*
x - 2*a)/b^2)*c^3*d + 1/8*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e
^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c^2
*d^2 + 1/8*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
- 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
- 2*a)/b^4)*c*d^3 + 1/80*(8*x^5 + 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e^(2*a)
+ 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 - 5*(2*b^4*x
^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4 + 1/8*c^4
*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 312 vs. $2(148) = 296$.
time = 0.36, size = 312, normalized size = 1.93

$$\frac{2b^5d^4x^5 + 10b^5cd^3x^4 + 20b^5c^2d^2x^3 + 20b^5c^3dx^2 + 10b^5c^4x - 5(2b^3d^4x^3 + 6b^3cd^3x^2 + 2b^3c^3d + 3bcd^3 + 3(2b^3c^2d^2 + bd^4)x)\cosh(bx+a)^2 + 5(2b^4d^4x^4 + 8b^4cd^3x^3 + 2b^4c^4 + 6b^2c^2d^2 + 3d^4 + 6(2b^4c^2d^2 + b^2d^4)x^2 + 4(2b^4c^3d + 3b^2cd^3)x)\cosh(bx+a)\sinh(bx+a) - 5(2b^3d^4x^3 + 6b^3cd^3x^2 + 2b^3c^3d + 3bcd^3 + 3(2b^3c^2d^2 + bd^4)x)\sinh(bx+a)^2}{20b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d*
x^2 + 10*b^5*c^4*x - 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b
*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 + 5*(2*b^4*d^4*x^4 +
8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2 + b^
2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) -
 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*b^3*c^
2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.
time = 0.58, size = 660, normalized size = 4.07

$$\begin{cases} -\frac{c^4 x \sinh^2(a+bx)}{2} + \frac{c^4 x \cosh^2(a+bx)}{2} - c^3 d x^2 \sinh^2(a+bx) + c^3 d x^2 \cosh^2(a+bx) - c^2 d^2 x^3 \sinh^2(a+bx) + c^2 d^2 x^3 \cosh^2(a+bx) - \frac{c d^3 x^4 \sinh^2(a+bx)}{2} + \frac{c d^3 x^4 \cosh^2(a+bx)}{2} - \frac{d^4 x^5 \sinh^2(a+bx)}{10} + \frac{d^4 x^5 \cosh^2(a+bx)}{10} + \frac{c^4 \sinh(a+bx)\cosh(a+bx)}{2b} + \frac{2c^3 d x \sinh(a+bx)\cosh(a+bx)}{b} + \frac{3c^2 d^2 x^2 \sinh(a+bx)\cosh(a+bx)}{b} + \frac{2 c d^3 x^3 \sinh(a+bx)\cosh(a+bx)}{b} + \frac{d^4 x^4 \sinh(a+bx)\cosh(a+bx)}{2b} - \frac{c^3 d \cosh^2(a+bx)}{b^2} - \frac{3c^2 d^2 x \sinh^2(a+bx)}{2b^2} - \frac{3c^2 d^2 x \cosh^2(a+bx)}{2b^2} - \frac{3 c d^3 x^2 \sinh^2(a+bx)}{2b^2} - \frac{3 c d^3 x^2 \cosh^2(a+bx)}{2b^2} - \frac{d^4 x^3 \sinh^2(a+bx)}{2b^2} - \frac{d^4 x^3 \cosh^2(a+bx)}{2b^2} + \frac{3c^2 d^2 \sinh(a+bx)\cosh(a+bx)}{2b^3} + \frac{3 c d^3 x \sinh(a+bx)\cosh(a+bx)}{b^3} + \frac{3 d^4 x^2 \sinh(a+bx)\cosh(a+bx)}{2b^3} - \frac{3 c d^3 \cosh^2(a+bx)}{2b^4} - \frac{3 d^4 x \sinh^2(a+bx)}{4b^4} - \frac{3 d^4 x \cosh^2(a+bx)}{4b^4} + \frac{3 d^4 \sinh(a+bx)\cosh(a+bx)}{4b^5} & \text{for } b \neq 0 \\ \left(c^4 x + 2c^3 d x^2 + 2c^2 d^2 x^3 + c d^3 x^4 + \frac{d^4 x^5}{5}\right)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**4*x*sinh(a + b*x)**2/2 + c**4*x*cosh(a + b*x)**2/2 - c**3*d*
x**2*sinh(a + b*x)**2 + c**3*d*x**2*cosh(a + b*x)**2 - c**2*d**2*x**3*sinh(
a + b*x)**2 + c**2*d**2*x**3*cosh(a + b*x)**2 - c*d**3*x**4*sinh(a + b*x)**
2/2 + c*d**3*x**4*cosh(a + b*x)**2/2 - d**4*x**5*sinh(a + b*x)**2/10 + d**4
*x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2*c**3
*d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*cosh(a
+ b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*sinh(a +
 b*x)*cosh(a + b*x)/(2*b) - c**3*d*cosh(a + b*x)**2/b**2 - 3*c**2*d**2*x*si
nh(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2) - 3*c*d**
3*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2/(2*b**2)
- d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)**2/(2*b**2)
 + 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*x*sinh(a + b
*x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) -
 3*c*d**3*cosh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)**2/(4*b**4) -
3*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*cosh(a + b*x)/(4*
b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**3*x**4
 + d**4*x**5/5)*cosh(a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(148) = 296$.
time = 0.42, size = 372, normalized size = 2.30

$$\frac{1}{10}\,d^4x^5 + \frac{1}{2}\,cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}\,c^4x + \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + 2\,b^4c^4 - 12\,b^3c^2d^2x + 6\,b^2d^4x^2 - 4\,b^3c^3d + 12\,b^2cd^3x + 6\,b^2c^2d^2 - 6\,bd^4x - 6\,bcd^3 + 3\,d^4)e^{(2\,bx+2\,a)}}{16\,b^5} - \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 8\,b^4c^3dx + 12\,b^3cd^3x^2 + 2\,b^4c^4 + 12\,b^3c^2d^2x + 6\,b^2d^4x^2 + 4\,b^3c^3d + 12\,b^2cd^3x + 6\,b^2c^2d^2 + 6\,bd^4x + 6\,bcd^3 + 3\,d^4)e^{(-2\,bx-2\,a)}}{16\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/16*(
2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*b^
4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2
 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3 + 3
*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*
c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^4*c^4
+ 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c
^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

Mupad [B]
time = 1.50, size = 332, normalized size = 2.05

$$\frac{c^4\,x}{2}+\frac{d^4\,x^5}{10}+c^3\,d\,x^2+\frac{c\,d^3\,x^4}{2}+\frac{c^4\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{3\,d^4\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{8\,b^5}+c^2\,d^2\,x^3-\frac{c^3\,d\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{3\,c\,d^3\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{4\,b^4}-\frac{3\,d^4\,x\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{4\,b^4}+\frac{3\,c^2\,d^2\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{4\,b^3}-\frac{d^4\,x^3\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{2\,b^2}+\frac{d^4\,x^4\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{4\,b}+\frac{3\,d^4\,x^2\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{4\,b^3}+\frac{3\,c^2\,d^2\,x^2\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{2\,b}+\frac{c^3\,d\,x\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{b}+\frac{3\,c\,d^3\,x\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{2\,b^3}-\frac{3\,c^2\,d^2\,x\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{2\,b^2}-\frac{3\,c\,d^3\,x^2\,\mathrm{cosh}\left(2\,a+2\,b\,x\right)}{2\,b^2}+\frac{c\,d^3\,x^3\,\mathrm{sinh}\left(2\,a+2\,b\,x\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^4,x)
```

```
[Out] (c^4*x)/2 + (d^4*x^5)/10 + c^3*d*x^2 + (c*d^3*x^4)/2 + (c^4*sinh(2*a + 2*b*
x))/(4*b) + (3*d^4*sinh(2*a + 2*b*x))/(8*b^5) + c^2*d^2*x^3 - (c^3*d*cosh(2
*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*cosh(
2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^3*co
sh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*sinh(2*a + 2*b*x))/(4*b) + (3*d^4*x^2*s
inh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*sinh(2*a + 2*b*x))/(2*b) + (c^3*
d*x*sinh(2*a + 2*b*x))/b + (3*c*d^3*x*sinh(2*a + 2*b*x))/(2*b^3) - (3*c^2*d
^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(2*b^2) +
 (c*d^3*x^3*sinh(2*a + 2*b*x))/b
```

3.9 $\int (c+dx)^3 \cosh^2(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{3cd^2x}{4b^2}+\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}-\frac{3d^3\cosh^2(a+bx)}{8b^4}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{3d^2(c+dx)\cosh(a+bx)\sinh(a}{4b^3}$$

```
[Out] 3/4*c*d^2*x/b^2+3/8*d^3*x^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cosh(b*x+a)^2/b^4-3
/4*d*(d*x+c)^2*cosh(b*x+a)^2/b^2+3/4*d^2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b^
3+1/2*(d*x+c)^3*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.06, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3392, 32, 3391}

$$-\frac{3d^3\cosh^2(a+bx)}{8b^4}+\frac{3d^2(c+dx)\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{3cd^2x}{4b^2}+\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x]^2,x]
```

```
[Out] (3*c*d^2*x)/(4*b^2) + (3*d^3*x^2)/(8*b^2) + (c + d*x)^4/(8*d) - (3*d^3*Cosh
[a + b*x]^2)/(8*b^4) - (3*d*(c + d*x)^2*Cosh[a + b*x]^2)/(4*b^2) + (3*d^2*(
c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^3*Cosh[a + b*x]*
Sinh[a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
```

```
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int (c+dx)^3\cosh^2(a+bx)\,dx = -\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c$$
$$=\frac{(c+dx)^4}{8d}-\frac{3d^3\cosh^2(a+bx)}{8b^4}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{3d^2(c+dx)\,\mathrm{co}}{}$$
$$=\frac{3cd^2x}{4b^2}+\frac{3d^3x^2}{8b^2}+\frac{(c+dx)^4}{8d}-\frac{3d^3\cosh^2(a+bx)}{8b^4}-\frac{3d(c+dx)^2\cosh^2(a+bx}{4b^2}$$

Mathematica [A]
time = 0.43, size = 104, normalized size = 0.78

$$\frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+2b(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\sinh(2(a+bx))}{16b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x]^2,x]
```

```
[Out] (2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
+ d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Sin
h[2*(a + b*x)])/(16*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(120) = 240$.
time = 0.77, size = 523, normalized size = 3.90 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d^3/b^3*(1/2*(b*x+a)^3*cosh(b*x+a)*sinh(b*x+a)+1/8*(b*x+a)^4-3/4*(b*x+
a)^2*cosh(b*x+a)^2+3/4*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+3/8*(b*x+a)^2-3/8*co
sh(b*x+a)^2)-3*d^3/b^3*a*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)
^3-1/2*(b*x+a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3*d
^2/b^2*c*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+a)*c
osh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)+3*d^3/b^3*a^2*(1/2*
(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)-6*d^2/b^2*
a*c*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+3
*d/b*c^2*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)
^2)-d^3/b^3*a^3*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+3*d^2/b^2*a^2*c
*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)-3*d/b*a*c^2*(1/2*cosh(b*x+a)*s
inh(b*x+a)+1/2*b*x+1/2*a)+c^3*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(120) = 240$.
time = 0.30, size = 263, normalized size = 1.96

$$\frac{3}{16}\left(4x^2 + \frac{\left(2bxe^{(2a)} - e^{(2a)}\right)e^{(2bx)}}{b^2} - \frac{(2bx+1)e^{(-2bx-2a)}}{b^2}\right)c^2d + \frac{1}{16}\left(8x^3 + \frac{3\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{b^3} - \frac{3\left(2b^2x^2 + 2bx + 1\right)e^{(-2bx-2a)}}{b^3}\right)cd^2 + \frac{1}{32}\left(4x^4 + \frac{\left(4b^3x^3e^{(2a)} - 6b^2x^2e^{(2a)} + 6bxe^{(2a)} - 3e^{(2a)}\right)e^{(2bx)}}{b^4} - \frac{\left(4b^3x^3 + 6b^2x^2 + 6bx + 3\right)e^{(-2bx-2a)}}{b^4}\right)d^3 + \frac{1}{8}c^3\left(4x + \frac{e^{(2bx+2a)}}{b} - \frac{e^{(-2bx-2a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c^2*d + 1/16*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
 e^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*c
*d^2 + 1/32*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2*a)
 - 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x
 - 2*a)/b^4)*d^3 + 1/8*c^3*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.42, size = 209, normalized size = 1.56

$$\frac{2b^4d^3x^4 + 8b^4cd^2x^3 + 12b^4c^2dx^2 + 8b^4c^3x - 3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3\right)\cosh\left(bx+a\right)^2 + 4\left(2b^3d^3x^3 + 6b^3cd^2x^2 + 2b^3c^3 + 3bcd^2 + 3\left(2b^3c^2d + bd^3\right)x\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right) - 3\left(2b^2d^3x^2 + 4b^2cd^2x + 2b^2c^2d + d^3\right)\sinh\left(bx+a\right)^2}{16b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x - 3*
(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 + 4*(2*
b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d + b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a) - 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*
b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(131) = 262$.
time = 0.40, size = 456, normalized size = 3.40

$$\begin{cases} -\frac{c^3x\sinh^2(a+bx)}{2} + \frac{c^3x\cosh^2(a+bx)}{2} - \frac{3c^2dx^2\sinh^2(a+bx)}{4} + \frac{3c^2dx^2\cosh^2(a+bx)}{4} - \frac{cd^2x^3\sinh^2(a+bx)}{2} + \frac{cd^2x^3\cosh^2(a+bx)}{2} - \frac{d^3x^4\sinh^2(a+bx)}{8} + \frac{d^3x^4\cosh^2(a+bx)}{8} + \frac{c^3\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{3c^2dx\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{3cd^2x^2\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{d^3x^3\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{3c^2d\cosh^2(a+bx)}{4b^2} - \frac{3cd^2x\sinh^2(a+bx)}{4b^2} - \frac{3cd^2x\cosh^2(a+bx)}{4b^2} - \frac{3d^3x^2\sinh^2(a+bx)}{8b^2} - \frac{3d^3x^2\cosh^2(a+bx)}{8b^2} + \frac{3cd^2\sinh(a+bx)\cosh(a+bx)}{4b^3} + \frac{3d^3x\sinh(a+bx)\cosh(a+bx)}{4b^3} - \frac{3d^3\cosh^2(a+bx)}{8b^4} & \text{for}\ b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**3*x*sinh(a + b*x)**2/2 + c**3*x*cosh(a + b*x)**2/2 - 3*c**2*
d*x**2*sinh(a + b*x)**2/4 + 3*c**2*d*x**2*cosh(a + b*x)**2/4 - c*d**2*x**3*
sinh(a + b*x)**2/2 + c*d**2*x**3*cosh(a + b*x)**2/2 - d**3*x**4*sinh(a + b*
x)**2/8 + d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*x)/(
2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*sinh(a
+ b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2*b) -
```

```
3*c**2*d*cosh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*b**2) -
 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**2/(8*b**
2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x)*cosh(a
+ b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) - 3*d**3*co
sh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**
3 + d**3*x**4/4)*cosh(a)**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(120) = 240$.
time = 0.42, size = 243, normalized size = 1.81

$$\frac{1}{8}\,d^3x^4 + \frac{1}{2}\,cd^2x^3 + \frac{3}{4}\,c^2dx^2 + \frac{1}{2}\,c^3x + \frac{(4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx - 6\,b^2d^3x^2 + 4\,b^3c^3 - 12\,b^2cd^2x - 6\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 3\,d^3)e^{(2\,bx+2\,a)}}{32\,b^4} - \frac{(4\,b^3d^3x^3 + 12\,b^3cd^2x^2 + 12\,b^3c^2dx + 6\,b^2d^3x^2 + 4\,b^3c^3 + 12\,b^2cd^2x + 6\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 3\,d^3)e^{(-2\,bx-2\,a)}}{32\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 1/32*(4*b^3*d^3*x
^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b^2
*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)/b^4
 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*x^2
+ 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3*d^3)
*e^(-2*b*x - 2*a)/b^4
```

Mupad [B]
time = 1.18, size = 229, normalized size = 1.71

$$\frac{4\,b^4\,c^3\,x - \frac{3\,d^3\,\cosh(2\,a+2\,b\,x)}{2} + 2\,b^3\,c^3\,\sinh(2\,a + 2\,b\,x) + b^4\,d^3\,x^4 - 3\,b^2\,c^2\,d\,\cosh(2\,a + 2\,b\,x) + 6\,b^4\,c^2\,d\,x^2 + 4\,b^4\,c\,d^2\,x^3 - 3\,b^2\,d^3\,x^2\,\cosh(2\,a + 2\,b\,x) + 2\,b^3\,d^3\,x^3\,\sinh(2\,a + 2\,b\,x) + 3\,b\,c\,d^2\,\sinh(2\,a + 2\,b\,x) + 3\,b\,d^3\,x\,\sinh(2\,a + 2\,b\,x) - 6\,b^2\,c\,d^2\,x\,\cosh(2\,a + 2\,b\,x) + 6\,b^3\,c^2\,d\,x\,\sinh(2\,a + 2\,b\,x) + 6\,b^3\,c\,d^2\,x^2\,\sinh(2\,a + 2\,b\,x)}{8\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^3,x)
```

```
[Out] (4*b^4*c^3*x - (3*d^3*cosh(2*a + 2*b*x))/2 + 2*b^3*c^3*sinh(2*a + 2*b*x) +
b^4*d^3*x^4 - 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d^2
*x^3 - 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) + 2*b^3*d^3*x^3*sinh(2*a + 2*b*x) +
3*b*c*d^2*sinh(2*a + 2*b*x) + 3*b*d^3*x*sinh(2*a + 2*b*x) - 6*b^2*c*d^2*x*c
osh(2*a + 2*b*x) + 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*sinh(2
*a + 2*b*x))/(8*b^4)
```

3.10 $\int (c+dx)^2 \cosh^2(a+bx)\,dx$

Optimal. Leaf size=95

$$\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3}+\frac{(c+dx)^2\cosh(a+bx)\sinh(a+b}{2b}$$

```
[Out] 1/4*d^2*x/b^2+1/6*(d*x+c)^3/d-1/2*d*(d*x+c)*cosh(b*x+a)^2/b^2+1/4*d^2*cosh(
b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b
```

Rubi [A]
time = 0.04, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 32, 2715, 8}

$$\frac{d^2\sinh(a+bx)\cosh(a+bx)}{4b^3}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x]^2,x]
```

```
[Out] (d^2*x)/(4*b^2) + (c + d*x)^3/(6*d) - (d*(c + d*x)*Cosh[a + b*x]^2)/(2*b^2)
 + (d^2*Cosh[a + b*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^2*Cosh[a + b*x]*S
inh[a + b*x])/(2*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cosh^2(a+bx)\,dx &= -\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c+\\ &= \frac{(c+dx)^3}{6d}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3}+\frac{(c+\\ &= \frac{d^2x}{4b^2}+\frac{(c+dx)^3}{6d}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3}-\end{aligned}$$

Mathematica [A]
time = 0.29, size = 75, normalized size = 0.79

$$\frac{4b^3x(3c^2+3cdx+d^2x^2)-6bd(c+dx)\cosh(2(a+bx))+3(d^2+2b^2(c+dx)^2)\sinh(2(a+bx))}{24b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x]^2,x]
[Out] (4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] +
3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 261 vs. $2(85) = 170$.
time = 0.78, size = 262, normalized size = 2.76

method	result
risch	$\frac{d^2x^3}{6}+\frac{dcx^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}+\frac{(2b^2d^2x^2+4b^2cdx+2b^2c^2-2bd^2x-2bcd+d^2)e^{2bx+2a}}{16b^3}-\frac{(2b^2d^2x^2+4b^2cdx+2b^2c^2+2}{16b}$
derivativedivides	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^3}{6}-\frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$
default	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^3}{6}-\frac{(bx+a)\left(\cosh^2(bx+a)\right)}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx+}{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
[Out] 1/b*(d^2/b^2*(1/2*(b*x+a)^2*cosh(b*x+a)*sinh(b*x+a)+1/6*(b*x+a)^3-1/2*(b*x+
a)*cosh(b*x+a)^2+1/4*cosh(b*x+a)*sinh(b*x+a)+1/4*b*x+1/4*a)-2*d^2/b^2*a*(1/
```

```
2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+2*d/b*c*
(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)^2)+d^2/b
^2*a^2*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)-2*d/b*a*c*(1/2*cosh(b*x+
a)*sinh(b*x+a)+1/2*b*x+1/2*a)+c^2*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*
a))
```

Maxima [A]
time = 0.27, size = 165, normalized size = 1.74

$$\frac{1}{8}\left(4\,x^2+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)cd+\frac{1}{48}\left(8\,x^3+\frac{3\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{b^3}-\frac{3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)d^2+\frac{1}{8}\,c^2\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b*
x - 2*a)/b^2)*c*d + 1/48*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^
(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*d^2
+ 1/8*c^2*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.36, size = 123, normalized size = 1.29

$$\frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x-3\,(bd^2x+bcd)\cosh\left(bx+a\right)^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-3\,(bd^2x+bcd)\sinh\left(bx+a\right)^2}{12\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*(b*d^2*x + b*c*d)*cos
h(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*x +
 a)*sinh(b*x + a) - 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(85) = 170$.
time = 0.26, size = 264, normalized size = 2.78

$$\begin{cases} -\frac{c^2x\sinh^2\left(a+bx\right)}{2}+\frac{c^2x\cosh^2\left(a+bx\right)}{2}-\frac{cdx^2\sinh^2\left(a+bx\right)}{2}+\frac{cdx^2\cosh^2\left(a+bx\right)}{2}-\frac{d^2x^3\sinh^2\left(a+bx\right)}{6}+\frac{d^2x^3\cosh^2\left(a+bx\right)}{6}+\frac{c^2\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}+\frac{cdx\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{b}+\frac{d^2x^2\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}-\frac{cd\cosh^2\left(a+bx\right)}{2b^2}-\frac{d^2x\sinh^2\left(a+bx\right)}{4b^2}-\frac{d^2x\cosh^2\left(a+bx\right)}{4b^2}+\frac{d^2\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{4b^3} & \text{for}\: b\neq 0\\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cosh^2\left(a\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c**2*x*sinh(a + b*x)**2/2 + c**2*x*cosh(a + b*x)**2/2 - c*d*x**
2*sinh(a + b*x)**2/2 + c*d*x**2*cosh(a + b*x)**2/2 - d**2*x**3*sinh(a + b*x
)**2/6 + d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cosh(a
+ b*x)/(2*b) - c*d*cosh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2/(4*b
```

```
**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a + b*x)/
(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)**2, True))
```

Giac [A]
time = 0.41, size = 136, normalized size = 1.43

$$\frac{1}{6}d^2x^3 + \frac{1}{2}cdx^2 + \frac{1}{2}c^2x + \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 - 2bd^2x - 2bcd + d^2)e^{(2bx+2a)}}{16b^3} - \frac{(2b^2d^2x^2 + 4b^2cdx + 2b^2c^2 + 2bd^2x + 2bcd + d^2)e^{(-2bx-2a)}}{16b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x +
 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*d
^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x - 2
*a)/b^3
```

Mupad [B]
time = 0.98, size = 127, normalized size = 1.34

$$\frac{c^2\,x}{2} + \frac{d^2\,x^3}{6} + \frac{c^2\,\mathrm{sinh}(2\,a+2\,b\,x)}{4\,b} + \frac{d^2\,\mathrm{sinh}(2\,a+2\,b\,x)}{8\,b^3} + \frac{c\,d\,x^2}{2} - \frac{d^2\,x\,\mathrm{cosh}(2\,a+2\,b\,x)}{4\,b^2} + \frac{d^2\,x^2\,\mathrm{sinh}(2\,a+2\,b\,x)}{4\,b} - \frac{c\,d\,\mathrm{cosh}(2\,a+2\,b\,x)}{4\,b^2} + \frac{c\,d\,x\,\mathrm{sinh}(2\,a+2\,b\,x)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^2,x)
```

```
[Out] (c^2*x)/2 + (d^2*x^3)/6 + (c^2*sinh(2*a + 2*b*x))/(4*b) + (d^2*sinh(2*a + 2
*b*x))/(8*b^3) + (c*d*x^2)/2 - (d^2*x*cosh(2*a + 2*b*x))/(4*b^2) + (d^2*x^2
*sinh(2*a + 2*b*x))/(4*b) - (c*d*cosh(2*a + 2*b*x))/(4*b^2) + (c*d*x*sinh(2
*a + 2*b*x))/(2*b)
```

3.11 $\int (c+dx)\cosh^2(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{cx}{2}+\frac{dx^2}{4}-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}$$

```
[Out] 1/2*c*x+1/4*d*x^2-1/4*d*cosh(b*x+a)^2/b^2+1/2*(d*x+c)*cosh(b*x+a)*sinh(b*x+
a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.071$, Rules used = {3391}

$$-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{cx}{2}+\frac{dx^2}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (c*x)/2 + (d*x^2)/4 - (d*Cosh[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cosh[a + b*x
]*Sinh[a + b*x])/(2*b)
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\cosh^2(a+bx)\,dx &= -\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int (c+dx)\,dx\\ &= \frac{cx}{2}+\frac{dx^2}{4}-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 51, normalized size = 0.93

$$\frac{-d\cosh(2(a+bx))+2b(2ac+bx(2c+dx)+(c+dx)\sinh(2(a+bx)))}{8b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x]^2,x]
```

```
[Out] (-(d*Cosh[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sinh[2*(
a + b*x)]))/(8*b^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.
time = 0.83, size = 103, normalized size = 1.87

method	result
risch	$\frac{d\,x^2}{4}+\frac{cx}{2}+\frac{(2bdx+2bc-d)\mathrm{e}^{2bx+2a}}{16b^2}-\frac{(2bdx+2bc+d)\mathrm{e}^{-2bx-2a}}{16b^2}$
derivativedivides	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^2}{4}-\frac{\left(\cosh^2(bx+a)\right)}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}\right.}{b}$
default	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^2}{4}-\frac{\left(\cosh^2(bx+a)\right)}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}\right.}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*(1/2*(b*x+a)*cosh(b*x+a)*sinh(b*x+a)+1/4*(b*x+a)^2-1/4*cosh(b*x+a)
^2)-d/b*a*(1/2*cosh(b*x+a)*sinh(b*x+a)+1/2*b*x+1/2*a)+c*(1/2*cosh(b*x+a)*si
nh(b*x+a)+1/2*b*x+1/2*a))
```

Maxima [A]
time = 0.26, size = 88, normalized size = 1.60

$$\frac{1}{16}\left(4\,x^2+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)d+\frac{1}{8}\,c\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*d + 1/8*c*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Fricas [A]
time = 0.39, size = 66, normalized size = 1.20

$$\frac{2\,b^2dx^2+4\,b^2cx-d\cosh\left(bx+a\right)^2+4\left(bdx+bc\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-d\sinh\left(bx+a\right)^2}{8\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*b^2*d*x^2 + 4*b^2*c*x - d*cosh(b*x + a)^2 + 4*(b*d*x + b*c)*cosh(b*x
 + a)*sinh(b*x + a) - d*sinh(b*x + a)^2)/b^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(49) = 98$.
time = 0.14, size = 126, normalized size = 2.29

$$\begin{cases} -\frac{cx\sinh^2(a+bx)}{2} + \frac{cx\cosh^2(a+bx)}{2} - \frac{dx^2\sinh^2(a+bx)}{4} + \frac{dx^2\cosh^2(a+bx)}{4} + \frac{c\sinh(a+bx)\cosh(a+bx)}{2b} + \frac{dx\sinh(a+bx)\cosh(a+bx)}{2b} - \frac{d\cosh^2(a+bx)}{4b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\cosh^2(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)**2,x)
```

```
[Out] Piecewise((-c*x*sinh(a + b*x)**2/2 + c*x*cosh(a + b*x)**2/2 - d*x**2*sinh(a
 + b*x)**2/4 + d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/(2
*b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*cosh(a + b*x)**2/(4*b**2),
Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**2, True))
```

Giac [A]
time = 0.40, size = 63, normalized size = 1.15

$$\frac{1}{4}\,dx^2 + \frac{1}{2}\,cx + \frac{(2\,bdx + 2\,bc - d)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bdx + 2\,bc + d)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*x^2 + 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/16
*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

Mupad [B]
time = 0.10, size = 58, normalized size = 1.05

$$\frac{b^2\,d\,x^2 - \frac{d\cosh(2\,a+2\,b\,x)}{2} + b\,c\sinh(2\,a + 2\,b\,x) + 2\,b^2\,c\,x + b\,d\,x\sinh(2\,a + 2\,b\,x)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x),x)
```

```
[Out] (b^2*d*x^2 - (d*cosh(2*a + 2*b*x))/2 + b*c*sinh(2*a + 2*b*x) + 2*b^2*c*x +
b*d*x*sinh(2*a + 2*b*x))/(4*b^2)
```

3.12 $\int \frac{\cosh^2(a+bx)}{c+dx} \, dx$

Optimal. Leaf size=78

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

```
[Out] 1/2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d+1/2*ln(d*x+c)/d+1/2*Shi(2*b*c/d+
2*b*x)*sinh(2*a-2*b*c/d)/d
```

Rubi [A]
time = 0.12, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3379, 3382}

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x),x]
```

```
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x
]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)}{c+dx}\,dx &= \int \left(\frac{1}{2(c+dx)}+\frac{\cosh(2a+2bx)}{2(c+dx)}\right)dx\\ &= \frac{\log(c+dx)}{2d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{c+dx}\,dx\\ &= \frac{\log(c+dx)}{2d}+\frac{1}{2}\cosh\left(2a-\frac{2bc}{d}\right)\int \frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx+\frac{1}{2}\sinh\left(2a-\frac{2bc}{d}\right)\int \frac{\mathrm{si}}{}\\ &= \frac{\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.82

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)+\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x),x]
[Out] (Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] + Sin
h[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Maple [A]
time = 3.58, size = 97, normalized size = 1.24

method	result	size
risch	$\frac{\ln(dx+c)}{2d}-\frac{\mathrm{e}^{-\frac{2(ad-bc)}{d}}\operatorname{expIntegral}\left(1,2bx+2a-\frac{2(ad-bc)}{d}\right)}{4d}-\frac{\mathrm{e}^{\frac{2ad-2bc}{d}}\operatorname{expIntegral}\left(1,-2bx-2a-\frac{2(-ad+bc)}{d}\right)}{4d}$	97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4
/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.30, size = 72, normalized size = 0.92

$$-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_1\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_1\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d}+\frac{\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a -
 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d + 1/2*log(d*x + c)/d
```

Fricas [A]
time = 0.37, size = 104, normalized size = 1.33

$$\frac{\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)+2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d
) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d)
 + 2*log(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.42, size = 68, normalized size = 0.87

$$\frac{\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+2\,\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-2
*a + 2*b*c/d) + 2*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(a+b\,x\right)}^2}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x),x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x), x)
```

3.13 $\int \frac{\cosh^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=81

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}+\frac{b\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

```
[Out] -cosh(b*x+a)^2/d/(d*x+c)+b*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^2+b*Chi(2
*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^2
```

Rubi [A]
time = 0.11, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3394, 12, 3384, 3379, 3382}

$$\frac{b\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^2}+\frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}-\frac{\cosh^2(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] -(Cosh[a + b*x]^2/(d*(c + d*x))) + (b*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[
2*a - (2*b*c)/d])/d^2 + (b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2
*b*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{(2ib)\int -\frac{i\sinh(2a+2bx)}{2(c+dx)}\,dx}{d} \\
&= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{b\int \frac{\sinh(2a+2bx)}{c+dx}\,dx}{d} \\
&= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{\left(b\cosh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,dx}{d} + \frac{\left(b\sinh\left(2a-\frac{2bc}{d}\right)\right)\int \frac{\cosh\left(\frac{2b}{}\right.}{c+}}{d} \\
&= -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{b\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}
\end{aligned}
$$

Mathematica [A]
time = 0.30, size = 75, normalized size = 0.93

$$
\frac{-\frac{d\cosh^2(a+bx)}{c+dx} + b\operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right) + b\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right)}{d^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] (-((d*Cosh[a + b*x]^2)/(c + d*x)) + b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[
2*a - (2*b*c)/d] + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])
/d^2
```

Maple [A]
time = 3.43, size = 152, normalized size = 1.88

method	result

risch	$-\frac{1}{2(dx+c)d}-\frac{b\,\mathrm{e}^{-2bx-2a}}{4d(bdx+bc)}+\frac{b\,\mathrm{e}^{-\frac{2(ad-bc)}{d}}\,\mathrm{expIntegral}\left(1,2bx+2a-\frac{2(ad-bc)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{4d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{2ad-2bc}{d}}\,\mathrm{expIntegral}\left(1,-\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(d*x+c)/d-1/4*b*exp(-2*b*x-2*a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-2*(a*d-b*
c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-1/
2*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.30, size = 88, normalized size = 1.09

$$-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d}-\frac{1}{2\,(d^2x+cd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) - 1
/2/(d^2*x + c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(81) = 162.
time = 0.37, size = 164, normalized size = 2.02

$$-\frac{d\cosh\left(bx+a\right)^2+d\sinh\left(bx+a\right)^2-\left((bdx+bc)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-\left((bdx+bc)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+(bdx+bc)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)+d}{2\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) - ((
b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*
sinh(-2*(b*c - a*d)/d) + d)/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 574 vs. 2(81) = 162.
time = 0.44, size = 574, normalized size = 7.09

$$-\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}+2\,b^3c\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}-2\,ab^2d\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}-2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}-2\,b^3c\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}+2\,ab^2d\mathrm{Ei}\left(\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{2\,(bc-ad)}{d}\right)}+b^2de^{\left(\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+b^2de^{\left(-\frac{2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+2\,b^2d\right)d^2}{4\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-2*((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) + 2
*b^3*c*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
*e^(2*(b*c - a*d)/d) - 2*a*b^2*d*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) - 2*(d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c))*b^2*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) - 2*b^3*c*Ei(2*((d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) + 2*a*b^2*d
*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*
(b*c - a*d)/d) + b^2*d*e^(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d
) + b^2*d*e^(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 2*b^2*d)
*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)
*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^2,x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^2, x)
```

3.14 $\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\, dx$

Optimal. Leaf size=112

$$-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2bc}{d}\right.}{d^3}$$

```
[Out] b^2*Chi(2*b*c/d+2*b*x)*cosh(2*a-2*b*c/d)/d^3-1/2*cosh(b*x+a)^2/d/(d*x+c)^2+
b^2*Shi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^3-b*cosh(b*x+a)*sinh(b*x+a)/d^2/
(d*x+c)
```

Rubi [A]
time = 0.13, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3395, 31, 3393, 3384, 3379, 3382}

$$\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}+\frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] -1/2*Cosh[a + b*x]^2/(d*(c + d*x)^2) + (b^2*Cosh[2*a - (2*b*c)/d]*CoshInteg
ral[(2*b*c)/d + 2*b*x])/d^3 - (b*Cosh[a + b*x]*Sinh[a + b*x])/(d^2*(c + d*x
)) + (b^2*Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
```

```
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx &= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}-\frac{b^2\int\frac{1}{c+dx}\,dx}{d^2}+\frac{(2b^2)\int\frac{\cosh^2(a+bx)}{c+dx}\,dx}{d^2}\\
&= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\log(c+dx)}{d^3}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}+\frac{(2b^2)\int\left(\frac{1}{2(c+dx)}+\dots\right)}{d^2}\\
&= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}+\frac{b^2\int\frac{\cosh(2a+2bx)}{c+dx}\,dx}{d^2}\\
&= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}+\frac{\left(b^2\cosh\left(2a-\frac{2bc}{d}\right)\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}\,d\dots}{d^2}\\
&= -\frac{\cosh^2(a+bx)}{2d(c+dx)^2}+\frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3}-\frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)}+\dots\end{aligned}$$

Mathematica [A]
time = 0.97, size = 102, normalized size = 0.91

$$\frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\operatorname{Chi}\left(\frac{2b(c+dx)}{d}\right)-\frac{d(d\cosh^2(a+bx)+b(c+dx)\sinh(2(a+bx)))}{(c+dx)^2}+2b^2\sinh\left(2a-\frac{2bc}{d}\right)\operatorname{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

```
[Out] (2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Cosh[a
 + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a -
(2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.
time = 3.56, size = 299, normalized size = 2.67

method	result
risch	$-\frac{1}{4(dx+c)^2 d}+\frac{b^3 e^{-2bx-2a} x}{4d(b^2d^2x^2+2b^2cdx+b^2c^2)}+\frac{b^3 e^{-2bx-2a} c}{4d^2(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 e^{-2bx-2a}}{8d(b^2d^2x^2+2b^2cdx+b^2c^2)}-\frac{b^2 e^{-\frac{2(ad-bc)}{d}} \text{expInt}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/(d*x+c)^2/d+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2
)*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b^2
*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*(a*
d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b
*x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*c)/d)
*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.31, size = 99, normalized size = 0.88

$$-\frac{1}{4\left(d^3x^2+2\,cd^2x+c^2d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(
3, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(
3, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.
time = 0.45, size = 278, normalized size = 2.48

$$-\frac{d^2\cosh(bx+a)^2+d^2\sinh(bx+a)^2+4\,(bd^2x+bcd)\cosh(bx+a)\sinh(bx+a)+d^2-2\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\text{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+(b^2d^2x^2+2\,b^2cdx+b^2c^2)\text{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-2\left((b^2d^2x^2+2\,b^2cdx+b^2c^2)\text{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(b^2d^2x^2+2\,b^2cdx+b^2c^2)\text{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh(
b*x + a)*sinh(b*x + a) + d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(
2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x + b
*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x +
 b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. 2(110) = 220.
time = 0.42, size = 330, normalized size = 2.95

$$\frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+4\,b^2c^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2c^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}-2\,bd^2xe^{(2\,bx+2\,a)}+2\,bd^2xe^{(-2\,bx-2\,a)}-2\,bcde^{(2\,bx+2\,a)}+2\,bcde^{(-2\,bx-2\,a)}-d^2e^{(2\,bx+2\,a)}-d^2e^{(-2\,bx-2\,a)}-2\,d^2}{8\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2*
Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d
) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2*(b*
d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2*x*e^
(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) - d^2*e
^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) - 2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d
^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\mathrm{cosh}\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^3,x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^3, x)
```

3.15 $\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$

Optimal. Leaf size=162

$$\frac{b^2}{3d^3(c+dx)}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}+\frac{2b^3\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4}-\frac{b\cosh(a+bx)\sinh(a}{3d^2(c+dx)^2}$$

```
[Out] 1/3*b^2/d^3/(d*x+c)-1/3*cosh(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*cosh(b*x+a)^2/d^3
/(d*x+c)+2/3*b^3*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^4+2/3*b^3*Chi(2*b*c
/d+2*b*x)*sinh(2*a-2*b*c/d)/d^4-1/3*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^2
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.438$, Rules used = {3395, 32, 3394, 12, 3384, 3379, 3382}

$$\frac{2b^3\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}+\frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] b^2/(3*d^3*(c + d*x)) - Cosh[a + b*x]^2/(3*d*(c + d*x)^3) - (2*b^2*Cosh[a +
 b*x]^2)/(3*d^3*(c + d*x)) + (2*b^3*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*
a - (2*b*c)/d])/(3*d^4) - (b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^
2) + (2*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(3*d^4)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
```

```
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx &= -\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}-\frac{b^2\int\frac{1}{(c+dx)^2}\,dx}{3d^2}+\frac{(2b^2)\int\frac{\cosh^2(a+bx)}{(c+dx)^2}\,d\\ &=\frac{b^2}{3d^3(c+dx)}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}+\frac{(4\\ &=\frac{b^2}{3d^3(c+dx)}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}+\frac{(2\\ &=\frac{b^2}{3d^3(c+dx)}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}-\frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2}+\frac{(2\\ &=\frac{b^2}{3d^3(c+dx)}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}-\frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)}+\frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right.}{3d^4}\end{aligned}$$

Mathematica [A]
time = 0.64, size = 121, normalized size = 0.75

$$\frac{4b^3\mathrm{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d\left(\left(d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+d(d+b(c+dx)\sinh(2(a+bx)))\right)}{(c+dx)^3}+4b^3\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2b(c+dx)}{d}\right)}{6d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

```
[Out] (4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 + 2
*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(d + b*(c + d*x)*Sinh[2*(a + b*x)])
))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d
])/(6*d^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. 2(150) = 300.
time = 3.50, size = 555, normalized size = 3.43

method	result
risch	$-\frac{1}{6(dx+c)^3 d}-\frac{b^5 \mathrm{e}^{-2bx-2a} x^2}{6d(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5 \mathrm{e}^{-2bx-2a} cx}{3d^2(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5 \mathrm{e}^{-2bx-2a} c}{6d^3(d^3x^3b^3+3b^3c\,d^2x^2+3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/(d*x+c)^3/d-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b
^3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^
2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^3+3
*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b^3*d
^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*a)/d^
2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(-2*b*x
-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/d^4*exp
(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b*x+2*a)/
(b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^4*exp(2*b
*x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+
b*c)/d)
```

Maxima [A]
time = 0.31, size = 110, normalized size = 0.68

$$-\frac{1}{6\left(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d\right)}-\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}-\frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d)
*exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/d)
*exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(150) = 300$.
time = 0.37, size = 409, normalized size = 2.52

$$-\frac{d^3 + (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh\left(bx + a\right)^2 + 2\,(bd^3x + bcd^2)\cosh\left(bx + a\right)\sinh\left(bx + a\right) + (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\sinh\left(bx + a\right)^2 - 2\,\left((b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right) - (b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right) - 2\,\left((b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right) + (b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + b^3c^3)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)}{6\,(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/6*(d^3 + (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x +
a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (2*b^2*d^3*x^2 +
 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 - 2*((b^3*d^3*x^3 + 3*b
^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^3*x^
3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*cosh
(-2*(b*c - a*d)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^
3*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d
*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*x^3 + 3*
c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2\left(a + bx\right)}{\left(c + dx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**4,x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 537 vs. $2(150) = 300$.
time = 0.40, size = 537, normalized size = 3.31

$$\frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3cd^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3c^2dx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 12\,b^3c^2dx\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} - 2\,b^2d^3x^2e^{(2\,bx+2\,a)} - 2\,b^2d^3x^2e^{(-2\,bx-2\,a)} + 4\,b^3c^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3c^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} - 4\,b^2cd^2xe^{(2\,bx+2\,a)} - 4\,b^2cd^2xe^{(-2\,bx-2\,a)} - 2\,b^2c^2de^{(2\,bx+2\,a)} - bd^3xe^{(2\,bx+2\,a)} - 2\,b^2c^2de^{(-2\,bx-2\,a)} + bd^3xe^{(-2\,bx-2\,a)} - bcd^2e^{(2\,bx+2\,a)} + bcd^2e^{(-2\,bx-2\,a)} - d^3e^{(2\,bx+2\,a)} - d^3e^{(-2\,bx-2\,a)} - 2\,d^3}{12\,(d^7x^3 + 3\,cd^6x^2 + 3\,c^2d^5x + c^3d^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^3
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x +
```

```
 b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(-2*
a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 12*
b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*e^(2*
b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x + b*c)/
d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d)
- 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) - 2*b^2*c^
2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*b*x - 2*a
) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*e^(-2*b*x
- 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) - 2*d^3)/(d^7*x^3 + 3*c
*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^4,x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^4, x)
```

3.16 $\int (c+dx)^4 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=225

$$-\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\cosh^3(a+$$

```
[Out] -160/9*d^3*(d*x+c)*cosh(b*x+a)/b^4-8/3*d*(d*x+c)^3*cosh(b*x+a)/b^2-8/27*d^3
*(d*x+c)*cosh(b*x+a)^3/b^4-4/9*d*(d*x+c)^3*cosh(b*x+a)^3/b^2+488/27*d^4*sin
h(b*x+a)/b^5+80/9*d^2*(d*x+c)^2*sinh(b*x+a)/b^3+2/3*(d*x+c)^4*sinh(b*x+a)/b
+4/9*d^2*(d*x+c)^2*cosh(b*x+a)^2*sinh(b*x+a)/b^3+1/3*(d*x+c)^4*cosh(b*x+a)^
2*sinh(b*x+a)/b+8/81*d^4*sinh(b*x+a)^3/b^5
```

Rubi [A]
time = 0.20, antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$\frac{8d^4\sinh^3(a+bx)}{81b^5}+\frac{488d^4\sinh(a+bx)}{27b^5}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}+\frac{80d^2(c+dx)^2\sinh(a+bx)}{9b^3}+\frac{4d^2(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{9b^3}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}+\frac{2(c+dx)^4\sinh(a+bx)}{3b}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^4*Cosh[a + b*x]^3,x]
```

```
[Out] (-160*d^3*(c + d*x)*Cosh[a + b*x])/(9*b^4) - (8*d*(c + d*x)^3*Cosh[a + b*x]
)/(3*b^2) - (8*d^3*(c + d*x)*Cosh[a + b*x]^3)/(27*b^4) - (4*d*(c + d*x)^3*C
osh[a + b*x]^3)/(9*b^2) + (488*d^4*Sinh[a + b*x])/(27*b^5) + (80*d^2*(c + d
*x)^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^4*Sinh[a + b*x])/(3*b) + (4*d^2
*(c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^4*Cosh[a +
 b*x]^2*Sinh[a + b*x])/(3*b) + (8*d^4*Sinh[a + b*x]^3)/(81*b^5)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^4\cosh^3(a+bx)\,dx &= -\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int \\ &= -\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2(c+dx)^4\sinh}{3b} \\ &= -\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4}-\frac{4d(c+dx)^3\cosh}{9b^2} \\ &= -\frac{16d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cos}{27b^4} \\ &= -\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\mathrm{co}}{27b} \\ &= -\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4}-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\mathrm{co}}{27b}\end{aligned}$$

Mathematica [A]
time = 0.60, size = 385, normalized size = 1.71

$$\frac{-972bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)-12bd(c+dx)\left(2d^2+3b^2(c+dx)^2\right)\cosh(3(a+bx))+243b^4c^4\sinh(a+bx)+2916b^2c^2d^2\sinh(a+bx)+5832d^4\sinh(a+bx)+972b^4c^3dx\sinh(a+bx)+5832b^2cd^3x\sinh(a+bx)+1458b^4c^2d^2x^2\sinh(a+bx)+2916b^2d^4x^2\sinh(a+bx)+972b^4cd^3x^3\sinh(a+bx)+243b^4d^4x^4\sinh(a+bx)+27b^4c^4\sinh(3(a+bx))+36b^2c^2d^2\sinh(3(a+bx))+8d^4\sinh(3(a+bx))+108b^4c^3dx\sinh(3(a+bx))+72b^2cd^3x\sinh(3(a+bx))+162b^4c^2d^2x^2\sinh(3(a+bx))+36b^2d^4x^2\sinh(3(a+bx))+108b^4cd^3x^3\sinh(3(a+bx))+27b^4d^4x^4\sinh(3(a+bx))}{324b^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^4*Cosh[a + b*x]^3,x]
```

```
[Out] (-972*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 12*b*d*(c + d
*x)*(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] + 243*b^4*c^4*Sinh[a + b*
x] + 2916*b^2*c^2*d^2*Sinh[a + b*x] + 5832*d^4*Sinh[a + b*x] + 972*b^4*c^3*
d*x*Sinh[a + b*x] + 5832*b^2*c*d^3*x*Sinh[a + b*x] + 1458*b^4*c^2*d^2*x^2*S
inh[a + b*x] + 2916*b^2*d^4*x^2*Sinh[a + b*x] + 972*b^4*c*d^3*x^3*Sinh[a +
b*x] + 243*b^4*d^4*x^4*Sinh[a + b*x] + 27*b^4*c^4*Sinh[3*(a + b*x)] + 36*b^
2*c^2*d^2*Sinh[3*(a + b*x)] + 8*d^4*Sinh[3*(a + b*x)] + 108*b^4*c^3*d*x*Sin
h[3*(a + b*x)] + 72*b^2*c*d^3*x*Sinh[3*(a + b*x)] + 162*b^4*c^2*d^2*x^2*Sin
h[3*(a + b*x)] + 36*b^2*d^4*x^2*Sinh[3*(a + b*x)] + 108*b^4*c*d^3*x^3*Sinh[
3*(a + b*x)] + 27*b^4*d^4*x^4*Sinh[3*(a + b*x)])/(324*b^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1262 vs. $2(205) = 410$.
time = 1.37, size = 1263, normalized size = 5.61

method	result
risch	$\frac{(27d^4x^4b^4+108b^4c\,d^3x^3+162b^4c^2d^2x^2-36b^3d^4x^3+108b^4c^3dx-108b^3c\,d^3x^2+27b^4c^4-108b^3c^2d^2x+36b^2d^4x^2-36b^3c^3d+72b^2c\,d^3x+36}{648b^5}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^4*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/972/b*(1/b^4*d^4*((3*b*x+3*a)^4*sinh(3*b*x+3*a)-4*(3*b*x+3*a)^3*cosh(3*b*
x+3*a)+12*(3*b*x+3*a)^2*sinh(3*b*x+3*a)-24*(3*b*x+3*a)*cosh(3*b*x+3*a)+24*s
inh(3*b*x+3*a))-12/b^4*d^4*a*((3*b*x+3*a)^3*sinh(3*b*x+3*a)-3*(3*b*x+3*a)^2
*cosh(3*b*x+3*a)+6*(3*b*x+3*a)*sinh(3*b*x+3*a)-6*cosh(3*b*x+3*a))+54/b^4*d^
4*a^2*((3*b*x+3*a)^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cosh(3*b*x+3*a)+2*sinh(3
*b*x+3*a))-108/b^4*d^4*a^3*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3*a))+81
/b^4*d^4*a^4*sinh(3*b*x+3*a)+12/b^3*c*d^3*((3*b*x+3*a)^3*sinh(3*b*x+3*a)-3*
(3*b*x+3*a)^2*cosh(3*b*x+3*a)+6*(3*b*x+3*a)*sinh(3*b*x+3*a)-6*cosh(3*b*x+3*
a))-108/b^3*c*d^3*a*((3*b*x+3*a)^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cosh(3*b*x
+3*a)+2*sinh(3*b*x+3*a))+324/b^3*c*d^3*a^2*((3*b*x+3*a)*sinh(3*b*x+3*a)-cos
h(3*b*x+3*a))-324/b^3*c*d^3*a^3*sinh(3*b*x+3*a)+54/b^2*c^2*d^2*((3*b*x+3*a)
^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cosh(3*b*x+3*a)+2*sinh(3*b*x+3*a))-324/b^2
*c^2*d^2*a*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3*a))+486/b^2*c^2*d^2*a^
2*sinh(3*b*x+3*a)+108/b*c^3*d*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3*a))
-324/b*c^3*d*a*sinh(3*b*x+3*a)+81*c^4*sinh(3*b*x+3*a))+3/4/b*(1/b^4*d^4*((b
*x+a)^4*sinh(b*x+a)-4*(b*x+a)^3*cosh(b*x+a)+12*(b*x+a)^2*sinh(b*x+a)-24*(b*
x+a)*cosh(b*x+a)+24*sinh(b*x+a))-4*d^4/b^4*a*((b*x+a)^3*sinh(b*x+a)-3*(b*x+
a)^2*cosh(b*x+a)+6*(b*x+a)*sinh(b*x+a)-6*cosh(b*x+a))+6/b^4*d^4*a^2*((b*x+a
)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-4/b^4*d^4*a^3*((b*x+a)
*sinh(b*x+a)-cosh(b*x+a))+1/b^4*d^4*a^4*sinh(b*x+a)+4/b^3*c*d^3*((b*x+a)^3*
sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+6*(b*x+a)*sinh(b*x+a)-6*cosh(b*x+a))-12
*d^3/b^3*a*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))+12
/b^3*c*d^3*a^2*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-4/b^3*c*d^3*a^3*sinh(b*x+a
)+6/b^2*c^2*d^2*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))
-12/b^2*c^2*d^2*a*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+6*d^2/b^2*a^2*c^2*sinh(
b*x+a)+4*d/b*c^3*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-4/b*c^3*d*a*sinh(b*x+a)+
c^4*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(205) = 410$.
time = 0.30, size = 644, normalized size = 2.86

$$\frac{1}{18}c^3d\left(\frac{(3bxe^{(3a)}-e^{(3a)})e^{(3bx)}}{b^2}+\frac{27(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right)+\frac{1}{24}c^4\left(\frac{e^{(3bx+3a)}}{b}+\frac{9e^{(bx+a)}}{b}-\frac{9e^{(-bx-a)}}{b}-\frac{e^{(-3bx-3a)}}{b}\right)+\frac{1}{36}c^2d^2\left(\frac{(9b^2x^2e^{(3a)}-6bxe^{(3a)}+2e^{(3a)})e^{(3bx)}}{b^3}+\frac{81(b^2x^2e^a-2bxe^a+2e^a)e^{(bx)}}{b^3}-\frac{81(b^2x^2+2bx+2)e^{(-bx-a)}}{b^3}-\frac{(9b^2x^2+6bx+2)e^{(-3bx-3a)}}{b^3}\right)+\frac{1}{54}cd^3\left(\frac{(9b^3x^3e^{(3a)}-9b^2x^2e^{(3a)}+6bxe^{(3a)}-2e^{(3a)})e^{(3bx)}}{b^4}+\frac{81(b^3x^3e^a-3b^2x^2e^a+6bxe^a-6e^a)e^{(bx)}}{b^4}-\frac{81(b^3x^3+3b^2x^2+6bx+6)e^{(-bx-a)}}{b^4}-\frac{(9b^3x^3+9b^2x^2+6bx+2)e^{(-3bx-3a)}}{b^4}\right)+\frac{1}{648}d^4\left(\frac{(27b^4x^4e^{(3a)}-36b^3x^3e^{(3a)}+36b^2x^2e^{(3a)}-24bxe^{(3a)}+8e^{(3a)})e^{(3bx)}}{b^5}+\frac{243(b^4x^4e^a-4b^3x^3e^a+12b^2x^2e^a-24bxe^a+24e^a)e^{(bx)}}{b^5}-\frac{243(b^4x^4+4b^3x^3+12b^2x^2+24bx+24)e^{(-bx-a)}}{b^5}-\frac{(27b^4x^4+36b^3x^3+36b^2x^2+24bx+8)e^{(-3bx-3a)}}{b^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^4*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(
-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(
3*a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x
 - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e
^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x
*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x -
a)/b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4) + 1/648*
d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*a) - 24*b*x
*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 + 243*(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12
*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4 + 4*b^3*x^3
+ 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 - (27*b^4*x^4 + 36*b^3*x^3 + 3
6*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(205) = 410$.
time = 0.38, size = 528, normalized size = 2.35

$$-\frac{12\,(3b^3d^4x^3+9b^3cd^3x^2+3b^3c^3d+2bcd^3+(9b^3c^2d^2+2bd^4)x)\cosh(bx+a)^3+36\,(3b^3d^4x^3+9b^3cd^3x^2+3b^3c^3d+2bcd^3+(9b^3c^2d^2+2bd^4)x)\cosh(bx+a)\sinh(bx+a)^2-(27b^4d^4x^4+108b^4cd^3x^3+27b^4c^4+36b^2c^2d^2+8d^4+18\,(9b^4c^2d^2+2b^2d^4)x^2+36\,(3b^4c^3d+2b^2cd^3)x)\sinh(bx+a)^3+972\,(b^3d^4x^3+3b^3cd^3x^2+b^3c^3d+6bcd^3+3\,(b^3c^2d^2+2bd^4)x)\cosh(bx+a)-3\,(81b^4d^4x^4+324b^4cd^3x^3+81b^4c^4+972b^2c^2d^2+1944d^4+486\,(b^4c^2d^2+2b^2d^4)x^2+(27b^4d^4x^4+108b^4cd^3x^3+27b^4c^4+36b^2c^2d^2+8d^4+18\,(9b^4c^2d^2+2b^2d^4)x^2+36\,(3b^4c^3d+2b^2cd^3)x)\cosh(bx+a)^2+324\,(b^4c^3d+6b^2cd^3)x)\sinh(bx+a)}{324\,b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/324*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*
b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^3 + 36*(3*b^3*d^4*x^3 + 9*b^3*c*d^3
*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)
*sinh(b*x + a)^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 36*b^
2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3*d +
2*b^2*c*d^3)*x)*sinh(b*x + a)^3 + 972*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*
c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a) - 3*(81*b^4*
d^4*x^4 + 324*b^4*c*d^3*x^3 + 81*b^4*c^4 + 972*b^2*c^2*d^2 + 1944*d^4 + 486
*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b
^4*c^4 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(
3*b^4*c^3*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)^2 + 324*(b^4*c^3*d + 6*b^2*c*d^
3)*x)*sinh(b*x + a))/b^5
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.
time = 0.82, size = 772, normalized size = 3.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**4*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**4*sinh(a + b*x)**3/(3*b) + c**4*sinh(a + b*x)*cosh(a + b*x
)**2/b - 8*c**3*d*x*sinh(a + b*x)**3/(3*b) + 4*c**3*d*x*sinh(a + b*x)*cosh(
a + b*x)**2/b - 4*c**2*d**2*x**2*sinh(a + b*x)**3/b + 6*c**2*d**2*x**2*sinh
(a + b*x)*cosh(a + b*x)**2/b - 8*c*d**3*x**3*sinh(a + b*x)**3/(3*b) + 4*c*d
**3*x**3*sinh(a + b*x)*cosh(a + b*x)**2/b - 2*d**4*x**4*sinh(a + b*x)**3/(3
*b) + d**4*x**4*sinh(a + b*x)*cosh(a + b*x)**2/b + 8*c**3*d*sinh(a + b*x)**
2*cosh(a + b*x)/(3*b**2) - 28*c**3*d*cosh(a + b*x)**3/(9*b**2) + 8*c**2*d**
2*x*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c**2*d**2*x*cosh(a + b*x)**3/(
3*b**2) + 8*c*d**3*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c*d**3*x**
2*cosh(a + b*x)**3/(3*b**2) + 8*d**4*x**3*sinh(a + b*x)**2*cosh(a + b*x)/(3
*b**2) - 28*d**4*x**3*cosh(a + b*x)**3/(9*b**2) - 80*c**2*d**2*sinh(a + b*x
)**3/(9*b**3) + 28*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) - 160*
c*d**3*x*sinh(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sinh(a + b*x)*cosh(a + b*x
)**2/(3*b**3) - 80*d**4*x**2*sinh(a + b*x)**3/(9*b**3) + 28*d**4*x**2*sinh(
a + b*x)*cosh(a + b*x)**2/(3*b**3) + 160*c*d**3*sinh(a + b*x)**2*cosh(a + b
*x)/(9*b**4) - 488*c*d**3*cosh(a + b*x)**3/(27*b**4) + 160*d**4*x*sinh(a +
b*x)**2*cosh(a + b*x)/(9*b**4) - 488*d**4*x*cosh(a + b*x)**3/(27*b**4) - 14
56*d**4*sinh(a + b*x)**3/(81*b**5) + 488*d**4*sinh(a + b*x)*cosh(a + b*x)**
2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x**3 + c*d**
3*x**4 + d**4*x**5/5)*cosh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(205) = 410.
time = 0.42, size = 654, normalized size = 2.91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d^
4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^2*
x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24*b*
d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 + 3/8*(b^4*d^4*x^4 + 4*b^4*
c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3*c*d^
3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 24*b^2*
c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x + a)/b^
5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3
+ 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x + 12*b^2*d^
4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c
*d^3 + 24*d^4)*e^(-b*x - a)/b^5 - 1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3
 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108*b^3*c*d^3*x
```

```
^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*c^3*d + 72*b^
2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)*e^(-3*b*x - 3
*a)/b^5
```

Mupad [B]
time = 1.35, size = 532, normalized size = 2.36

$$\frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(27\,b^4\,c^4+252\,b^2\,c^2\,d^2+488\,d^4\right)}{27\,b^5}-\frac{2\,\sinh(a+b\,x)^3\,\left(27\,b^4\,c^4+360\,b^2\,c^2\,d^2+728\,d^4\right)}{81\,b^5}-\frac{4\,\cosh(a+b\,x)^3\,\left(21\,b^2\,c^3\,d+122\,c\,d^3\right)}{27\,b^4}+\frac{8\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^4}-\frac{28\,d^4\,x^3\,\cosh(a+b\,x)^3}{9\,b^2}-\frac{4\,x\,\cosh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d^2+122\,d^4\right)}{27\,b^4}-\frac{2\,d^4\,x^4\,\sinh(a+b\,x)^3}{3\,b}-\frac{8\,x\,\sinh(a+b\,x)^3\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^3}-\frac{4\,x^2\,\sinh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^3}+\frac{2\,x^2\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d^2+14\,d^4\right)}{3\,b^3}-\frac{28\,c\,d^3\,x^2\,\cosh(a+b\,x)^3}{3\,b^2}+\frac{d^4\,x^4\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b}+\frac{8\,d^4\,x^3\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{3\,b^2}-\frac{8\,c\,d^3\,x^3\,\sinh(a+b\,x)^3}{3\,b}+\frac{8\,x\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^4}+\frac{4\,x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(3\,b^2\,c^3\,d+14\,c\,d^3\right)}{3\,b^3}+\frac{4\,c\,d^3\,x^3\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b}+\frac{8\,c\,d^3\,x^2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^4,x)
```

```
[Out] (cosh(a + b*x)^2*sinh(a + b*x)*(488*d^4 + 27*b^4*c^4 + 252*b^2*c^2*d^2))/(2
7*b^5) - (2*sinh(a + b*x)^3*(728*d^4 + 27*b^4*c^4 + 360*b^2*c^2*d^2))/(81*b
^5) - (4*cosh(a + b*x)^3*(122*c*d^3 + 21*b^2*c^3*d))/(27*b^4) + (8*cosh(a +
 b*x)*sinh(a + b*x)^2*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^4) - (28*d^4*x^3*cosh(
a + b*x)^3)/(9*b^2) - (4*x*cosh(a + b*x)^3*(122*d^4 + 63*b^2*c^2*d^2))/(27*
b^4) - (2*d^4*x^4*sinh(a + b*x)^3)/(3*b) - (8*x*sinh(a + b*x)^3*(20*c*d^3 +
 3*b^2*c^3*d))/(9*b^3) - (4*x^2*sinh(a + b*x)^3*(20*d^4 + 9*b^2*c^2*d^2))/(
9*b^3) + (2*x^2*cosh(a + b*x)^2*sinh(a + b*x)*(14*d^4 + 9*b^2*c^2*d^2))/(3*
b^3) - (28*c*d^3*x^2*cosh(a + b*x)^3)/(3*b^2) + (d^4*x^4*cosh(a + b*x)^2*si
nh(a + b*x))/b + (8*d^4*x^3*cosh(a + b*x)*sinh(a + b*x)^2)/(3*b^2) - (8*c*d
^3*x^3*sinh(a + b*x)^3)/(3*b) + (8*x*cosh(a + b*x)*sinh(a + b*x)^2*(20*d^4
+ 9*b^2*c^2*d^2))/(9*b^4) + (4*x*cosh(a + b*x)^2*sinh(a + b*x)*(14*c*d^3 +
3*b^2*c^3*d))/(3*b^3) + (4*c*d^3*x^3*cosh(a + b*x)^2*sinh(a + b*x))/b + (8*
c*d^3*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2
```

3.17 $\int (c+dx)^3 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=175

$$-\frac{40d^3\cosh(a+bx)}{9b^4}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{40d^2(c+dx)}{9}$$

```
[Out] -40/9*d^3*cosh(b*x+a)/b^4-2*d*(d*x+c)^2*cosh(b*x+a)/b^2-2/27*d^3*cosh(b*x+a
)^3/b^4-1/3*d*(d*x+c)^2*cosh(b*x+a)^3/b^2+40/9*d^2*(d*x+c)*sinh(b*x+a)/b^3+
2/3*(d*x+c)^3*sinh(b*x+a)/b+2/9*d^2*(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/b^3+1
/3*(d*x+c)^3*cosh(b*x+a)^2*sinh(b*x+a)/b
```

Rubi [A]
time = 0.12, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2718, 3391}

$$-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{40d^3\cosh(a+bx)}{9b^4}+\frac{40d^2(c+dx)\sinh(a+bx)}{9b^3}+\frac{2d^2(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{9b^3}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}+\frac{2(c+dx)^3\sinh(a+bx)}{3b}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Cosh[a + b*x]^3,x]
```

```
[Out] (-40*d^3*Cosh[a + b*x])/(9*b^4) - (2*d*(c + d*x)^2*Cosh[a + b*x])/b^2 - (2*
d^3*Cosh[a + b*x]^3)/(27*b^4) - (d*(c + d*x)^2*Cosh[a + b*x]^3)/(3*b^2) + (
40*d^2*(c + d*x)*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^3*Sinh[a + b*x])/(3*
b) + (2*d^2*(c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(9*b^3) + ((c + d*x)^3
*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\cosh^3(a+bx)\,dx &= -\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (\\ &= -\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2(c+dx)^3\sinh(a+bx)}{3b}\\ &= -\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-\frac{d(c+dx)^2\cosh^3(a+bx}{3b^2}\\ &= -\frac{4d^3\cosh(a+bx)}{9b^4}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-d(c\\ &= -\frac{40d^3\cosh(a+bx)}{9b^4}-\frac{2d(c+dx)^2\cosh(a+bx)}{b^2}-\frac{2d^3\cosh^3(a+bx)}{27b^4}-d(c\end{aligned}$$

Mathematica [A]
time = 0.64, size = 122, normalized size = 0.70

$$\frac{-486d(2d^2+b^2(c+dx)^2)\cosh(a+bx)-2d(2d^2+9b^2(c+dx)^2)\cosh(3(a+bx))+12b(c+dx)\left(82d^2+15b^2(c+dx)^2+(2d^2+3b^2(c+dx)^2)\cosh(2(a+bx))\right)\sinh(a+bx)}{216b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Cosh[a + b*x]^3,x]
```

```
[Out] (-486*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*d*(2*d^2 + 9*b^2*(c + d
*x)^2)*Cosh[3*(a + b*x)] + 12*b*(c + d*x)*(82*d^2 + 15*b^2*(c + d*x)^2 + (2
*d^2 + 3*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(216*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 708 vs. $2(161) = 322$.
time = 1.28, size = 709, normalized size = 4.05

method	result
risch	$\frac{(9d^3x^3b^3+27b^3cd^2x^2+27b^3c^2dx-9b^2d^3x^2+9b^3c^3-18b^2cd^2x-9b^2c^2d+6bd^3x+6bcd^2-2d^3)e^{3bx+3a}}{216b^4}+\frac{3(d^3x^3b^3+3b^3cd^2x^2+3b^3c}{}$

default	$\frac{d^3\left((3bx+3a)^3\sinh(3bx+3a)-3(3bx+3a)^2\cosh(3bx+3a)+6(3bx+3a)\sinh(3bx+3a)-6\cosh(3bx+3a)\right)}{b^3}-\frac{9d^3a\left((3bx+3a)^2\sinh(3bx+3a)-2(3bx+3a)\cos\right.}{b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/324/b*(1/b^3*d^3*((3*b*x+3*a)^3*sinh(3*b*x+3*a)-3*(3*b*x+3*a)^2*cosh(3*b*
x+3*a)+6*(3*b*x+3*a)*sinh(3*b*x+3*a)-6*cosh(3*b*x+3*a))-9/b^3*d^3*a*((3*b*x
+3*a)^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cosh(3*b*x+3*a)+2*sinh(3*b*x+3*a))+27
/b^3*d^3*a^2*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3*a))-27/b^3*d^3*a^3*s
inh(3*b*x+3*a)+9*c*d^2/b^2*((3*b*x+3*a)^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cos
h(3*b*x+3*a)+2*sinh(3*b*x+3*a))-54/b^2*c*d^2*a*((3*b*x+3*a)*sinh(3*b*x+3*a)
-cosh(3*b*x+3*a))+81/b^2*c*d^2*a^2*sinh(3*b*x+3*a)+27/b*c^2*d*((3*b*x+3*a)*
sinh(3*b*x+3*a)-cosh(3*b*x+3*a))-81/b*c^2*d*a*sinh(3*b*x+3*a)+27*c^3*sinh(3
*b*x+3*a))+3/4/b*(1/b^3*d^3*((b*x+a)^3*sinh(b*x+a)-3*(b*x+a)^2*cosh(b*x+a)+
6*(b*x+a)*sinh(b*x+a)-6*cosh(b*x+a))-3/b^3*d^3*a*((b*x+a)^2*sinh(b*x+a)-2*(
b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))+3*d^3/b^3*a^2*((b*x+a)*sinh(b*x+a)-cosh(b
*x+a))-1/b^3*d^3*a^3*sinh(b*x+a)+3*d^2/b^2*c*((b*x+a)^2*sinh(b*x+a)-2*(b*x+
a)*cosh(b*x+a)+2*sinh(b*x+a))-6*d^2/b^2*a*c*((b*x+a)*sinh(b*x+a)-cosh(b*x+a
))+3/b^2*c*d^2*a^2*sinh(b*x+a)+3/b*c^2*d*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))-
3/b*c^2*d*a*sinh(b*x+a)+c^3*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(161) = 322$.
time = 0.29, size = 439, normalized size = 2.51

$$\frac{1}{24}c^2d\left(\frac{\left(3bxe^{(3a)}-e^{(3a)}\right)e^{(3bx)}}{b^2}+\frac{27\left(bxe^a-e^a\right)e^{(bx)}}{b^2}-\frac{27\left(bx+1\right)e^{(-bx-a)}}{b^2}-\frac{\left(3bx+1\right)e^{(-3bx-3a)}}{b^2}\right)+\frac{1}{24}c^3\left(\frac{e^{(3bx+3a)}}{b}+\frac{9e^{(bx+a)}}{b}-\frac{9e^{(-bx-a)}}{b}-\frac{e^{(-3bx-3a)}}{b}\right)+\frac{1}{72}cd^2\left(\frac{\left(9b^2x^2e^{(3a)}-6bxe^{(3a)}+2e^{(3a)}\right)e^{(3bx)}}{b^3}+\frac{81\left(b^2x^2e^a-2bxe^a+2e^a\right)e^{(bx)}}{b^3}-\frac{81\left(b^2x^2+2bx+2\right)e^{(-bx-a)}}{b^3}-\frac{\left(9b^2x^2+6bx+2\right)e^{(-3bx-3a)}}{b^3}\right)+\frac{1}{216}d^3\left(\frac{\left(9b^3x^3e^{(3a)}-9b^2x^2e^{(3a)}+6bxe^{(3a)}-2e^{(3a)}\right)e^{(3bx)}}{b^4}+\frac{81\left(b^3x^3e^a-3b^2x^2e^a+6bxe^a-6e^a\right)e^{(bx)}}{b^4}-\frac{81\left(b^3x^3+3b^2x^2+6bx+6\right)e^{(-bx-a)}}{b^4}-\frac{\left(9b^3x^3+9b^2x^2+6bx+2\right)e^{(-3bx-3a)}}{b^4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^
(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^3*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(
-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*
a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(
b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x -
 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b*x*e^(3
*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^
a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-b*x - a)/
b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(161) = 322$.
time = 0.39, size = 343, normalized size = 1.96

$$-\frac{\left(9b^2d^3x^2+18b^2cd^2x+9b^2c^2d+2d^3\right)\cosh\left(bx+a\right)^3+3\left(9b^2d^3x^2+18b^2cd^2x+9b^2c^2d+2d^3\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2-3\left(3b^3d^3x^3+9b^3cd^2x^2+3b^3c^3+2bcd^2+\left(9b^3c^2d+2bd^3\right)x\right)\sinh\left(bx+a\right)^3+243\left(b^2d^3x^2+2b^2cd^2x+b^2c^2d+2d^3\right)\cosh\left(bx+a\right)-9\left(9b^3d^3x^3+27b^3cd^2x^2+9b^3c^3+54bcd^2+\left(3b^3d^3x^3+9b^3cd^2x^2+3b^3c^3+2bcd^2+\left(9b^3c^2d+2bd^3\right)x\right)\cosh\left(bx+a\right)^2+27\left(b^3c^2d+2bd^3\right)x\right)\sinh\left(bx+a\right)}{108b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x + a
)^3 + 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x + a
)*sinh(b*x + a)^2 - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b*c*
d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*sinh(b*x + a)^3 + 243*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - 9*(9*b^3*d^3*x^3 + 27*b^3*c*d
^2*x^2 + 9*b^3*c^3 + 54*b*c*d^2 + (3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*
c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^2 + 27*(b^3*c^2*
d + 2*b*d^3)*x)*sinh(b*x + a))/b^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.
time = 0.54, size = 495, normalized size = 2.83

$$\begin{cases} -\frac{2c^3\sinh^3(a+bx)}{3b} + \frac{c^3\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2c^2dx\sinh^3(a+bx)}{b} + \frac{3c^2dx\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2cd^2x^2\sinh^3(a+bx)}{b} + \frac{3cd^2x^2\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2d^3x^3\sinh^3(a+bx)}{3b} + \frac{d^3x^3\sinh(a+bx)\cosh^2(a+bx)}{b} + \frac{2c^2d\sinh^2(a+bx)\cosh(a+bx)}{b^2} - \frac{7c^2d\cosh^3(a+bx)}{3b^2} + \frac{4cd^2x\sinh^2(a+bx)\cosh(a+bx)}{b^2} - \frac{14cd^2x\cosh^3(a+bx)}{3b^2} + \frac{2d^3x^2\sinh^2(a+bx)\cosh(a+bx)}{b^2} - \frac{7d^3x^2\cosh^3(a+bx)}{3b^2} - \frac{40cd^2\sinh^3(a+bx)}{9b^3} + \frac{14cd^2\sinh(a+bx)\cosh^2(a+bx)}{3b^3} - \frac{40d^3x\sinh^3(a+bx)}{9b^3} + \frac{14d^3x\sinh(a+bx)\cosh^2(a+bx)}{3b^3} + \frac{40d^3\sinh^2(a+bx)\cosh(a+bx)}{9b^4} - \frac{122d^3\cosh^3(a+bx)}{27b^4} & \text{for } b \neq 0 \\ \left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right)\cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**3*sinh(a + b*x)**3/(3*b) + c**3*sinh(a + b*x)*cosh(a + b*x
)**2/b - 2*c**2*d*x*sinh(a + b*x)**3/b + 3*c**2*d*x*sinh(a + b*x)*cosh(a +
b*x)**2/b - 2*c*d**2*x**2*sinh(a + b*x)**3/b + 3*c*d**2*x**2*sinh(a + b*x)*
cosh(a + b*x)**2/b - 2*d**3*x**3*sinh(a + b*x)**3/(3*b) + d**3*x**3*sinh(a
+ b*x)*cosh(a + b*x)**2/b + 2*c**2*d*sinh(a + b*x)**2*cosh(a + b*x)/b**2 -
7*c**2*d*cosh(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sinh(a + b*x)**2*cosh(a + b
*x)/b**2 - 14*c*d**2*x*cosh(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sinh(a + b*x
)**2*cosh(a + b*x)/b**2 - 7*d**3*x**2*cosh(a + b*x)**3/(3*b**2) - 40*c*d**2
*sinh(a + b*x)**3/(9*b**3) + 14*c*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*b*
*3) - 40*d**3*x*sinh(a + b*x)**3/(9*b**3) + 14*d**3*x*sinh(a + b*x)*cosh(a
+ b*x)**2/(3*b**3) + 40*d**3*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4) - 122*
d**3*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*
d**2*x**3 + d**3*x**4/4)*cosh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(161) = 322$.
time = 0.42, size = 414, normalized size = 2.37

$$\frac{(9b^3d^3x^3 + 27b^3cd^2x^2 + 27b^3c^2dx - 9b^2d^3x^2 + 9b^3c^3 - 18b^2cd^2x - 9b^2c^2d + 6bd^3x + 6bcd^2 - 2d^3)e^{(3bx+3a)}}{216b^4} + \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx - 3b^2d^3x^2 + b^3c^3 - 6b^2cd^2x - 3b^2c^2d + 6bd^3x + 6bcd^2 - 6d^3)e^{(bx+a)}}{8b^4} - \frac{3(b^3d^3x^3 + 3b^3cd^2x^2 + 3b^3c^2dx + 3b^2d^3x^2 + b^3c^3 + 6b^2cd^2x + 3b^2c^2d + 6bd^3x + 6bcd^2 + 6d^3)e^{(-bx-a)}}{8b^4} - \frac{(9b^3d^3x^3 + 27b^3cd^2x^2 + 27b^3c^2dx + 9b^2d^3x^2 + 9b^3c^3 + 18b^2cd^2x + 9b^2c^2d + 6bd^3x + 6bcd^2 + 2d^3)e^{(-3bx-3a)}}{216b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)*e
```

```
^(3*b*x + 3*a)/b^4 + 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3
*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^
2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2
*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x +
6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x
^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x + 9*b^2*c^
2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

Mupad [B]
time = 1.14, size = 364, normalized size = 2.08

$$\frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(3\,b^2\,c^3+14\,c\,d^2\right)}{3\,b^3}-\frac{2\,\sinh(a+b\,x)^3\,\left(3\,b^2\,c^3+20\,c\,d^2\right)}{9\,b^3}-\frac{\cosh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d+122\,d^3\right)}{27\,b^4}+\frac{2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^4}-\frac{2\,x\,\sinh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^3}-\frac{7\,d^3\,x^2\,\cosh(a+b\,x)^3}{3\,b^2}-\frac{2\,d^3\,x^3\,\sinh(a+b\,x)^3}{3\,b}-\frac{14\,c\,d^2\,x\,\cosh(a+b\,x)^3}{3\,b^2}+\frac{x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d+14\,d^3\right)}{3\,b^3}+\frac{d^3\,x^3\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b}+\frac{2\,d^3\,x^2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2}-\frac{2\,c\,d^2\,x^2\,\sinh(a+b\,x)^3}{b}+\frac{3\,c\,d^2\,x^2\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b}+\frac{4\,c\,d^2\,x\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^3,x)
```

```
[Out] (cosh(a + b*x)^2*sinh(a + b*x)*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (2*sinh(a
+ b*x)^3*(20*c*d^2 + 3*b^2*c^3))/(9*b^3) - (cosh(a + b*x)^3*(122*d^3 + 63*b
^2*c^2*d))/(27*b^4) + (2*cosh(a + b*x)*sinh(a + b*x)^2*(20*d^3 + 9*b^2*c^2*
d))/(9*b^4) - (2*x*sinh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (7*d^3
*x^2*cosh(a + b*x)^3)/(3*b^2) - (2*d^3*x^3*sinh(a + b*x)^3)/(3*b) - (14*c*d
^2*x*cosh(a + b*x)^3)/(3*b^2) + (x*cosh(a + b*x)^2*sinh(a + b*x)*(14*d^3 +
9*b^2*c^2*d))/(3*b^3) + (d^3*x^3*cosh(a + b*x)^2*sinh(a + b*x))/b + (2*d^3*
x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2 - (2*c*d^2*x^2*sinh(a + b*x)^3)/b +
(3*c*d^2*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b + (4*c*d^2*x*cosh(a + b*x)*si
nh(a + b*x)^2)/b^2
```

3.18 $\int (c+dx)^2 \cosh^3(a+bx)\,dx$

Optimal. Leaf size=123

$$-\frac{4d(c+dx)\cosh(a+bx)}{3b^2}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{14d^2\sinh(a+bx)}{9b^3}+\frac{2(c+dx)^2\sinh(a+bx)}{3b}+\frac{(c+d$$

```
[Out] -4/3*d*(d*x+c)*cosh(b*x+a)/b^2-2/9*d*(d*x+c)*cosh(b*x+a)^3/b^2+14/9*d^2*sin
h(b*x+a)/b^3+2/3*(d*x+c)^2*sinh(b*x+a)/b+1/3*(d*x+c)^2*cosh(b*x+a)^2*sinh(b
*x+a)/b+2/27*d^2*sinh(b*x+a)^3/b^3
```

Rubi [A]
time = 0.08, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 3377, 2717, 2713}

$$\frac{2d^2\sinh^3(a+bx)}{27b^3}+\frac{14d^2\sinh(a+bx)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}-\frac{4d(c+dx)\cosh(a+bx)}{3b^2}+\frac{2(c+dx)^2\sinh(a+bx)}{3b}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

```
[Out] (-4*d*(c + d*x)*Cosh[a + b*x])/(3*b^2) - (2*d*(c + d*x)*Cosh[a + b*x]^3)/(9
*b^2) + (14*d^2*Sinh[a + b*x])/(9*b^3) + (2*(c + d*x)^2*Sinh[a + b*x])/(3*b
) + ((c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b) + (2*d^2*Sinh[a + b*x
]^3)/(27*b^3)
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\cosh^3(a+bx)\,dx &= -\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\cosh^2(a+bx)\sinh(a+bx)}{3b}+\frac{2}{3}\int (c \\ &= -\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{2(c+dx)^2\sinh(a+bx)}{3b}+\frac{(c+dx)^2\cosh^2(a+}{3b} \\ &= -\frac{4d(c+dx)\cosh(a+bx)}{3b^2}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{2d^2\sinh(a+bx)}{9b^3}+ \\ &= -\frac{4d(c+dx)\cosh(a+bx)}{3b^2}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{14d^2\sinh(a+bx)}{9b^3}-\end{aligned}$$

Mathematica [A]
time = 0.37, size = 93, normalized size = 0.76

$$\frac{-162bd(c+dx)\cosh(a+bx)-6bd(c+dx)\cosh(3(a+bx))+2(82d^2+45b^2(c+dx)^2+(2d^2+9b^2(c+dx)^2)\cosh(2(a+bx)))\sinh(a+bx)}{108b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

```
[Out] (-162*b*d*(c + d*x)*Cosh[a + b*x] - 6*b*d*(c + d*x)*Cosh[3*(a + b*x)] + 2*(
82*d^2 + 45*b^2*(c + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)]
)*Sinh[a + b*x])/(108*b^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(111) = 222$.
time = 1.26, size = 340, normalized size = 2.76

method	result
risch	$\frac{(9b^2d^2x^2+18b^2cdx+9b^2c^2-6b\,d^2x-6bcd+2d^2)e^{3bx+3a}}{216b^3}+\frac{3(b^2d^2x^2+2b^2cdx+b^2c^2-2b\,d^2x-2bcd+2d^2)e^{bx+a}}{8b^3}-\frac{3(b^2d^2x^2+2b^2cdx}{}$
default	$\frac{\frac{d^2\left((3bx+3a)^2\sinh(3bx+3a)-2(3bx+3a)\cosh(3bx+3a)+2\sinh(3bx+3a)\right)}{b^2}-\frac{6d^2a((3bx+3a)\sinh(3bx+3a)-\cosh(3bx+3a))}{b^2}+\frac{9d^2a^2\sinh(3bx+3a)}{b^2}+\frac{6ca}{}}{108b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/108/b*(d^2/b^2*((3*b*x+3*a)^2*sinh(3*b*x+3*a)-2*(3*b*x+3*a)*cosh(3*b*x+3*
a)+2*sinh(3*b*x+3*a))-6/b^2*d^2*a*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3
```

```
*a))+9/b^2*d^2*a^2*sinh(3*b*x+3*a)+6/b*c*d*((3*b*x+3*a)*sinh(3*b*x+3*a)-cos
h(3*b*x+3*a))-18/b*c*d*a*sinh(3*b*x+3*a)+9*c^2*sinh(3*b*x+3*a))+3/4/b*(d^2/
b^2*((b*x+a)^2*sinh(b*x+a)-2*(b*x+a)*cosh(b*x+a)+2*sinh(b*x+a))-2/b^2*d^2*a
*((b*x+a)*sinh(b*x+a)-cosh(b*x+a))+1/b^2*d^2*a^2*sinh(b*x+a)+2/b*c*d*((b*x+
a)*sinh(b*x+a)-cosh(b*x+a))-2*d/b*a*c*sinh(b*x+a)+sinh(b*x+a)*c^2)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(111) = 222$.
time = 0.28, size = 272, normalized size = 2.21

$$\frac{1}{36}\,cd\left(\frac{\left(3\,bxe^{(3\,a)} - e^{(3\,a)}\right)e^{(3\,bx)}}{b^2} + \frac{27\,(bxe^{a} - e^{a})e^{(bx)}}{b^2} - \frac{27\,(bx+1)e^{(-bx-a)}}{b^2} - \frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right) + \frac{1}{24}\,c^2\left(\frac{e^{(3\,bx+3\,a)}}{b} + \frac{9\,e^{(bx+a)}}{b} - \frac{9\,e^{(-bx-a)}}{b} - \frac{e^{(-3\,bx-3\,a)}}{b}\right) + \frac{1}{216}\,d^2\left(\frac{\left(9\,b^2x^2e^{(3\,a)} - 6\,bxe^{(3\,a)} + 2\,e^{(3\,a)}\right)e^{(3\,bx)}}{b^3} + \frac{81\,(b^2x^2e^{a} - 2\,bxe^{a} + 2\,e^{a})e^{(bx)}}{b^3} - \frac{81\,(b^2x^2 + 2\,bx + 2)e^{(-bx-a)}}{b^3} - \frac{(9\,b^2x^2 + 6\,bx + 2)e^{(-3\,bx-3\,a)}}{b^3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(b
*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c^2*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3
*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3*a))
*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81*(b^2
*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*
a)/b^3)
```

Fricas [**A**]
time = 0.35, size = 199, normalized size = 1.62

$$-\frac{6\,(bd^2x + bcd)\cosh\left(bx + a\right)^3 + 18\,(bd^2x + bcd)\cosh\left(bx + a\right)\sinh\left(bx + a\right)^2 - (9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 2\,d^2)\sinh\left(bx + a\right)^3 + 162\,(bd^2x + bcd)\cosh\left(bx + a\right) - 3\left(27\,b^2d^2x^2 + 54\,b^2cdx + 27\,b^2c^2 + (9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 2\,d^2)\cosh\left(bx + a\right)^2 + 54\,d^2\right)\sinh\left(bx + a\right)}{108\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/108*(6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*d^2*x + b*c*d)*cosh(b*x
 + a)*sinh(b*x + a)^2 - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2)*
sinh(b*x + a)^3 + 162*(b*d^2*x + b*c*d)*cosh(b*x + a) - 3*(27*b^2*d^2*x^2 +
 54*b^2*c*d*x + 27*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*
d^2)*cosh(b*x + a)^2 + 54*d^2)*sinh(b*x + a))/b^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.
time = 0.34, size = 284, normalized size = 2.31

$$\begin{cases} -\frac{2c^2\sinh^3(a+bx)}{3b} + \frac{c^2\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{4cdx\sinh^3(a+bx)}{3b} + \frac{2cdx\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2d^2x^2\sinh^3(a+bx)}{3b} + \frac{d^2x^2\sinh(a+bx)\cosh^2(a+bx)}{b} + \frac{4cd\sinh^2(a+bx)\cosh(a+bx)}{3b^2} - \frac{14cd\cosh^3(a+bx)}{9b^2} + \frac{4d^2x\sinh^2(a+bx)\cosh(a+bx)}{3b^2} - \frac{14d^2x\cosh^3(a+bx)}{9b^2} - \frac{40d^2\sinh^3(a+bx)}{27b^3} + \frac{14d^2\sinh(a+bx)\cosh^2(a+bx)}{9b^3} & \text{for } b \neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cosh^3(a) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*cosh(b*x+a)**3,x)
```

```
[Out] Piecewise((-2*c**2*sinh(a + b*x)**3/(3*b) + c**2*sinh(a + b*x)*cosh(a + b*x
)**2/b - 4*c*d*x*sinh(a + b*x)**3/(3*b) + 2*c*d*x*sinh(a + b*x)*cosh(a + b*
x)**2/b - 2*d**2*x**2*sinh(a + b*x)**3/(3*b) + d**2*x**2*sinh(a + b*x)*cosh
(a + b*x)**2/b + 4*c*d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 14*c*d*cos
h(a + b*x)**3/(9*b**2) + 4*d**2*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) -
 14*d**2*x*cosh(a + b*x)**3/(9*b**2) - 40*d**2*sinh(a + b*x)**3/(27*b**3) +
 14*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*x + c*d
*x**2 + d**2*x**3/3)*cosh(a)**3, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(111) = 222$.
time = 0.42, size = 230, normalized size = 1.87

$$\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2-6\,bd^2x-6\,bcd+2\,d^2)e^{(3\,bx+3\,a)}}{216\,b^3}+\frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,bd^2x-2\,bcd+2\,d^2)e^{(bx+a)}}{8\,b^3}-\frac{3\,(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,bd^2x+2\,bcd+2\,d^2)e^{(-bx-a)}}{8\,b^3}-\frac{(9\,b^2d^2x^2+18\,b^2cdx+9\,b^2c^2+6\,bd^2x+6\,bcd+2\,d^2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] 1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*d
^2)*e^(3*b*x + 3*a)/b^3 + 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^
2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 - 1/216*(9*b^2*d^2*x
^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x - 3*
a)/b^3
```

Mupad [B]
time = 1.22, size = 183, normalized size = 1.49

$$\frac{\frac{3\,d^2\,\sinh(a+b\,x)}{2}+\frac{d^2\,\sinh(3\,a+3\,b\,x)}{54}+\frac{3\,b^2\,c^2\,\sinh(a+b\,x)}{4}+\frac{b^2\,c^2\,\sinh(3\,a+3\,b\,x)}{12}+\frac{3\,b^2\,d^2\,x^2\,\sinh(a+b\,x)}{4}-\frac{b\,c\,d\,\cosh(3\,a+3\,b\,x)}{18}-\frac{3\,b\,d^2\,x\,\cosh(a+b\,x)}{2}+\frac{b^2\,d^2\,x^2\,\sinh(3\,a+3\,b\,x)}{12}-\frac{b\,d^2\,x\,\cosh(3\,a+3\,b\,x)}{18}-\frac{3\,b\,c\,d\,\cosh(a+b\,x)}{2}+\frac{b^2\,c\,d\,x\,\sinh(3\,a+3\,b\,x)}{6}+\frac{3\,b^2\,c\,d\,x\,\sinh(a+b\,x)}{2}}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^2,x)
```

```
[Out] ((3*d^2*sinh(a + b*x))/2 + (d^2*sinh(3*a + 3*b*x))/54 + (3*b^2*c^2*sinh(a +
 b*x))/4 + (b^2*c^2*sinh(3*a + 3*b*x))/12 + (3*b^2*d^2*x^2*sinh(a + b*x))/4
 - (b*c*d*cosh(3*a + 3*b*x))/18 - (3*b*d^2*x*cosh(a + b*x))/2 + (b^2*d^2*x^
2*sinh(3*a + 3*b*x))/12 - (b*d^2*x*cosh(3*a + 3*b*x))/18 - (3*b*c*d*cosh(a
+ b*x))/2 + (b^2*c*d*x*sinh(3*a + 3*b*x))/6 + (3*b^2*c*d*x*sinh(a + b*x))/2
)/b^3
```

3.19 $\int (c+dx)\cosh^3(a+bx)\,dx$

Optimal. Leaf size=75

$$-\frac{2d\cosh(a+bx)}{3b^2}-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{2(c+dx)\sinh(a+bx)}{3b}+\frac{(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{3b}$$

```
[Out] -2/3*d*cosh(b*x+a)/b^2-1/9*d*cosh(b*x+a)^3/b^2+2/3*(d*x+c)*sinh(b*x+a)/b+1/
3*(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3391, 3377, 2718}

$$-\frac{d\cosh^3(a+bx)}{9b^2}-\frac{2d\cosh(a+bx)}{3b^2}+\frac{2(c+dx)\sinh(a+bx)}{3b}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] (-2*d*Cosh[a + b*x])/(3*b^2) - (d*Cosh[a + b*x]^3)/(9*b^2) + (2*(c + d*x)*S
inh[a + b*x])/(3*b) + ((c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\int (c+dx)\cosh^3(a+bx)\,dx = -\frac{d\cosh^3(a+bx)}{9b^2} + \frac{(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{2}{3}\int (c+dx)\cosh($$
$$= -\frac{d\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\cosh^2(a+bx)\sinh(a+$$
$$= -\frac{2d\cosh(a+bx)}{3b^2} - \frac{d\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\cos}{}$$

Mathematica [A]
time = 0.17, size = 52, normalized size = 0.69

$$-\frac{27d\cosh(a+bx)+d\cosh(3(a+bx))-3b(c+dx)(9\sinh(a+bx)+\sinh(3(a+bx)))}{36b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Cosh[a + b*x]^3,x]
```

```
[Out] -1/36*(27*d*Cosh[a + b*x] + d*Cosh[3*(a + b*x)] - 3*b*(c + d*x)*(9*Sinh[a +
 b*x] + Sinh[3*(a + b*x)]))/b^2
```

Maple [A]
time = 1.34, size = 124, normalized size = 1.65

method	result
risch	$\frac{(3bdx+3bc-d)e^{3bx+3a}}{72b^2}+\frac{3(bdx+bc-d)e^{bx+a}}{8b^2}-\frac{3(bdx+bc+d)e^{-bx-a}}{8b^2}-\frac{(3bdx+3bc+d)e^{-3bx-3a}}{72b^2}$
default	$\frac{\frac{d((3bx+3a)\sinh(3bx+3a)-\cosh(3bx+3a))}{b}-\frac{3ad\sinh(3bx+3a)}{b}+3c\sinh(3bx+3a)}{36b}+\frac{\frac{3d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{4b}-\frac{3ad\sinh(bx+a)}{4b}+\frac{3c\,\mathrm{si}}{}}{b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/36/b*(1/b*d*((3*b*x+3*a)*sinh(3*b*x+3*a)-cosh(3*b*x+3*a))-3*a*d/b*sinh(3*
b*x+3*a)+3*c*sinh(3*b*x+3*a))+3/4/b*(1/b*d*((b*x+a)*sinh(b*x+a)-cosh(b*x+a)
)-a*d/b*sinh(b*x+a)+c*sinh(b*x+a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. 2(67) = 134.
time = 0.28, size = 143, normalized size = 1.91

$$\frac{1}{72}d\left(\frac{(3bxe^{(3a)}-e^{(3a)})e^{(3bx)}}{b^2}+\frac{27(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3bx+1)e^{(-3bx-3a)}}{b^2}\right)+\frac{1}{24}c\left(\frac{e^{(3bx+3a)}}{b}+\frac{9e^{(bx+a)}}{b}-\frac{9e^{(-bx-a)}}{b}-\frac{e^{(-3bx-3a)}}{b}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")

[Out] 1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2) + 1/24*c*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*b*x - 3*a)/b)

Fricas [A]
time = 0.39, size = 95, normalized size = 1.27

$$-\frac{d\cosh\left(bx+a\right)^{3}+3\,d\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-3\left(bdx+bc\right)\sinh\left(bx+a\right)^{3}+27\,d\cosh\left(bx+a\right)-9\left(3\,bdx+\left(bdx+bc\right)\cosh\left(bx+a\right)^{2}+3\,bc\right)\sinh\left(bx+a\right)}{36\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")

[Out] -1/36*(d*cosh(b*x + a)^3 + 3*d*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*d*x + b*c)*sinh(b*x + a)^3 + 27*d*cosh(b*x + a) - 9*(3*b*d*x + (b*d*x + b*c)*cosh(b*x + a)^2 + 3*b*c)*sinh(b*x + a))/b^2

Sympy [A]
time = 0.21, size = 126, normalized size = 1.68

$$\begin{cases} -\frac{2c\sinh^{3}\left(a+bx\right)}{3b}+\frac{c\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{b}-\frac{2dx\sinh^{3}\left(a+bx\right)}{3b}+\frac{dx\sinh\left(a+bx\right)\cosh^{2}\left(a+bx\right)}{b}+\frac{2d\sinh^{2}\left(a+bx\right)\cosh\left(a+bx\right)}{3b^{2}}-\frac{7d\cosh^{3}\left(a+bx\right)}{9b^{2}} & \text{for}\: b\neq 0 \\ \left(cx+\frac{dx^{2}}{2}\right)\cosh^{3}\left(a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cosh(b*x+a)**3,x)

[Out] Piecewise((-2*c*sinh(a + b*x)**3/(3*b) + c*sinh(a + b*x)*cosh(a + b*x)**2/b - 2*d*x*sinh(a + b*x)**3/(3*b) + d*x*sinh(a + b*x)*cosh(a + b*x)**2/b + 2*d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 7*d*cosh(a + b*x)**3/(9*b**2), Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**3, True))

Giac [A]
time = 0.41, size = 98, normalized size = 1.31

$$\frac{(3\,bdx+3\,bc-d)e^{(3\,bx+3\,a)}}{72\,b^{2}}+\frac{3\,(bdx+bc-d)e^{(bx+a)}}{8\,b^{2}}-\frac{3\,(bdx+bc+d)e^{(-bx-a)}}{8\,b^{2}}-\frac{(3\,bdx+3\,bc+d)e^{(-3\,bx-3\,a)}}{72\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")

[Out] 1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 + 3/8*(b*d*x + b*c - d)*e^(b*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 - 1/72*(3*b*d*x + 3*b*c + d)*e^(-3*b*x - 3*a)/b^2

Mupad [B]
time = 0.22, size = 77, normalized size = 1.03

$$\frac{\frac{3\,c\,\sinh(a+b\,x)}{4}+\frac{c\,\sinh(3\,a+3\,b\,x)}{12}+\frac{d\,x\,\sinh(3\,a+3\,b\,x)}{12}+\frac{3\,d\,x\,\sinh(a+b\,x)}{4}}{b}-\frac{d\cosh(3\,a+3\,b\,x)}{36\,b^2}-\frac{3\,d\cosh(a+b\,x)}{4\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x),x)
```

```
[Out] ((3*c*sinh(a + b*x))/4 + (c*sinh(3*a + 3*b*x))/12 + (d*x*sinh(3*a + 3*b*x))
/12 + (3*d*x*sinh(a + b*x))/4)/b - (d*cosh(3*a + 3*b*x))/(36*b^2) - (3*d*co
sh(a + b*x))/(4*b^2)
```

3.20 $\int \frac{\cosh^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=121

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\sinh\left(3a-\frac{3bc}{d}\right.$$

```
[Out] 1/4*Chi(3*b*c/d+3*b*x)*cosh(3*a-3*b*c/d)/d+3/4*Chi(b*c/d+b*x)*cosh(a-b*c/d)
/d+1/4*Shi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d+3/4*Shi(b*c/d+b*x)*sinh(a-b*c
/d)/d
```

Rubi [A]
time = 0.18, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3384, 3379, 3382}

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sinh\left(3a-\frac{3bc}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*
c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d) + (3*Sinh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/(4*d) + (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d
 + 3*b*x])/(4*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(a+bx)}{c+dx}\,dx &= \int \left(\frac{3\cosh(a+bx)}{4(c+dx)}+\frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx\\ &= \frac{1}{4}\int \frac{\cosh(3a+3bx)}{c+dx}\,dx+\frac{3}{4}\int \frac{\cosh(a+bx)}{c+dx}\,dx\\ &= \frac{1}{4}\cosh\left(3a-\frac{3bc}{d}\right)\int \frac{\cosh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx+\frac{1}{4}\left(3\cosh\left(a-\frac{bc}{d}\right)\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}\\ &= \frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}(}{4d}\end{aligned}$$

Mathematica [A]
time = 0.17, size = 102, normalized size = 0.84

$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)+3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3b(c+dx)}{d}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x),x]
```

```
[Out] (3*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + Cosh[3*a - (3*b*c)/d]*Cosh
Integral[(3*b*(c + d*x))/d] + 3*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)]
 + Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Maple [A]
time = 2.92, size = 166, normalized size = 1.37

method	result
risch	$-\frac{e^{-\frac{3(ad-bc)}{d}}\,\text{expIntegral}\left(1,3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d}-\frac{3\,e^{-\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{8d}-\frac{3\,e^{\frac{ad-bc}{d}}\,\text{expIntegral}\left(1,-bx-a-\right.}{8d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b*
c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)
/d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 117, normalized size = 0.97

$$-\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_1\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_1\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a +
b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 3/8*e^(a - b*c/d)*exp_integral_
e(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x +
c)*b/d)/d
```

Fricas [A]
time = 0.38, size = 186, normalized size = 1.54

$$\frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/8*(3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) +
(Ei(3*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) + 3
*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) + (Ei(3*
(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cosh^3\left(a+bx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.43, size = 112, normalized size = 0.93

$$\frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}+3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*Ei((b*d*x + b*c)/d)*e^(a -
 b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + Ei(-3*(b*d*x + b*c)/d)*e^
(-3*a + 3*b*c/d))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x),x)
```

```
[Out] int(cosh(a + b*x)^3/(c + d*x), x)
```

3.21 $\int \frac{\cosh^3(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=145

$$-\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{3b\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d^2}+\frac{3b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d^2}+\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}\right.}{4d^2}$$

```
[Out] -cosh(b*x+a)^3/d/(d*x+c)+3/4*b*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^2+3/4*b*cosh(
3*a-3*b*c/d)*Shi(3*b*c/d+3*b*x)/d^2+3/4*b*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c
/d)/d^2+3/4*b*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^2
```

Rubi [A]
time = 0.18, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3394, 3384, 3379, 3382}

$$\frac{3b\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}+\frac{3b\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2}+\frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}-\frac{\cosh^3(a+bx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] -(Cosh[a + b*x]^3/(d*(c + d*x))) + (3*b*CoshIntegral[(3*b*c)/d + 3*b*x]*Sin
h[3*a - (3*b*c)/d])/(4*d^2) + (3*b*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*
c)/d])/(4*d^2) + (3*b*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(4*d^2
) + (3*b*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(4*d^2)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx &= -\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{(3ib)\int\left(-\frac{i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{d}\\
&= -\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{(3b)\int\frac{\sinh(a+bx)}{c+dx}\,dx}{4d}+\frac{(3b)\int\frac{\sinh(3a+3bx)}{c+dx}\,dx}{4d}\\
&= -\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{\left(3b\cosh\left(3a-\frac{3bc}{d}\right)\right)\int\frac{\sinh\left(\frac{3bc}{d}+3bx\right)}{c+dx}\,dx}{4d}+\frac{\left(3b\cosh\left(a-\frac{bc}{d}\right)\right)\int\frac{\sinh\left(\frac{b}{c}\right.}{c+}}{4d}\\
&= -\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{3b\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d^2}+\frac{3b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d^2}+
\end{aligned}$$

Mathematica [A]
time = 0.36, size = 196, normalized size = 1.35

$$-\frac{3\cosh(a)\cosh(bx)}{4d(c+dx)}-\frac{\cosh(3a)\cosh(3bx)}{4d(c+dx)}-\frac{3\sinh(a)\sinh(bx)}{4d(c+dx)}-\frac{\sinh(3a)\sinh(3bx)}{4d(c+dx)}-\frac{3b\left(-2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)-2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)-2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)-2\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)\right)}{8d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] (-3*Cosh[a]*Cosh[b*x])/(4*d*(c + d*x)) - (Cosh[3*a]*Cosh[3*b*x])/(4*d*(c +
d*x)) - (3*Sinh[a]*Sinh[b*x])/(4*d*(c + d*x)) - (Sinh[3*a]*Sinh[3*b*x])/(4*
d*(c + d*x)) - (3*b*(-2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*c)/
d] - 2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] - 2*Cosh[a - (b*c)/d]*
SinhIntegral[(b*c)/d + b*x] - 2*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/
d + 3*b*x]))/(8*d^2)
```

Maple [A]
time = 2.95, size = 271, normalized size = 1.87

method	result

risch	$-\frac{b\,e^{-3bx-3a}}{8d(bdx+bc)}+\frac{3b\,e^{-\frac{3(ad-bc)}{d}}\operatorname{expIntegral}\left(1,3bx+3a-\frac{3(ad-bc)}{d}\right)}{8d^2}-\frac{3b\,e^{-bx-a}}{8d(bdx+bc)}+\frac{3b\,e^{-\frac{ad-bc}{d}}\operatorname{expIntegral}\left(1,bx+a-\frac{ad-bc}{d}\right)}{8d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*b*exp(-3*b*x-3*a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*b
*x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d-b
*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)-3/8*b/d^2*e
xp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c/d+b
*x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Maxima [A]
time = 0.32, size = 145, normalized size = 1.00

$$-\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}-\frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d}-\frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d}-\frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) -
3/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - 3/8*e^(
a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a - 3
*b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(137) = 274$.
time = 0.35, size = 305, normalized size = 2.10

$$-\frac{2\,d\cosh\left(bx+a\right)^3+6\,d\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+6\,d\cosh\left(bx+a\right)-3\left((bdx+bc){\rm Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc){\rm Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc){\rm Ei}\left(\frac{3\,(bdx+bc)}{d}\right)-(bdx+bc){\rm Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)-3\left((bdx+bc){\rm Ei}\left(\frac{bdx+bc}{d}\right)+(bdx+bc){\rm Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)-3\left((bdx+bc){\rm Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+(bdx+bc){\rm Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{3\,(bc-ad)}{d}\right)}{8\,(d^3x+cd^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/8*(2*d*cosh(b*x + a)^3 + 6*d*cosh(b*x + a)*sinh(b*x + a)^2 + 6*d*cosh(b*
x + a) - 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x +
b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) - (b
*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) - 3*((b*d*x + b*
c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a
*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3*(b*d*
x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(137) = 274$.
time = 0.50, size = 1075, normalized size = 7.41

$$-\frac{\left(3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{3(bc-ad)}{d}\right)}+3b^3c\,\mathrm{Ei}\left(-\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{3(bc-ad)}{d}\right)}-3ab^2d\,\mathrm{Ei}\left(-\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{3(bc-ad)}{d}\right)}+3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+3b^3c\,\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}-3ab^2d\,\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}-3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-3b^3c\,\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+3ab^2d\,\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}-3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{3(bc-ad)}{d}\right)}-3b^3c\,\mathrm{Ei}\left(\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{3(bc-ad)}{d}\right)}+3ab^2d\,\mathrm{Ei}\left(\frac{3\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(-\frac{3(bc-ad)}{d}\right)}+b^2de^{\left(\frac{3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+3b^2de^{\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+3b^2de^{\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}+b^2de^{\left(-\frac{3(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right)}\right)d^2}{8\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)d^4+bcd^4-ad^5\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-3*((d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) + 3
*b^3*c*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)
*e^(3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/
(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)
) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 3*b^3*c*Ei(-((d*x + c)*(b - b*c/(d*x
+ c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) - 3*a*b^2*d*Ei(-((d
*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/
d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(((d*x + c)*(b -
 b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - 3*b^3*
c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*
c - a*d)/d) + 3*a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
 b*c - a*d)/d)*e^(-(b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))*b^2*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*
d)/d)*e^(-3*(b*c - a*d)/d) - 3*b^3*c*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a
*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) + 3*a*b^2*d*Ei(3*((d*x +
 c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d
) + b^2*d*e^(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 3*b^2*d*e
^((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + 3*b^2*d*e^(-(d*x + c)*
(b - b*c/(d*x + c) + a*d/(d*x + c))/d) + b^2*d*e^(-3*(d*x + c)*(b - b*c/(d*
x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^2,x)
[Out] int(cosh(a + b*x)^3/(c + d*x)^2, x)
```

3.22 $\int \frac{\cosh^3(a+bx)}{(c+dx)^3} \, dx$

Optimal. Leaf size=184

$$-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}+\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b\cosh^2(a+bx)\sinh(}{2d^2(c+dx)}$$

```
[Out] 9/8*b^2*Chi(3*b*c/d+3*b*x)*cosh(3*a-3*b*c/d)/d^3+3/8*b^2*Chi(b*c/d+b*x)*cos
h(a-b*c/d)/d^3-1/2*cosh(b*x+a)^3/d/(d*x+c)^2+9/8*b^2*Shi(3*b*c/d+3*b*x)*sin
h(3*a-3*b*c/d)/d^3+3/8*b^2*Shi(b*c/d+b*x)*sinh(a-b*c/d)/d^3-3/2*b*cosh(b*x+
a)^2*sinh(b*x+a)/d^2/(d*x+c)
```

Rubi [A]

time = 0.24, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3395, 3384, 3379, 3382, 3393}

$$\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}+\frac{3b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}+\frac{9b^2\sinh\left(3a-\frac{3bc}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] -1/2*Cosh[a + b*x]^3/(d*(c + d*x)^2) + (3*b^2*Cosh[a - (b*c)/d]*CoshIntegra
l[(b*c)/d + b*x])/(8*d^3) + (9*b^2*Cosh[3*a - (3*b*c)/d]*CoshIntegral[(3*b*
c)/d + 3*b*x])/(8*d^3) - (3*b*Cosh[a + b*x]^2*Sinh[a + b*x])/(2*d^2*(c + d*
x)) + (3*b^2*Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/(8*d^3) + (9*b^
2*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/(8*d^3)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx &= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} - \frac{(3b^2)\int\frac{\cosh(a+bx)}{c+dx}\,dx}{d^2} + \frac{(9b^2)\int \frac{\cos}{2}\\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} + \frac{(9b^2)\int\left(\frac{3\cosh(a+bx)}{4(c+dx)} + \frac{\cosh(3a+3bx)}{4(c+dx)}\right.}{2d^2}\\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} -\\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} - \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{d^3} - \frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)} -\\
&= -\frac{\cosh^3(a+bx)}{2d(c+dx)^2} + \frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3} + \frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Chi}\left(\frac{3bc}{d}+3b\right.}{8d^3}
\end{aligned}$$

Mathematica [A]

time = 0.61, size = 218, normalized size = 1.18

$$-\frac{6d\cosh(bx)(d\cosh(a)+b(c+dx)\sinh(a))+2d\cosh(3bx)(d\cosh(3a)+3b(c+dx)\sinh(3a))+6d(b(c+dx)\cosh(a)+d\sinh(a))\sinh(bx)+2d(3b(c+dx)\cosh(3a)+d\sinh(3a))\sinh(3bx)-6b^2(c+dx)^2\left(\cosh\left(a-\frac{bc}{d}\right)\operatorname{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+3\cosh\left(3a-\frac{3bc}{d}\right)\operatorname{Chi}\left(\frac{3b(c+dx)}{d}\right)+\sinh\left(a-\frac{bc}{d}\right)\operatorname{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+3\sinh\left(3a-\frac{3bc}{d}\right)\operatorname{Shi}\left(\frac{3b(c+dx)}{d}\right)\right)}{16d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^3,x]
```

```
[Out] -1/16*(6*d*Cosh[b*x]*(d*Cosh[a] + b*(c + d*x)*Sinh[a]) + 2*d*Cosh[3*b*x]*(d
*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a]) + 6*d*(b*(c + d*x)*Cosh[a] + d*Sinh[a
])*Sinh[b*x] + 2*d*(3*b*(c + d*x)*Cosh[3*a] + d*Sinh[3*a])*Sinh[3*b*x] - 6*
b^2*(c + d*x)^2*(Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + 3*Cosh[3*a -
 (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d] + Sinh[a - (b*c)/d]*SinhIntegra
l[b*(c/d + x)] + 3*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d]))/
(d^3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(172) = 344$.
time = 2.92, size = 562, normalized size = 3.05

method	result
risch	$\frac{3b^3 e^{-3bx-3a} x}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} + \frac{3b^3 e^{-3bx-3a} c}{16d^2(b^2d^2x^2+2b^2cdx+b^2c^2)} - \frac{b^2 e^{-3bx-3a}}{16d(b^2d^2x^2+2b^2cdx+b^2c^2)} - \frac{9b^2 e^{-\frac{3(ad-bc)}{d}} \operatorname{expIntegral}\left(1,3bx+3\right.}{16d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+3/16*b^3*exp
(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/16*b^2*exp(-3*b*x-3*
a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)*Ei(
1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+
b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-3/1
6*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*exp(-(a*
d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-3/1
6*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-
(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/d^3*exp(3*
b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*
d+b*c)/d)
```

Maxima [A]
time = 0.34, size = 145, normalized size = 0.79

$$-\frac{e^{\left(-3a+\frac{3bc}{d}\right)}E_3\left(\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2 d} - \frac{3e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8(dx+c)^2 d} - \frac{3e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8(dx+c)^2 d} - \frac{e^{\left(3a-\frac{3bc}{d}\right)}E_3\left(-\frac{3(dx+c)b}{d}\right)}{8(dx+c)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d)
- 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - 3/8
*e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e^(3
*a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(172) = 344$.
time = 0.38, size = 527, normalized size = 2.86

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*d^2*cosh(b*x + a)^3 + 6*d^2*cosh(b*x + a)*sinh(b*x + a)^2 + 6*(b*d
^2*x + b*c*d)*sinh(b*x + a)^3 + 6*d^2*cosh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*
c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c
*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) + 6*(b*d^2*x + b*c*d + 3*
(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a) - 3*((b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*
Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*
d^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**3,x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(172) = 344$.
time = 0.44, size = 602, normalized size = 3.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*d^2*x^2
*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e^(
-a + b*c/d) + 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 18*
```

```
b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 6*b^2*c*d*x*Ei((b*d*x +
 b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) +
18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei(3*(b*
d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/
d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 9*b^2*c^2*Ei(-3*(b*d*x
 + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) - 3*b*d^2*x*e^(b*
x + a) + 3*b*d^2*x*e^(-b*x - a) + 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*b*c*d*e^(3
*b*x + 3*a) - 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) + 3*b*c*d*e^(-3*b*
x - 3*a) - d^2*e^(3*b*x + 3*a) - 3*d^2*e^(b*x + a) - 3*d^2*e^(-b*x - a) - d
^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^3,x)
```

```
[Out] int(cosh(a + b*x)^3/(c + d*x)^3, x)
```

3.23 $\int x^3 \cosh^4(a+bx)\,dx$

Optimal. Leaf size=172

$$\frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{45x\cosh(a+bx)}{64b^3}$$

```
[Out] 45/128*x^2/b^2+3/32*x^4-45/128*cosh(b*x+a)^2/b^4-9/16*x^2*cosh(b*x+a)^2/b^2
-3/128*cosh(b*x+a)^4/b^4-3/16*x^2*cosh(b*x+a)^4/b^2+45/64*x*cosh(b*x+a)*sin
h(b*x+a)/b^3+3/8*x^3*cosh(b*x+a)*sinh(b*x+a)/b+3/32*x*cosh(b*x+a)^3*sinh(b*
x+a)/b^3+1/4*x^3*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A]
time = 0.10, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3392, 30, 3391}

$$-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{45\cosh^2(a+bx)}{128b^4}+\frac{3x\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{45x\sinh(a+bx)\cosh(a+bx)}{64b^3}-\frac{3x^2\cosh^4(a+bx)}{16b^2}-\frac{9x^2\cosh^2(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3x^3\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{45x^2}{128b^2}+\frac{3x^4}{32}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Cosh[a + b*x]^4,x]
```

```
[Out] (45*x^2)/(128*b^2) + (3*x^4)/32 - (45*Cosh[a + b*x]^2)/(128*b^4) - (9*x^2*C
osh[a + b*x]^2)/(16*b^2) - (3*Cosh[a + b*x]^4)/(128*b^4) - (3*x^2*Cosh[a +
b*x]^4)/(16*b^2) + (45*x*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^3*Cos
h[a + b*x]*Sinh[a + b*x])/(8*b) + (3*x*Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b
^3) + (x^3*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
```

```
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\begin{aligned}
\int x^3\cosh^4(a+bx)\,dx &= -\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int x^3\cosh^2(a+bx)\,dx+ \\
&= -\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3x^3\cosh(a+bx)\sinh}{8b} \\
&= \frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh^4(a+bx)}{16b^2} \\
&= \frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4}-\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4}-\frac{3x^2\cosh}{1}
\end{aligned}$$

Mathematica [A]
time = 0.30, size = 100, normalized size = 0.58

$$\frac{-192(1+2b^2x^2)\cosh(2(a+bx))-3(1+8b^2x^2)\cosh(4(a+bx))+4bx(24b^3x^3+32(3+2b^2x^2)\sinh(2(a+bx))+(3+8b^2x^2)\sinh(4(a+bx)))}{1024b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Cosh[a + b*x]^4,x]
[Out] (-192*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*x
)] + 4*b*x*(24*b^3*x^3 + 32*(3 + 2*b^2*x^2)*Sinh[2*(a + b*x)] + (3 + 8*b^2*
x^2)*Sinh[4*(a + b*x)]))/(1024*b^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(152) = 304$.
time = 1.31, size = 367, normalized size = 2.13

method	result
risch	$\frac{3x^4}{32}+\frac{(32b^3x^3-24b^2x^2+12bx-3)e^{4bx+4a}}{2048b^4}+\frac{(4b^3x^3-6b^2x^2+6bx-3)e^{2bx+2a}}{32b^4}-\frac{(4b^3x^3+6b^2x^2+6bx+3)e^{-2bx-2a}}{32b^4}-\frac{(32b^3x^3+}{}$
default	$\frac{3x^4}{32}+\frac{(2bx+2a)^3\sinh(2bx+2a)-3(2bx+2a)^2\cosh(2bx+2a)+6(2bx+2a)\sinh(2bx+2a)-6\cosh(2bx+2a)-6a\left((2bx+2a)^2\sinh(2bx+\right.}{32b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 3/32*x^4+1/32/b^4*((2*b*x+2*a)^3*sinh(2*b*x+2*a)-3*(2*b*x+2*a)^2*cosh(2*b*x
+2*a)+6*(2*b*x+2*a)*sinh(2*b*x+2*a)-6*cosh(2*b*x+2*a)-6*a*((2*b*x+2*a)^2*si
nh(2*b*x+2*a)-2*(2*b*x+2*a)*cosh(2*b*x+2*a)+2*sinh(2*b*x+2*a))+12*a^2*((2*b
```

```
*x+2*a)*sinh(2*b*x+2*a)-cosh(2*b*x+2*a))-8*a^3*sinh(2*b*x+2*a))+1/2048/b^4*
((4*b*x+4*a)^3*sinh(4*b*x+4*a)-3*(4*b*x+4*a)^2*cosh(4*b*x+4*a)+6*(4*b*x+4*a
)*sinh(4*b*x+4*a)-6*cosh(4*b*x+4*a)-12*a*((4*b*x+4*a)^2*sinh(4*b*x+4*a)-2*(
4*b*x+4*a)*cosh(4*b*x+4*a)+2*sinh(4*b*x+4*a))+48*a^2*((4*b*x+4*a)*sinh(4*b*
x+4*a)-cosh(4*b*x+4*a))-64*a^3*sinh(4*b*x+4*a))
```

Maxima [A]
time = 0.27, size = 176, normalized size = 1.02

$$\frac{3}{32}\,x^{4}+\frac{\left(32\,b^{3}x^{3}e^{(4\,a)}-24\,b^{2}x^{2}e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^{4}}+\frac{\left(4\,b^{3}x^{3}e^{(2\,a)}-6\,b^{2}x^{2}e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^{4}}-\frac{\left(4\,b^{3}x^{3}+6\,b^{2}x^{2}+6\,bx+3\right)e^{(-2\,bx-2\,a)}}{32\,b^{4}}-\frac{\left(32\,b^{3}x^{3}+24\,b^{2}x^{2}+12\,bx+3\right)e^{(-4\,bx-4\,a)}}{2048\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a)
 - 3*e^(4*a))*e^(4*b*x)/b^4 + 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) +
 6*b*x*e^(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/32*(4*b^3*x^3 + 6*b^2*x^2 + 6
*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b*x +
 3)*e^(-4*b*x - 4*a)/b^4
```

Fricas [A]
time = 0.35, size = 195, normalized size = 1.13

$$\frac{96\,b^{4}x^{4}-3\,\left(8\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{4}+16\,\left(8\,b^{3}x^{3}+3\,bx\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{3}-3\,\left(8\,b^{2}x^{2}+1\right)\sinh\left(bx+a\right)^{4}-192\,\left(2\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{2}-6\,\left(64\,b^{2}x^{2}+3\,\left(8\,b^{2}x^{2}+1\right)\cosh\left(bx+a\right)^{2}+32\right)\sinh\left(bx+a\right)^{2}+16\,\left(\left(8\,b^{3}x^{3}+3\,bx\right)\cosh\left(bx+a\right)^{3}+16\,\left(2\,b^{3}x^{3}+3\,bx\right)\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{1024\,b^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/1024*(96*b^4*x^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 16*(8*b^3*x^3 + 3*
b*x)*cosh(b*x + a)*sinh(b*x + a)^3 - 3*(8*b^2*x^2 + 1)*sinh(b*x + a)^4 - 19
2*(2*b^2*x^2 + 1)*cosh(b*x + a)^2 - 6*(64*b^2*x^2 + 3*(8*b^2*x^2 + 1)*cosh(
b*x + a)^2 + 32)*sinh(b*x + a)^2 + 16*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a)^3
+ 16*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A]
time = 0.66, size = 253, normalized size = 1.47

$$\begin{cases} \frac{3x^{4}\sinh^{4}{\left(a+bx\right)}}{32}-\frac{3x^{4}\sinh^{2}{\left(a+bx\right)}\cosh^{2}{\left(a+bx\right)}}{16}+\frac{3x^{4}\cosh^{4}{\left(a+bx\right)}}{32}-\frac{3x^{3}\sinh^{3}{\left(a+bx\right)}\cosh{\left(a+bx\right)}}{8b}+\frac{5x^{3}\sinh{\left(a+bx\right)}\cosh^{3}{\left(a+bx\right)}}{8b}+\frac{45x^{2}\sinh^{4}{\left(a+bx\right)}}{128b^{2}}-\frac{9x^{2}\sinh^{2}{\left(a+bx\right)}\cosh^{2}{\left(a+bx\right)}}{64b^{2}}-\frac{51x^{2}\cosh^{4}{\left(a+bx\right)}}{128b^{2}}-\frac{45x\sinh^{3}{\left(a+bx\right)}\cosh{\left(a+bx\right)}}{64b^{3}}+\frac{51x\sinh{\left(a+bx\right)}\cosh^{3}{\left(a+bx\right)}}{64b^{3}}+\frac{45\sinh^{4}{\left(a+bx\right)}}{256b^{4}}-\frac{51\cosh^{4}{\left(a+bx\right)}}{256b^{4}} & \text{for}\: b\neq 0 \\ \frac{x^{4}\cosh^{4}{\left(a\right)}}{4} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**4*sinh(a + b*x)**4/32 - 3*x**4*sinh(a + b*x)**2*cosh(a + b*
x)**2/16 + 3*x**4*cosh(a + b*x)**4/32 - 3*x**3*sinh(a + b*x)**3*cosh(a + b*
x)/(8*b) + 5*x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 45*x**2*sinh(a + b
*x)**4/(128*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2) - 51
```

```
*x**2*cosh(a + b*x)**4/(128*b**2) - 45*x*sinh(a + b*x)**3*cosh(a + b*x)/(64
*b**3) + 51*x*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) + 45*sinh(a + b*x)**
4/(256*b**4) - 51*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cosh(a)**4/
4, True))
```

Giac [A]
time = 0.41, size = 150, normalized size = 0.87

$$\frac{3}{32}\,x^4 + \frac{(32\,b^3x^3 - 24\,b^2x^2 + 12\,bx - 3)e^{(4\,bx+4\,a)}}{2048\,b^4} + \frac{(4\,b^3x^3 - 6\,b^2x^2 + 6\,bx - 3)e^{(2\,bx+2\,a)}}{32\,b^4} - \frac{(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3)e^{(-2\,bx-2\,a)}}{32\,b^4} - \frac{(32\,b^3x^3 + 24\,b^2x^2 + 12\,bx + 3)e^{(-4\,bx-4\,a)}}{2048\,b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b^
4 + 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/32*(4*
b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x^3
+ 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Mupad [B]
time = 0.37, size = 129, normalized size = 0.75

$$\frac{3\,x^4}{32} - \frac{\frac{3\cosh(2\,a+2\,b\,x)}{16} + \frac{3\cosh(4\,a+4\,b\,x)}{1024} + b^2\left(\frac{3\,x^2\cosh(2\,a+2\,b\,x)}{8} + \frac{3\,x^2\cosh(4\,a+4\,b\,x)}{128}\right) - b\left(\frac{3\,x\sinh(2\,a+2\,b\,x)}{8} + \frac{3\,x\sinh(4\,a+4\,b\,x)}{256}\right) - b^3\left(\frac{x^3\sinh(2\,a+2\,b\,x)}{4} + \frac{x^3\sinh(4\,a+4\,b\,x)}{32}\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*cosh(a + b*x)^4,x)
```

```
[Out] (3*x^4)/32 - ((3*cosh(2*a + 2*b*x))/16 + (3*cosh(4*a + 4*b*x))/1024 + b^2*(
(3*x^2*cosh(2*a + 2*b*x))/8 + (3*x^2*cosh(4*a + 4*b*x))/128) - b*((3*x*sinh
(2*a + 2*b*x))/8 + (3*x*sinh(4*a + 4*b*x))/256) - b^3*((x^3*sinh(2*a + 2*b*
x))/4 + (x^3*sinh(4*a + 4*b*x))/32))/b^4
```

3.24 $\int x^2 \cosh^4(a+bx)\,dx$

Optimal. Leaf size=134

$$\frac{15x}{64b^2}+\frac{x^3}{8}-\frac{3x\cosh^2(a+bx)}{8b^2}-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3}+\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{8b}$$

```
[Out] 15/64*x/b^2+1/8*x^3-3/8*x*cosh(b*x+a)^2/b^2-1/8*x*cosh(b*x+a)^4/b^2+15/64*c
osh(b*x+a)*sinh(b*x+a)/b^3+3/8*x^2*cosh(b*x+a)*sinh(b*x+a)/b+1/32*cosh(b*x+
a)^3*sinh(b*x+a)/b^3+1/4*x^2*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A]
time = 0.07, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3392, 30, 2715, 8}

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{32b^3}+\frac{15\sinh(a+bx)\cosh(a+bx)}{64b^3}-\frac{x\cosh^4(a+bx)}{8b^2}-\frac{3x\cosh^2(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3x^2\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{15x}{64b^2}+\frac{x^3}{8}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Cosh[a + b*x]^4,x]
```

```
[Out] (15*x)/(64*b^2) + x^3/8 - (3*x*Cosh[a + b*x]^2)/(8*b^2) - (x*Cosh[a + b*x]^
4)/(8*b^2) + (15*Cosh[a + b*x]*Sinh[a + b*x])/(64*b^3) + (3*x^2*Cosh[a + b*
x]*Sinh[a + b*x])/(8*b) + (Cosh[a + b*x]^3*Sinh[a + b*x])/(32*b^3) + (x^2*C
osh[a + b*x]^3*Sinh[a + b*x])/(4*b)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
```

```
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rubi steps

$$\int x^2\cosh^4(a+bx)\,dx = -\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int x^2\cosh^2(a+bx)\,dx+\frac{\int c$$
$$= -\frac{3x\cosh^2(a+bx)}{8b^2}-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+$$
$$= \frac{x^3}{8}-\frac{3x\cosh^2(a+bx)}{8b^2}-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3}+\frac{3x^2\cos$$
$$= \frac{15x}{64b^2}+\frac{x^3}{8}-\frac{3x\cosh^2(a+bx)}{8b^2}-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3}+$$

Mathematica [A]
time = 0.12, size = 90, normalized size = 0.67

$$\frac{32b^3x^3-64bx\cosh(2(a+bx))-4bx\cosh(4(a+bx))+32\sinh(2(a+bx))+64b^2x^2\sinh(2(a+bx))+\sinh(4(a+bx))+8b^2x^2\sinh(4(a+bx))}{256b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Cosh[a + b*x]^4,x]
[Out] (32*b^3*x^3 - 64*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 32*Sinh[
2*(a + b*x)] + 64*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x^2
*Sinh[4*(a + b*x)])/(256*b^3)
```

Maple [A]
time = 1.26, size = 213, normalized size = 1.59

method	result
risch	$\frac{x^3}{8}+\frac{(8b^2x^2-4bx+1)\,\mathrm{e}^{4bx+4a}}{512b^3}+\frac{(2b^2x^2-2bx+1)\,\mathrm{e}^{2bx+2a}}{16b^3}-\frac{(2b^2x^2+2bx+1)\,\mathrm{e}^{-2bx-2a}}{16b^3}-\frac{(8b^2x^2+4bx+1)\,\mathrm{e}^{-4bx-4a}}{512b^3}$
default	$\frac{x^3}{8}+\frac{(2bx+2a)^2\sinh(2bx+2a)-2(2bx+2a)\cosh(2bx+2a)+2\sinh(2bx+2a)-4a((2bx+2a)\sinh(2bx+2a)-\cosh(2bx+2a))+4a^2\sinh(2}{16b^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
[Out] 1/8*x^3+1/16/b^3*((2*b*x+2*a)^2*sinh(2*b*x+2*a)-2*(2*b*x+2*a)*cosh(2*b*x+2*
a)+2*sinh(2*b*x+2*a)-4*a*((2*b*x+2*a)*sinh(2*b*x+2*a)-cosh(2*b*x+2*a))+4*a^
2*sinh(2*b*x+2*a))+1/512/b^3*((4*b*x+4*a)^2*sinh(4*b*x+4*a)-2*(4*b*x+4*a)*c
```

```
osh(4*b*x+4*a)+2*sinh(4*b*x+4*a)-8*a*((4*b*x+4*a)*sinh(4*b*x+4*a)-cosh(4*b*
x+4*a))+16*a^2*sinh(4*b*x+4*a))
```

Maxima [A]
time = 0.27, size = 132, normalized size = 0.99

$$\frac{1}{8}\,x^3+\frac{\left(8\,b^2x^2e^{(4\,a)}-4\,bxe^{(4\,a)}+e^{(4\,a)}\right)e^{(4\,bx)}}{512\,b^3}+\frac{\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 1/8*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^3
 + 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/16*
(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1
)*e^(-4*b*x - 4*a)/b^3
```

Fricas [A]
time = 0.34, size = 147, normalized size = 1.10

$$\frac{8\,b^3x^3-bx\cosh\left(bx+a\right)^4-bx\sinh\left(bx+a\right)^4+\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-16\,bx\cosh\left(bx+a\right)^2-2\left(3\,bx\cosh\left(bx+a\right)^2+8\,bx\right)\sinh\left(bx+a\right)^2+\left(\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)^3+16\left(2\,b^2x^2+1\right)\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{64\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/64*(8*b^3*x^3 - b*x*cosh(b*x + a)^4 - b*x*sinh(b*x + a)^4 + (8*b^2*x^2 +
1)*cosh(b*x + a)*sinh(b*x + a)^3 - 16*b*x*cosh(b*x + a)^2 - 2*(3*b*x*cosh(b
*x + a)^2 + 8*b*x)*sinh(b*x + a)^2 + ((8*b^2*x^2 + 1)*cosh(b*x + a)^3 + 16*
(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A]
time = 0.45, size = 209, normalized size = 1.56

$$\begin{cases} \frac{x^3\sinh^4(a+bx)}{8}-\frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{4}+\frac{x^3\cosh^4(a+bx)}{8}-\frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{5x^2\sinh(a+bx)\cosh^3(a+bx)}{8b}+\frac{15x\sinh^4(a+bx)}{64b^2}-\frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{32b^2}-\frac{17x\cosh^4(a+bx)}{64b^2}-\frac{15\sinh^3(a+bx)\cosh(a+bx)}{64b^3}+\frac{17\sinh(a+bx)\cosh^3(a+bx)}{64b^3} & \text{for}\ b\neq 0 \\ \frac{x^3\cosh^4(a)}{3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((x**3*sinh(a + b*x)**4/8 - x**3*sinh(a + b*x)**2*cosh(a + b*x)**2
/4 + x**3*cosh(a + b*x)**4/8 - 3*x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b)
+ 5*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 15*x*sinh(a + b*x)**4/(64*b
**2) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - 17*x*cosh(a + b*x)
**4/(64*b**2) - 15*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 17*sinh(a + b
*x)*cosh(a + b*x)**3/(64*b**3), Ne(b, 0)), (x**3*cosh(a)**4/3, True))
```

Giac [A]
time = 0.41, size = 118, normalized size = 0.88

$$\frac{1}{8}\,x^3+\frac{\left(8\,b^2x^2-4\,bx+1\right)e^{(4\,bx+4\,a)}}{512\,b^3}+\frac{\left(2\,b^2x^2-2\,bx+1\right)e^{(2\,bx+2\,a)}}{16\,b^3}-\frac{\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{16\,b^3}-\frac{\left(8\,b^2x^2+4\,bx+1\right)e^{(-4\,bx-4\,a)}}{512\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 1/8*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/16*(2*b^2*x
^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*
x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Mupad [B]
time = 0.25, size = 94, normalized size = 0.70

$$\frac{\frac{\sinh(2\,a+2\,b\,x)}{8}+\frac{\sinh(4\,a+4\,b\,x)}{256}-b\,\left(\frac{x\cosh(2\,a+2\,b\,x)}{4}+\frac{x\cosh(4\,a+4\,b\,x)}{64}\right)+b^2\,\left(\frac{x^2\sinh(2\,a+2\,b\,x)}{4}+\frac{x^2\sinh(4\,a+4\,b\,x)}{32}\right)}{b^3}+\frac{x^3}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(a + b*x)^4,x)
```

```
[Out] (sinh(2*a + 2*b*x)/8 + sinh(4*a + 4*b*x)/256 - b*((x*cosh(2*a + 2*b*x))/4 +
 (x*cosh(4*a + 4*b*x))/64) + b^2*((x^2*sinh(2*a + 2*b*x))/4 + (x^2*sinh(4*a
 + 4*b*x))/32))/b^3 + x^3/8
```

3.25 $\int x\cosh^4(a+bx)\,dx$

Optimal. Leaf size=80

$$\frac{3x^2}{16}-\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

```
[Out] 3/16*x^2-3/16*cosh(b*x+a)^2/b^2-1/16*cosh(b*x+a)^4/b^2+3/8*x*cosh(b*x+a)*si
nh(b*x+a)/b+1/4*x*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Rubi [A]
time = 0.03, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3391, 30}

$$-\frac{\cosh^4(a+bx)}{16b^2}-\frac{3\cosh^2(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3x\sinh(a+bx)\cosh(a+bx)}{8b}+\frac{3x^2}{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*Cosh[a + b*x]^4,x]
```

```
[Out] (3*x^2)/16 - (3*Cosh[a + b*x]^2)/(16*b^2) - Cosh[a + b*x]^4/(16*b^2) + (3*x
*Cosh[a + b*x]*Sinh[a + b*x])/(8*b) + (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*
b)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rubi steps

$$\begin{aligned}\int x\cosh^4(a+bx)\,dx &= -\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\cosh^3(a+bx)\sinh(a+bx)}{4b}+\frac{3}{4}\int x\cosh^2(a+bx)\,dx\\ &= -\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(a+b\\ &= \frac{3x^2}{16}-\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(\end{aligned}$$

Mathematica [A]
time = 0.18, size = 53, normalized size = 0.66

$$-\frac{16\cosh(2(a+bx))+\cosh(4(a+bx))-4bx(6bx+8\sinh(2(a+bx))+\sinh(4(a+bx)))}{128b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Cosh[a + b*x]^4,x]
```

```
[Out] -1/128*(16*Cosh[2*(a + b*x)] + Cosh[4*(a + b*x)] - 4*b*x*(6*b*x + 8*Sinh[2*
(a + b*x)] + Sinh[4*(a + b*x)]))/b^2
```

Maple [A]
time = 1.22, size = 101, normalized size = 1.26

method	result	s
risch	$\frac{3x^2}{16}+\frac{(4bx-1)e^{4bx+4a}}{256b^2}+\frac{(2bx-1)e^{2bx+2a}}{16b^2}-\frac{(2bx+1)e^{-2bx-2a}}{16b^2}-\frac{(4bx+1)e^{-4bx-4a}}{256b^2}$	8
default	$\frac{3x^2}{16}+\frac{(2bx+2a)\sinh(2bx+2a)-\cosh(2bx+2a)-2a\sinh(2bx+2a)}{8b^2}+\frac{(4bx+4a)\sinh(4bx+4a)-\cosh(4bx+4a)-4a\sinh(4bx+4a)}{128b^2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

```
[Out] 3/16*x^2+1/8/b^2*((2*b*x+2*a)*sinh(2*b*x+2*a)-cosh(2*b*x+2*a)-2*a*sinh(2*b*
x+2*a))+1/128/b^2*((4*b*x+4*a)*sinh(4*b*x+4*a)-cosh(4*b*x+4*a)-4*a*sinh(4*b
*x+4*a))
```

Maxima [A]
time = 0.29, size = 96, normalized size = 1.20

$$\frac{3}{16}x^2+\frac{\left(4\,bxe^{(4\,a)}-e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2}+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2}-\frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="maxima")
```

```
[Out] 3/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/16*(2*b*x*e^(2
*a) - e^(2*a))*e^(2*b*x)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/25
6*(4*b*x + 1)*e^(-4*b*x - 4*a)/b^2
```

Fricas [A]
time = 0.35, size = 114, normalized size = 1.42

$$\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+24\,b^2x^2-\cosh\left(bx+a\right)^4-\sinh\left(bx+a\right)^4-2\left(3\,\cosh\left(bx+a\right)^2+8\right)\sinh\left(bx+a\right)^2-16\,\cosh\left(bx+a\right)^2+16\left(bx\cosh\left(bx+a\right)^3+4\,bx\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)}{128\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 24*b^2*x^2 - cosh(b*x + a)^4
- sinh(b*x + a)^4 - 2*(3*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 - 16*cosh(b*x
 + a)^2 + 16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/b^2
```

Sympy [A]
time = 0.29, size = 138, normalized size = 1.72

$$\begin{cases} \frac{3x^2\sinh^4(a+bx)}{16} - \frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8} + \frac{3x^2\cosh^4(a+bx)}{16} - \frac{3x\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5x\sinh(a+bx)\cosh^3(a+bx)}{8b} + \frac{3\sinh^4(a+bx)}{32b^2} - \frac{5\cosh^4(a+bx)}{32b^2} & \text{for } b \neq 0 \\ \frac{x^2\cosh^4(a)}{2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)**4,x)
```

```
[Out] Piecewise((3*x**2*sinh(a + b*x)**4/16 - 3*x**2*sinh(a + b*x)**2*cosh(a + b*
x)**2/8 + 3*x**2*cosh(a + b*x)**4/16 - 3*x*sinh(a + b*x)**3*cosh(a + b*x)/(
8*b) + 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 3*sinh(a + b*x)**4/(32*b*
*2) - 5*cosh(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cosh(a)**4/2, True))
```

Giac [A]
time = 0.42, size = 86, normalized size = 1.08

$$\frac{3}{16}\,x^2 + \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} + \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*cosh(b*x+a)^4,x, algorithm="giac")
```

```
[Out] 3/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/16*(2*b*x - 1)*e^(2*b*
x + 2*a)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/256*(4*b*x + 1)*e^
(-4*b*x - 4*a)/b^2
```

Mupad [B]
time = 0.15, size = 68, normalized size = 0.85

$$\frac{3\,x^2}{16} - \frac{\frac{3\,{\cosh(a+b\,x)}^2}{16} + \frac{{\cosh(a+b\,x)}^4}{16} - b\,\left(\frac{x\,\sinh(a+b\,x)\,{\cosh(a+b\,x)}^3}{4} + \frac{3\,x\,\sinh(a+b\,x)\,\cosh(a+b\,x)}{8}\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(a + b*x)^4,x)
```

```
[Out] (3*x^2)/16 - ((3*cosh(a + b*x)^2)/16 + cosh(a + b*x)^4/16 - b*((x*cosh(a +
b*x)^3*sinh(a + b*x))/4 + (3*x*cosh(a + b*x)*sinh(a + b*x))/8))/b^2
```

3.26 $\int (c+dx)^3 \text{sech}(a+bx)\, dx$

Optimal. Leaf size=179

$$\frac{2(c+dx)^3 \text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2 \text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2 \text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{6id^2(c+dx}{}$$

```
[Out] 2*(d*x+c)^3*arctan(exp(b*x+a))/b-3*I*d*(d*x+c)^2*polylog(2,-I*exp(b*x+a))/b
^2+3*I*d*(d*x+c)^2*polylog(2,I*exp(b*x+a))/b^2+6*I*d^2*(d*x+c)*polylog(3,-I
*exp(b*x+a))/b^3-6*I*d^2*(d*x+c)*polylog(3,I*exp(b*x+a))/b^3-6*I*d^3*polylo
g(4,-I*exp(b*x+a))/b^4+6*I*d^3*polylog(4,I*exp(b*x+a))/b^4
```

Rubi [A]
time = 0.09, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {4265, 2611, 6744, 2320, 6724}

$$\frac{2(c+dx)^3 \text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{6id^3 \text{Li}_4\left(-ie^{a+bx}\right)}{b^4}+\frac{6id^3 \text{Li}_4\left(ie^{a+bx}\right)}{b^4}+\frac{6id^2(c+dx) \text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{6id^2(c+dx) \text{Li}_3\left(ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2 \text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{3id(c+dx)^2 \text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x],x]
```

```
[Out] (2*(c + d*x)^3*ArcTan[E^(a + b*x)])/b - ((3*I)*d*(c + d*x)^2*PolyLog[2, (-I
)*E^(a + b*x)])/b^2 + ((3*I)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 +
 ((6*I)*d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((6*I)*d^2*(c + d
*x)*PolyLog[3, I*E^(a + b*x)])/b^3 - ((6*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)
])/b^4 + ((6*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\text{sech}(a+bx)\,dx = \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{(3id)\int (c+dx)^2\log\left(1-ie^{a+bx}\right)\,dx}{b} + \frac{(3id)\int (c\cdots}{}$$

$$= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(ie^{a-\cdots}\right.}{b^2}$$

$$= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(ie^{a-\cdots}\right.}{b^2}$$

$$= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(ie^{a-\cdots}\right.}{b^2}$$

$$= \frac{2(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{3id(c+dx)^2\text{Li}_2\left(ie^{a-\cdots}\right.}{b^2}$$

Mathematica [A]
time = 2.15, size = 343, normalized size = 1.92

$$\frac{i\left(-2ib^3c^3\text{ArcTan}\left(e^{a+bx}\right)+3b^3c^2dx\log\left(1-ie^{a+bx}\right)+3b^3cd^2x^2\log\left(1-ie^{a+bx}\right)+b^3d^3x^3\log\left(1-ie^{a+bx}\right)-3b^3c^2dx\log\left(1+ie^{a+bx}\right)-3b^3cd^2x^2\log\left(1+ie^{a+bx}\right)-b^3d^3x^3\log\left(1+ie^{a+bx}\right)-3b^2d(c+dx)^2\text{PolyLog}\left(2,-ie^{a+bx}\right)+3b^2d(c+dx)^2\text{PolyLog}\left(2,ie^{a+bx}\right)+6bcd^2\text{PolyLog}\left(3,-ie^{a+bx}\right)+6bd^3x\text{PolyLog}\left(3,-ie^{a+bx}\right)-6bcd^2\text{PolyLog}\left(3,ie^{a+bx}\right)-6bd^3x\text{PolyLog}\left(3,ie^{a+bx}\right)-6d^3\text{PolyLog}\left(4,-ie^{a+bx}\right)+6d^3\text{PolyLog}\left(4,ie^{a+bx}\right)\right)}{b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x],x]
```

```
[Out] (I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*x
)] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a +
b*x)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^
(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*d*(c + d*x)^2*PolyL
og[2, (-I)*E^(a + b*x)] + 3*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)] + 6
*b*c*d^2*PolyLog[3, (-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(a + b*
x)] - 6*b*c*d^2*PolyLog[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E^(a + b
*x)] - 6*d^3*PolyLog[4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a + b*x)]
))/b^4
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3 \operatorname{sech}(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^3*sech(b*x+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*c^3*arctan(e^(-b*x - a))/b + 2*integrate((d^3*x^3*e^a + 3*c*d^2*x^2*e^a
+ 3*c^2*d*x*e^a)*e^(b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [**B**] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(146) = 292.
time = 0.41, size = 497, normalized size = 2.78

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (6*I*d^3*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*I*d^3*polylog(4,
 -I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x
- I*b^2*c^2*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(I*b^2*d^3*x^2
+ 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a))
+ (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*a^3*d^3)*log(cosh(b*x
```

```
+ a) + sinh(b*x + a) + I) + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2
 + I*a^3*d^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^3*d^3*x^3 - 3*
I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*a
^3*d^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^3*d^3*x^3 + 3*I*b
^3*c*d^2*x^2 + 3*I*b^3*c^2*d*x + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*a^3*
d^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 6*(I*b*d^3*x + I*b*c*d^2
)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(-I*b*d^3*x - I*b*c*d^2
)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \operatorname{sech}(a+bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a),x)
[Out] Integral((c + d*x)**3*sech(a + b*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^3*sech(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^3}{\mathrm{cosh}\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cosh(a + b*x),x)
[Out] int((c + d*x)^3/cosh(a + b*x), x)
```

3.27 $\int (c+dx)^2 \text{sech}(a+bx)\,dx$

Optimal. Leaf size=119

$$\frac{2(c+dx)^2\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{2id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}+\frac{2id^2\text{PolyLog}(}{b^3}$$

```
[Out] 2*(d*x+c)^2*arctan(exp(b*x+a))/b-2*I*d*(d*x+c)*polylog(2,-I*exp(b*x+a))/b^2
+2*I*d*(d*x+c)*polylog(2,I*exp(b*x+a))/b^2+2*I*d^2*polylog(3,-I*exp(b*x+a))
/b^3-2*I*d^2*polylog(3,I*exp(b*x+a))/b^3
```

Rubi [A]
time = 0.06, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4265, 2611, 2320, 6724}

$$\frac{2(c+dx)^2\text{ArcTan}\left(e^{a+bx}\right)}{b}+\frac{2id^2\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{2id^2\text{Li}_3\left(ie^{a+bx}\right)}{b^3}-\frac{2id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x],x]
```

```
[Out] (2*(c + d*x)^2*ArcTan[E^(a + b*x)])/b - ((2*I)*d*(c + d*x)*PolyLog[2, (-I)*
E^(a + b*x)])/b^2 + ((2*I)*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)])/b^2 + ((2
*I)*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((2*I)*d^2*PolyLog[3, I*E^(a +
b*x)])/b^3
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
```

```
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\text{sech}(a+bx)\,dx &= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{(2id)\int (c+dx)\log\left(1-ie^{a+bx}\right)\,dx}{b}+\frac{(2id)\int (c+\\
&= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{Li}_2\left(ie^{a+bx}\right.}{b^2}\\
&= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{Li}_2\left(ie^{a+bx}\right.}{b^2}\\
&= \frac{2(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{2id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{2id(c+dx)\text{Li}_2\left(ie^{a+bx}\right.}{b^2}\end{aligned}$$

Mathematica [A]
time = 1.10, size = 199, normalized size = 1.67

$$\frac{i\left(-2ib^2c^2\text{ArcTan}\left(e^{a+bx}\right)+2b^2cdx\log\left(1-ie^{a+bx}\right)+b^2d^2x^2\log\left(1-ie^{a+bx}\right)-2b^2cdx\log\left(1+ie^{a+bx}\right)-b^2d^2x^2\log\left(1+ie^{a+bx}\right)-2bd(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bd(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)+2d^2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2d^2\text{PolyLog}\left(3,ie^{a+bx}\right)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x],x]
```

```
[Out] (I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + 2*b^2*c*d*x*Log[1 - I*E^(a + b*x)]
 + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b^2*c*d*x*Log[1 + I*E^(a + b*x)]
- b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, (-I)*E^(a
 + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)] + 2*d^2*PolyLog[3, (-I
)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a + b*x)]))/b^3
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{2}\text{sech}\left(bx+a\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^2*sech(b*x+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] -2*c^2*arctan(e^(-b*x - a))/b + 2*integrate((d^2*x^2*e^a + 2*c*d*x*e^a)*e^(
b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(96) = 192.
time = 0.39, size = 305, normalized size = 2.56

$$\frac{-2i\,d^2\mathrm{polylog}(3, i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right))+2i\,d^2\mathrm{polylog}(3, -i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right))-2\left(-i\,bd^2x-i\,bcd\right)\mathrm{Li}_2(i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right))-2\left(i\,bd^2x+i\,bcd\right)\mathrm{Li}_2(-i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right))+\left(i\,b^2c^2-2i\,abcd+i\,a^2d^2\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)+i\right)+\left(-i\,b^2c^2+2i\,abcd-i\,a^2d^2\right)\log\left(\cosh\left(bx+a\right)+\sinh\left(bx+a\right)-i\right)+\left(-i\,b^2d^2x^2-2i\,b^2cdx-2i\,abcd+i\,a^2d^2\right)\log\left(i\,\cosh\left(bx+a\right)+i\,\sinh\left(bx+a\right)+1\right)+\left(i\,b^2d^2x^2+2i\,b^2cdx+2i\,abcd-i\,a^2d^2\right)\log\left(-i\,\cosh\left(bx+a\right)-i\,\sinh\left(bx+a\right)+1\right)}{b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (-2*I*d^2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*I*d^2*polylog(3
, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(I*co
sh(b*x + a) + I*sinh(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cosh(b*x
+ a) - I*sinh(b*x + a)) + (I*b^2*c^2 - 2*I*a*b*c*d + I*a^2*d^2)*log(cosh(b*
x + a) + sinh(b*x + a) + I) + (-I*b^2*c^2 + 2*I*a*b*c*d - I*a^2*d^2)*log(co
sh(b*x + a) + sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*
b*c*d + I*a^2*d^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^2*d^2*
x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*log(-I*cosh(b*x + a) - I*sin
h(b*x + a) + 1))/b^3
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c+dx\right)^2\operatorname{sech}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)**2*sech(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^2*sech(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cosh\left(a+b\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cosh(a + b*x),x)
[Out] int((c + d*x)^2/cosh(a + b*x), x)
```

3.28 $\int (c+dx)\text{sech}(a+bx)\,dx$

Optimal. Leaf size=61

$$\frac{2(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

```
[Out] 2*(d*x+c)*arctan(exp(b*x+a))/b-I*d*polylog(2,-I*exp(b*x+a))/b^2+I*d*polylog
(2,I*exp(b*x+a))/b^2
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {4265, 2317, 2438}

$$\frac{2(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x],x]
```

```
[Out] (2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a + b*x)])/b^
2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int (c+dx)\text{sech}(a+bx)\,dx = \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{(id)\int \log\left(1-ie^{a+bx}\right)\,dx}{b} + \frac{(id)\int \log\left(1+ie^{a+bx}\right)}{b}$$
$$= \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{(id)\text{Subst}\left(\int \frac{\log(1-ix)}{x}\,dx, x, e^{a+bx}\right)}{b^2} + \frac{(id)\text{Subst}\left(\int \frac{1}{}\right.}{}$$
$$= \frac{2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{b^2} + \frac{id\text{Li}_2\left(ie^{a+bx}\right)}{b^2}$$

Mathematica [B] Both result and optimal contain complex but leaf count is larger than twice the leaf count of optimal. 127 vs. $2(61) = 122$.
time = 0.12, size = 127, normalized size = 2.08

$$\frac{bc\text{ArcTan}(\sinh(a+bx)) + \frac{1}{2}d\left(-\left((-2ia+\pi-2ibx)\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)\right)+(-2ia+\pi)\log\left(\cot\left(\frac{1}{4}(2ia+\pi+2ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{a+bx}\right)-\text{PolyLog}\left(2,ie^{a+bx}\right)\right)\right)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x],x]
```

```
[Out] (b*c*ArcTan[Sinh[a + b*x]] + (d*(-(((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E
^(a + b*x)] - Log[1 + I*E^(a + b*x)])) + ((-2*I)*a + Pi)*Log[Cot[((2*I)*a +
 Pi + (2*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E
^(a + b*x)])))/2)/b^2
```

Maple [A]
time = 0.96, size = 101, normalized size = 1.66

method	result
derivativedivides	$\frac{\frac{d\left(i(bx+a)\left(\ln\left(1-ie^{bx+a}\right)-\ln\left(1+ie^{bx+a}\right)\right)-i\,\text{dilog}\left(1+ie^{bx+a}\right)+i\,\text{dilog}\left(1-ie^{bx+a}\right)\right)}{b}-\frac{2da\arctan\left(e^{bx+a}\right)}{b}+2c\arctan\left(e^{bx+a}\right)}{b}$
default	$\frac{\frac{d\left(i(bx+a)\left(\ln\left(1-ie^{bx+a}\right)-\ln\left(1+ie^{bx+a}\right)\right)-i\,\text{dilog}\left(1+ie^{bx+a}\right)+i\,\text{dilog}\left(1-ie^{bx+a}\right)\right)}{b}-\frac{2da\arctan\left(e^{bx+a}\right)}{b}+2c\arctan\left(e^{bx+a}\right)}{b}$
risch	$\frac{2c\arctan\left(e^{bx+a}\right)}{b}-\frac{id\ln\left(1+ie^{bx+a}\right)x}{b}-\frac{id\ln\left(1+ie^{bx+a}\right)a}{b^2}+\frac{id\ln\left(1-ie^{bx+a}\right)x}{b}+\frac{id\ln\left(1-ie^{bx+a}\right)a}{b^2}-\frac{id\,\text{dilog}(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/b*(d/b*(I*(b*x+a)*(ln(1-I*exp(b*x+a))-ln(1+I*exp(b*x+a)))-I*dilog(1+I*exp
(b*x+a))+I*dilog(1-I*exp(b*x+a)))-2*d/b*a*arctan(exp(b*x+a))+2*c*arctan(exp
(b*x+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="maxima")
```

```
[Out] 2*d*integrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x) - 2*c*arctan(e^(-b*x
- a))/b
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(48) = 96$.
time = 0.39, size = 157, normalized size = 2.57

$$\frac{i\,d\mathrm{Li}_2(i\cosh(bx+a)+i\sinh(bx+a))-i\,d\mathrm{Li}_2(-i\cosh(bx+a)-i\sinh(bx+a))+(i\,bc-i\,ad)\log(\cosh(bx+a)+\sinh(bx+a)+i)+(-i\,bc+i\,ad)\log(\cosh(bx+a)+\sinh(bx+a)-i)+(-i\,bdx-i\,ad)\log(i\cosh(bx+a)+i\sinh(bx+a)+1)+(i\,bdx+i\,ad)\log(-i\cosh(bx+a)-i\sinh(bx+a)+1)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="fricas")
```

```
[Out] (I*d*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - I*d*dilog(-I*cosh(b*x + a)
- I*sinh(b*x + a)) + (I*b*c - I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) + I)
 + (-I*b*c + I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b*d*x - I*
a*d)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b*d*x + I*a*d)*log(-I*
cosh(b*x + a) - I*sinh(b*x + a) + 1))/b^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\operatorname{sech}(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sech(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{c+d\,x}{\cosh(a+b\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cosh(a + b*x),x)
```

```
[Out] int((c + d*x)/cosh(a + b*x), x)
```

3.29 $\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/(d*x+c),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]/(c + d*x),x]
[Out] Defer[Int][Sech[a + b*x]/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 2.99, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/(c + d*x),x]
[Out] Integrate[Sech[a + b*x]/(c + d*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c),x)
[Out] int(sech(b*x+a)/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)/(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="fricas")
[Out] integral(sech(b*x + a)/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x)
[Out] Integral(sech(a + b*x)/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c),x, algorithm="giac")
[Out] integrate(sech(b*x + a)/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\cosh\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*(c + d*x)),x)
```

```
[Out] int(1/(cosh(a + b*x)*(c + d*x)), x)
```

3.30 $\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=17

$$\operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{(c+dx)^2}, x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/(d*x+c)^2,x)
```

Rubi [A]
time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} \, dx = \int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [A]
time = 6.20, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/(c + d*x)^2,x]
```

```
[Out] Integrate[Sech[a + b*x]/(c + d*x)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c)^2,x)
[Out] int(sech(b*x+a)/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
[Out] integrate(sech(b*x + a)/(d*x + c)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
[Out] integral(sech(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)**2,x)
[Out] Integral(sech(a + b*x)/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="giac")
[Out] integrate(sech(b*x + a)/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\cosh\left(a+b\,x\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*(c + d*x)^2),x)
```

```
[Out] int(1/(cosh(a + b*x)*(c + d*x)^2), x)
```

3.31 $\int (c+dx)^3 \mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=103

$$\frac{(c+dx)^3}{b}-\frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{3d^2(c+dx)\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{3d^3\text{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4}+\cdot$$

```
[Out] (d*x+c)^3/b-3*d*(d*x+c)^2*ln(1+exp(2*b*x+2*a))/b^2-3*d^2*(d*x+c)*polylog(2,
-exp(2*b*x+2*a))/b^3+3/2*d^3*polylog(3,-exp(2*b*x+2*a))/b^4+(d*x+c)^3*tanh(
b*x+a)/b
```

Rubi [A]
time = 0.14, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4269, 3799, 2221, 2611, 2320, 6724}

$$\frac{3d^3\text{Li}_3\left(-e^{2(a+bx)}\right)}{2b^4}-\frac{3d^2(c+dx)\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}-\frac{3d(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{(c+dx)^3}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x]^2,x]
```

```
[Out] (c + d*x)^3/b - (3*d*(c + d*x)^2*Log[1 + E^(2*(a + b*x))])/b^2 - (3*d^2*(c
+ d*x)*PolyLog[2, -E^(2*(a + b*x))])/b^3 + (3*d^3*PolyLog[3, -E^(2*(a + b*x
))])/(2*b^4) + ((c + d*x)^3*Tanh[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
- 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
f, g, n}, x] && GtQ[m, 0]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3\operatorname{sech}^2(a+bx)\,dx &= \frac{(c+dx)^3\tanh(a+bx)}{b}-\frac{(3d)\int (c+dx)^2\tanh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^3}{b}+\frac{(c+dx)^3\tanh(a+bx)}{b}-\frac{(6d)\int \frac{e^{2(a+bx)}(c+dx)^2}{1+e^{2(a+bx)}}\,dx}{b}\\ &= \frac{(c+dx)^3}{b}-\frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2}+\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{(6d^2)}{}\\ &= \frac{(c+dx)^3}{b}-\frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}+\frac{(c}{}\\ &= \frac{(c+dx)^3}{b}-\frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}+\frac{(c}{}\\ &= \frac{(c+dx)^3}{b}-\frac{3d(c+dx)^2\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{3d^2(c+dx)\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}+\frac{3d}{}\end{aligned}$$

Mathematica [A]
time = 1.36, size = 135, normalized size = 1.31

$$\frac{d\left(\frac{4b^3e^{2a}x(3c^2+3cdx+d^2x^2)}{1+e^{2a}}-6b^2(c+dx)^2\log\left(1+e^{2(a+bx)}\right)-6bd(c+dx)\mathrm{PolyLog}\left(2,-e^{2(a+bx)}\right)+3d^2\mathrm{PolyLog}\left(3,-e^{2(a+bx)}\right)\right)}{2b^4}+\frac{(c+dx)^3\operatorname{sech}(a)\operatorname{sech}(a+bx)\sinh(bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x]^2,x]
```

```
[Out] (d*((4*b^3*E^(2*a)*x*(3*c^2 + 3*c*d*x + d^2*x^2))/(1 + E^(2*a)) - 6*b^2*(c
+ d*x)^2*Log[1 + E^(2*(a + b*x))] - 6*b*d*(c + d*x)*PolyLog[2, -E^(2*(a + b
*x))] + 3*d^2*PolyLog[3, -E^(2*(a + b*x))]))/(2*b^4) + ((c + d*x)^3*Sech[a]
*Sech[a + b*x]*Sinh[b*x])/b
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(101) = 202$.
time = 1.52, size = 298, normalized size = 2.89

method	result
risch	$-\frac{2\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{\left(e^{2bx+2a}+1\right)b}+\frac{6d\,c^2\ln\left(e^{bx+a}\right)}{b^2}-\frac{3d\,c^2\ln\left(e^{2bx+2a}+1\right)}{b^2}+\frac{6d^3a^2\ln\left(e^{bx+a}\right)}{b^4}+\frac{2d^3x^3}{b}-\frac{6d^3a^2x}{b^3}-\frac{4d^3a^3}{b^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(exp(2*b*x+2*a)+1)/b+6/b^2*d*c^2*ln(
exp(b*x+a))-3/b^2*d*c^2*ln(exp(2*b*x+2*a)+1)+6/b^4*d^3*a^2*ln(exp(b*x+a))+2
/b*d^3*x^3-6/b^3*d^3*a^2*x-4/b^4*d^3*a^3-3/b^2*d^3*ln(exp(2*b*x+2*a)+1)*x^2
-3/b^3*d^3*polylog(2,-exp(2*b*x+2*a))*x+3/2*d^3*polylog(3,-exp(2*b*x+2*a))/
b^4-12/b^3*d^2*a*c*ln(exp(b*x+a))+6/b*d^2*c*x^2+12/b^2*d^2*a*c*x+6/b^3*d^2*
c*a^2-6/b^2*d^2*c*ln(exp(2*b*x+2*a)+1)*x-3/b^3*d^2*c*polylog(2,-exp(2*b*x+2
*a))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(100) = 200$.
time = 0.39, size = 238, normalized size = 2.31

$$3\,c^2d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right)-\frac{3\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)cd^2}{b^3}+\frac{2\,c^3}{b\left(e^{(-2\,bx-2\,a)}+1\right)}-\frac{2\left(d^3x^3+3\,cd^2x^2\right)}{be^{(2\,bx+2\,a)}+b}-\frac{3\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)\right)d^3}{2\,b^4}+\frac{2\left(b^3d^3x^3+3\,b^3cd^2x^2\right)}{b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a)
 + 1)*e^(-2*a))/b^2) - 3*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*x
+ 2*a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) + 1)) - 2*(d^3*x^3 + 3*c*d^
2*x^2)/(b*e^(2*b*x + 2*a) + b) - 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1) +
2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))*d^3/b^4 + 2*(
b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Fricas [C] Result contains complex when optimal does not.
time = 0.43, size = 1332, normalized size = 12.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 - 2*(b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*
cosh(b*x + a)^2 - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^
2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) - 2*(b^3*d^3
*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^
3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cosh(b*
x + a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^3*x + b
*c*d^2)*sinh(b*x + a)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + 6*(b*d^
3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 + 2*(b*d^3*x + b*c*d^2)
*cosh(b*x + a)*sinh(b*x + a) + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(-
I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 +
 (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)^2 + 2*(b^2*c^2*d - 2*a*b
*c*d^2 + a^2*d^3)*cosh(b*x + a)*sinh(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 +
a^2*d^3)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) + I) + 3*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x
 + a)^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)*sinh(b*x + a)
 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*log(cosh(b*x + a) +
 sinh(b*x + a) - I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^
3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cosh(b*x + a)^2 +
 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*cosh(b*x + a)*sinh
(b*x + a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3)*sinh(b*x
+ a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + 3*(b^2*d^3*x^2 + 2*b^2
*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d
^2 - a^2*d^3)*cosh(b*x + a)^2 + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^
2 - a^2*d^3)*cosh(b*x + a)*sinh(b*x + a) + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2
*a*b*c*d^2 - a^2*d^3)*sinh(b*x + a)^2)*log(-I*cosh(b*x + a) - I*sinh(b*x +
a) + 1) - 6*(d^3*cosh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*
sinh(b*x + a)^2 + d^3)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(d
^3*cosh(b*x + a)^2 + 2*d^3*cosh(b*x + a)*sinh(b*x + a) + d^3*sinh(b*x + a)^
2 + d^3)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/(b^4*cosh(b*x + a)
^2 + 2*b^4*cosh(b*x + a)*sinh(b*x + a) + b^4*sinh(b*x + a)^2 + b^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^3 \operatorname{sech}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sech(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{\cosh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cosh(a + b*x)^2,x)
```

```
[Out] int((c + d*x)^3/cosh(a + b*x)^2, x)
```

3.32 $\int (c+dx)^2 \mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=73

$$\frac{(c+dx)^2}{b}-\frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2}-\frac{d^2\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3}+\frac{(c+dx)^2\tanh(a+bx)}{b}$$

```
[Out] (d*x+c)^2/b-2*d*(d*x+c)*ln(1+exp(2*b*x+2*a))/b^2-d^2*polylog(2,-exp(2*b*x+2
*a))/b^3+(d*x+c)^2*tanh(b*x+a)/b
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {4269, 3799, 2221, 2317, 2438}

$$-\frac{d^2\text{Li}_2\left(-e^{2(a+bx)}\right)}{b^3}-\frac{2d(c+dx)\log\left(e^{2(a+bx)}+1\right)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{(c+dx)^2}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x]^2,x]
```

```
[Out] (c + d*x)^2/b - (2*d*(c + d*x)*Log[1 + E^(2*(a + b*x))])/b^2 - (d^2*PolyLog
[2, -E^(2*(a + b*x))])/b^3 + ((c + d*x)^2*Tanh[a + b*x])/b
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
```

```
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\mathrm{sech}^2(a+bx)\,dx &= \frac{(c+dx)^2\tanh(a+bx)}{b} - \frac{(2d)\int (c+dx)\tanh(a+bx)\,dx}{b}\\ &= \frac{(c+dx)^2}{b} + \frac{(c+dx)^2\tanh(a+bx)}{b} - \frac{(4d)\int \frac{e^{2(a+bx)}(c+dx)}{1+e^{2(a+bx)}}\,dx}{b}\\ &= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tanh(a+bx)}{b} + \frac{(2d^2)\\ &= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} + \frac{(c+dx)^2\tanh(a+bx)}{b} + \frac{d^2\mathrm{Sub}\\ &= \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{d^2\mathrm{Li}_2\left(-e^{2(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\,\mathrm{ta}}{b}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.84, size = 186, normalized size = 2.55

$$\frac{\mathrm{sech}(a)\left(2bcd(-\cosh(a)\log(\cosh(a+bx))+bx\sinh(a))+d^2\left(\cosh(a)\left(i\pi\log\left(1+e^{2bx}\right)-2bx\log\left(1-e^{-2\left(bx+\tanh^{-1}(\coth(a))\right)}\right)-i\pi(bx+\log(\cosh(bx)))-2\tanh^{-1}(\coth(a))\left(bx+\log\left(1-e^{-2\left(bx+\tanh^{-1}(\coth(a))\right)}\right)-\log\left(i\sinh\left(bx+\tanh^{-1}(\coth(a))\right)\right)\right)+\mathrm{PolyLog}\left(2,e^{-2\left(bx+\tanh^{-1}(\coth(a))\right)}\right)\right)+b^2e^{-\tanh^{-1}(\coth(a))}x^2\sqrt{-\mathrm{csch}^2(a)}\sinh(a)\right)+b^2(c+dx)^2\mathrm{sech}(a+bx)\sinh(bx)\right)}{b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x]^2,x]
```

```
[Out] (Sech[a]*(2*b*c*d*(-(Cosh[a]*Log[Cosh[a + b*x]]) + b*x*Sinh[a]) + d^2*(Cosh
[a]*(I*Pi*Log[1 + E^(2*b*x)] - 2*b*x*Log[1 - E^(-2*(b*x + ArcTanh[Coth[a]])
)] - I*Pi*(b*x + Log[Cosh[b*x]]) - 2*ArcTanh[Coth[a]]*(b*x + Log[1 - E^(-2*
(b*x + ArcTanh[Coth[a]]))] - Log[I*Sinh[b*x + ArcTanh[Coth[a]]]]) + PolyLog
[2, E^(-2*(b*x + ArcTanh[Coth[a]]))]) + (b^2*x^2*Sqrt[-Csch[a]^2]*Sinh[a])/
E^ArcTanh[Coth[a]]) + b^2*(c + d*x)^2*Sech[a + b*x]*Sinh[b*x]))/b^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(73) = 146.
time = 1.36, size = 159, normalized size = 2.18

method	result
risch	$-\frac{2(d^2x^2+2cdx+c^2)}{(e^{2bx+2a}+1)b}-\frac{2dc\ln(e^{2bx+2a}+1)}{b^2}+\frac{4dc\ln(e^{bx+a})}{b^2}+\frac{2d^2x^2}{b}+\frac{4d^2ax}{b^2}+\frac{2d^2a^2}{b^3}-\frac{2d^2\ln(e^{2bx+2a}+1)x}{b^2}-\frac{d^2\,\mathrm{polylo}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2*(d^2*x^2+2*c*d*x+c^2)/(exp(2*b*x+2*a)+1)/b-2/b^2*d*c*ln(exp(2*b*x+2*a)+1
)+4/b^2*d*c*ln(exp(b*x+a))+2/b*d^2*x^2+4/b^2*d^2*a*x+2/b^3*d^2*a^2-2/b^2*d^
2*ln(exp(2*b*x+2*a)+1)*x-d^2*polylog(2,-exp(2*b*x+2*a))/b^3-4/b^3*d^2*a*ln(
exp(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -2*d^2*(x^2/(b*e^(2*b*x + 2*a) + b) - 2*integrate(x/(b*e^(2*b*x + 2*a) + b)
, x)) + 2*c*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x
+ 2*a) + 1)*e^(-2*a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [C] Result contains complex when optimal does not.
time = 0.47, size = 715, normalized size = 9.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a^2
*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
- a^2*d^2)*sinh(b*x + a)^2 + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sin
h(b*x + a) + d^2*sinh(b*x + a)^2 + d^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x
+ a)) + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^2*sinh
(b*x + a)^2 + d^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (b*c*d - a*d
^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(b*c*d - a*d^2)*cosh(b*x + a)*sinh
(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x +
 a) + I) + (b*c*d - a*d^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(b*c*d - a*
d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cos
h(b*x + a) + sinh(b*x + a) - I) + (b*d^2*x + a*d^2 + (b*d^2*x + a*d^2)*cosh
```

```
(b*x + a)^2 + 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^2*x +
a*d^2)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (b*d^2
*x + a*d^2 + (b*d^2*x + a*d^2)*cosh(b*x + a)^2 + 2*(b*d^2*x + a*d^2)*cosh(b
*x + a)*sinh(b*x + a) + (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(-I*cosh(b*x
+ a) - I*sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*cosh(b*x + a)*sin
h(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^2 \operatorname{sech}^2(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a)**2,x)
[Out] Integral((c + d*x)**2*sech(a + b*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2*sech(b*x + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c + d\,x)^2}{\cosh(a + b\,x)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cosh(a + b*x)^2,x)
[Out] int((c + d*x)^2/cosh(a + b*x)^2, x)
```

3.33 $\int (c+dx)\text{sech}^2(a+bx)\,dx$

Optimal. Leaf size=29

$$-\frac{d\log(\cosh(a+bx))}{b^2}+\frac{(c+dx)\tanh(a+bx)}{b}$$

```
[Out] -d*ln(cosh(b*x+a))/b^2+(d*x+c)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4269, 3556}

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\log(\cosh(a+bx))}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x]^2,x]
```

```
[Out] -((d*Log[Cosh[a + b*x]])/b^2) + ((c + d*x)*Tanh[a + b*x])/b
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int (c+dx)\text{sech}^2(a+bx)\,dx &= \frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\int \tanh(a+bx)\,dx}{b}\\ &= -\frac{d\log(\cosh(a+bx))}{b^2}+\frac{(c+dx)\tanh(a+bx)}{b}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 51, normalized size = 1.76

$$-\frac{d\log(\cosh(a+bx))}{b^2}+\frac{dx\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b}+\frac{dx\tanh(a)}{b}+\frac{c\tanh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x]^2,x]
```

```
[Out] -((d*Log[Cosh[a + b*x]])/b^2) + (d*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b + (
d*x*Tanh[a])/b + (c*Tanh[a + b*x])/b
```

Maple [A]
time = 0.84, size = 57, normalized size = 1.97

method	result	size
risch	$\frac{2dx}{b}+\frac{2da}{b^2}-\frac{2(dx+c)}{(e^{2bx+2a}+1)b}-\frac{d\ln(e^{2bx+2a}+1)}{b^2}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/b*x+2*d/b^2*a-2*(d*x+c)/(exp(2*b*x+2*a)+1)/b-d/b^2*ln(exp(2*b*x+2*a)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 72 vs. 2(29) = 58.
time = 0.29, size = 72, normalized size = 2.48

$$d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b}-\frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right)+\frac{2\,c}{b\left(e^{(-2\,bx-2\,a)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a) + 1)*
e^(-2*a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a) + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 161 vs. 2(29) = 58.
time = 0.41, size = 161, normalized size = 5.55

$$\frac{2\,bdx\cosh\left(bx+a\right)^2+4\,bdx\cosh\left(bx+a\right)\sinh\left(bx+a\right)+2\,bdx\sinh\left(bx+a\right)^2-2\,bc-\left(d\cosh\left(bx+a\right)^2+2\,d\cosh\left(bx+a\right)\sinh\left(bx+a\right)+d\sinh\left(bx+a\right)^2+d\right)\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b^2\cosh\left(bx+a\right)^2+2\,b^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2\sinh\left(bx+a\right)^2+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] (2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*si
nh(b*x + a)^2 - 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x + a
) + d*sinh(b*x + a)^2 + d)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x +
a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*sinh(b
*x + a)^2 + b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)\operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(29) = 58.
time = 0.41, size = 78, normalized size = 2.69

$$\frac{2\,bdxe^{(2\,bx+2\,a)}-de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right)-2\,bc-d\log\left(e^{(2\,bx+2\,a)}+1\right)}{b^2e^{(2\,bx+2\,a)}+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="giac")
```

```
[Out] (2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) - 2*b
*c - d*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*x + 2*a) + b^2)
```

Mupad [B]
time = 0.09, size = 50, normalized size = 1.72

$$\frac{2\,d\,x}{b}-\frac{2\,(c+d\,x)}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{d\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cosh(a + b*x)^2,x)
```

```
[Out] (2*d*x)/b - (2*(c + d*x))/(b*(exp(2*a + 2*b*x) + 1)) - (d*log(exp(2*a)*exp(
2*b*x) + 1))/b^2
```

3.34 $\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)^2/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]^2/(c + d*x),x]
```

```
[Out] Defer[Int][Sech[a + b*x]^2/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$$

Mathematica [A]
time = 16.08, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^2/(c + d*x),x]
```

```
[Out] Integrate[Sech[a + b*x]^2/(c + d*x), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2/(d*x+c),x)
```

```
[Out] int(sech(b*x+a)^2/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) + 2*
b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x)), x) - 2/(b*d*x + b*c + (b*d*x*e
^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{2}\left(a + bx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)**2/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2/(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh\left(a+b\,x\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*(c + d*x)),x)
```

```
[Out] int(1/(cosh(a + b*x)^2*(c + d*x)), x)
```

3.35 $\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2} dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\text{sech}^2(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)^2/(d*x+c)^2,x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]^2/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2} dx = \int \frac{\text{sech}^2(a+bx)}{(c+dx)^2} dx$$

Mathematica [A]
time = 16.32, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^2/(c + d*x)^2,x]
```

```
[Out] Integrate[Sech[a + b*x]^2/(c + d*x)^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(bx+a)^2}{(dx+c)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^2/(d*x+c)^2,x)
```

```
[Out] int(sech(b*x+a)^2/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3*
x^3*e^(2*a) + 3*b*c*d^2*x^2*e^(2*a) + 3*b*c^2*d*x*e^(2*a) + b*c^3*e^(2*a))*
e^(2*b*x)), x) - 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) + 2*
b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**2/(d*x+c)**2,x)
```

```
[Out] Integral(sech(a + b*x)**2/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^2*(c + d*x)^2),x)
[Out] int(1/(cosh(a + b*x)^2*(c + d*x)^2), x)
```

3.36 $\int (c+dx)^3 \mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=296

$$-\frac{6d^2(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\text{ArcTan}\left(e^{a+bx}\right)}{b}+\frac{3id^3\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4}-\frac{3id(c+dx)^2\text{PolyLog}(2}{2b^2}$$

```
[Out] -6*d^2*(d*x+c)*arctan(exp(b*x+a))/b^3+(d*x+c)^3*arctan(exp(b*x+a))/b+3*I*d^
3*polylog(2,-I*exp(b*x+a))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-I*exp(b*x+a))/b
^2-3*I*d^3*polylog(2,I*exp(b*x+a))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,I*exp(b*
x+a))/b^2+3*I*d^2*(d*x+c)*polylog(3,-I*exp(b*x+a))/b^3-3*I*d^2*(d*x+c)*poly
log(3,I*exp(b*x+a))/b^3-3*I*d^3*polylog(4,-I*exp(b*x+a))/b^4+3*I*d^3*polylo
g(4,I*exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*sech(b*x+a)/b^2+1/2*(d*x+c)^3*sech(b*
x+a)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.15, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 8, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {4271, 4265, 2317, 2438, 2611, 6744, 2320, 6724}

$$-\frac{6d^2(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\text{ArcTan}\left(e^{a+bx}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4}-\frac{3id^3\text{Li}_2\left(ie^{a+bx}\right)}{b^4}-\frac{3id^3\text{Li}_4\left(-ie^{a+bx}\right)}{b^4}+\frac{3id^3\text{Li}_4\left(ie^{a+bx}\right)}{b^4}+\frac{3id^2(c+dx)\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{3id^2(c+dx)\text{Li}_3\left(ie^{a+bx}\right)}{b^3}-\frac{3id(c+dx)^2\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2}+\frac{3id(c+dx)^2\text{Li}_2\left(ie^{a+bx}\right)}{2b^2}+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*Sech[a + b*x]^3,x]
```

```
[Out] (-6*d^2*(c + d*x)*ArcTan[E^(a + b*x)])/b^3 + ((c + d*x)^3*ArcTan[E^(a + b*x
)])/b + ((3*I)*d^3*PolyLog[2, (-I)*E^(a + b*x)])/b^4 - (((3*I)/2)*d*(c + d*
x)^2*PolyLog[2, (-I)*E^(a + b*x)])/b^2 - ((3*I)*d^3*PolyLog[2, I*E^(a + b*x
)])/b^4 + (((3*I)/2)*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)])/b^2 + ((3*I)*
d^2*(c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - ((3*I)*d^2*(c + d*x)*Poly
Log[3, I*E^(a + b*x)])/b^3 - ((3*I)*d^3*PolyLog[4, (-I)*E^(a + b*x)])/b^4 +
 ((3*I)*d^3*PolyLog[4, I*E^(a + b*x)])/b^4 + (3*d*(c + d*x)^2*Sech[a + b*x]
)/(2*b^2) + ((c + d*x)^3*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
```

```
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
```

```
d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx = \frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\text{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int (c+$$
$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{3d(c+dx)^2\text{sech}(a$$
$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{3id(c+dx)^2\text{Li}_2(-}{2b^2}$$
$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4}$$
$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4}$$
$$= -\frac{6d^2(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b^3}+\frac{(c+dx)^3\tan^{-1}\left(e^{a+bx}\right)}{b}+\frac{3id^3\text{Li}_2\left(-ie^{a+bx}\right)}{b^4}$$

Mathematica [A]
time = 20.95, size = 455, normalized size = 1.54

$$\frac{i(-2ib^3c^3\text{ArcTan}(e^{a+bx})+12ibcd^2\text{ArcTan}(e^{a+bx})+3b^3c^2dx\log(1-ie^{a+bx})-6bd^3x\log(1-ie^{a+bx})+3b^3cd^2x^2\log(1-ie^{a+bx})+b^3d^3x^3\log(1-ie^{a+bx})-3b^3c^2dx\log(1+ie^{a+bx})+6bd^3x\log(1+ie^{a+bx})-3b^3cd^2x^2\log(1+ie^{a+bx})-b^3d^3x^3\log(1+ie^{a+bx})-3d(-2d^2+b^2(c+dx)^2)\text{PolyLog}(2,-ie^{a+bx})+3d(-2d^2+b^2(c+dx)^2)\text{PolyLog}(2,ie^{a+bx})+6bcd^2\text{PolyLog}(3,-ie^{a+bx})+6bd^3x\text{PolyLog}(3,-ie^{a+bx})-6bcd^2\text{PolyLog}(3,ie^{a+bx})-6bd^3x\text{PolyLog}(3,ie^{a+bx})-6d^3\text{PolyLog}(4,-ie^{a+bx})+6d^3\text{PolyLog}(4,ie^{a+bx}))+b^2(c+dx)^2\text{sech}(a+bx)(3d+b(c+dx)\tanh(a+bx))}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*Sech[a + b*x]^3,x]
```

```
[Out] (I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + (12*I)*b*c*d^2*ArcTan[E^(a + b*x)]
 + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*x)] - 6*b*d^3*x*Log[1 - I*E^(a + b*x)]
+ 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a + b*x
)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] + 6*b*d^3*x*Log[1 + I*E^(a + b*x)
] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(a + b
*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(a + b*x)] + 3*d*(-
2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, I*E^(a + b*x)] + 6*b*c*d^2*PolyLog[3, (
-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*c*d^2*PolyL
og[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E^(a + b*x)] - 6*d^3*PolyLog[
4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a + b*x)]) + b^2*(c + d*x)^2*S
ech[a + b*x]*(3*d + b*(c + d*x)*Tanh[a + b*x]))/(2*b^4)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^3\,\text{sech}(bx+a)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*sech(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^3*sech(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^3*integrate(x^3*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*c
*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*c^2*
d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^3*(arctan(e^(
-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) + e
^(-4*b*x - 4*a) + 1))) - 6*d^3*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a)
 + b^2), x) - 6*c*d^2*arctan(e^(b*x + a))/b^3 + ((b*d^3*x^3*e^(3*a) + 3*c^2
*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)*x*e^(3*a
))*e^(3*b*x) - (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x^2*e^a + 3
*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x
+ 2*a) + b^2)
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4785 vs. 2(242) = 484.
time = 0.46, size = 4785, normalized size = 16.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 +
 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*cosh(b*x + a)^3 + 6*(b^3*d^3*x^3 + b^3*c^3
+ 3*b^2*c^2*d + 3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x
)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^3*d^3*x^3 + b^3*c^3 + 3*b^2*c^2*d +
3*(b^3*c*d^2 + b^2*d^3)*x^2 + 3*(b^3*c^2*d + 2*b^2*c*d^2)*x)*sinh(b*x + a)^
3 - 2*(b^3*d^3*x^3 + b^3*c^3 - 3*b^2*c^2*d + 3*(b^3*c*d^2 - b^2*d^3)*x^2 +
3*(b^3*c^2*d - 2*b^2*c*d^2)*x)*cosh(b*x + a) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*
c*d^2*x - I*b^2*c^2*d + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2
*I*d^3)*cosh(b*x + a)^4 + 4*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d
 + 2*I*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2
*x - I*b^2*c^2*d + 2*I*d^3)*sinh(b*x + a)^4 + 2*I*d^3 + 2*(-I*b^2*d^3*x^2 -
 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cosh(b*x + a)^2 + 2*(-I*b^2*d^3*x
```

^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3 + 3*(-I*b^2*d^3*x^2 - 2*I*b^2*
c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((-I*
b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cosh(b*x + a)^3 + (-
I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*x - I*b^2*c^2*d + 2*I*d^3)*cosh(b*x + a))*sin
h(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(I*b^2*d^3*x^2 + 2
*I*b^2*c*d^2*x + I*b^2*c^2*d + (I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2
*d - 2*I*d^3)*cosh(b*x + a)^4 + 4*(I*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*
c^2*d - 2*I*d^3)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*d^3*x^2 + 2*I*b^2*c
*d^2*x + I*b^2*c^2*d - 2*I*d^3)*sinh(b*x + a)^4 - 2*I*d^3 + 2*(I*b^2*d^3*x^
2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cosh(b*x + a)^2 + 2*(I*b^2*d^3
*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3 + 3*(I*b^2*d^3*x^2 + 2*I*b^2
*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((I*
b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cosh(b*x + a)^3 + (I
*b^2*d^3*x^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d - 2*I*d^3)*cosh(b*x + a))*sinh
(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (I*b^3*c^3 - 3*I*a*b
^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2
- 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^4 - 4*(-I*b^3*c^3 + 3*I*a*b
^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a)*sinh(b*
x + a)^3 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 -
6*a)*d^3)*sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 - 2*(-I*b^3*c^3 + 3*I*a*b^2*c
^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^2 - 2*(-I*b
^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*(-
I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*co
sh(b*x + a)^2)*sinh(b*x + a)^2 - 4*((-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^
2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^3 + (-I*b^3*c^3 + 3*I*a*b
^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a))*sinh(b
*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + (-I*b^3*c^3 + 3*I*a*b^2*c
^2*d - 3*I*(a^2 - 2)*b*c*d^2 + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2
)*b*c*d^2 + I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^4 - 4*(I*b^3*c^3 - 3*I*a*b^2*c
^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a)*sinh(b*x +
a)^3 + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*(a^2 - 2)*b*c*d^2 + I*(a^3 - 6*a
)*d^3)*sinh(b*x + a)^4 + I*(a^3 - 6*a)*d^3 - 2*(I*b^3*c^3 - 3*I*a*b^2*c^2*d
 + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^2 - 2*(I*b^3*c^
3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3 + 3*(I*b^3*
c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x
 + a)^2)*sinh(b*x + a)^2 - 4*((I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*(a^2 - 2)*
b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a)^3 + (I*b^3*c^3 - 3*I*a*b^2*c^2*d
 + 3*I*(a^2 - 2)*b*c*d^2 - I*(a^3 - 6*a)*d^3)*cosh(b*x + a))*sinh(b*x + a))
*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x
^2 - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^
2 - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d
- 2*b*d^3)*x)*cosh(b*x + a)^4 - 4*(I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x^2 + 3*I*
a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b^3*c^2*d - 2*b*d^
3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^3*d^3*x^3 - 3*I*b^3*c*d^2*x^2 -
 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*(a^3 - 6*a)*d^3 - 3*I*(b^3*c^2*d - 2

```
*b*d^3)*x)*sinh(b*x + a)^4 - I*(a^3 - 6*a)*d^3 - 2*(I*b^3*d^3*x^3 + 3*I*b^3
*c*d^2*x^2 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*I*(b
^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 - 2*(I*b^3*d^3*x^3 + 3*I*b^3*c*d^2*x
^2 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 + 3*(I*b^3*d^3*x
^3 + 3*I*b^3*c*d^2*x^2 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*
d^3 + 3*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^2 + 3*I*(b^3*c^2*d - 2*b*d
^3)*x)*sinh(b*x + a)^2 - 3*I*(b^3*c^2*d - 2*b*d^3)*x - 4*((I*b^3*d^3*x^3 +
3*I*b^3*c*d^2*x^2 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I*(a^3 - 6*a)*d^3 +
 3*I*(b^3*c^2*d - 2*b*d^3)*x)*cosh(b*x + a)^3 + (I*b^3*d^3*x^3 + 3*I*b^3*c*
d^2*x^2 + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^2 + I...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^3 \operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**3*sech(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3*sech(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c+d\,x\right)}^3}{{\mathrm{cosh}\left(a+b\,x\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/cosh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)^3/cosh(a + b*x)^3, x)
```

3.37 $\int (c+dx)^2 \mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=175

$$\frac{(c+dx)^2\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{d^2\text{ArcTan}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id(c+dx)\text{PolyLog}(2}{b^2}$$

```
[Out] (d*x+c)^2*arctan(exp(b*x+a))/b-d^2*arctan(sinh(b*x+a))/b^3-I*d*(d*x+c)*poly
log(2,-I*exp(b*x+a))/b^2+I*d*(d*x+c)*polylog(2,I*exp(b*x+a))/b^2+I*d^2*poly
log(3,-I*exp(b*x+a))/b^3-I*d^2*polylog(3,I*exp(b*x+a))/b^3+d*(d*x+c)*sech(b
*x+a)/b^2+1/2*(d*x+c)^2*sech(b*x+a)*tanh(b*x+a)/b
```

Rubi [A]
time = 0.11, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {4271, 3855, 4265, 2611, 2320, 6724}

$$-\frac{d^2\text{ArcTan}(\sinh(a+bx))}{b^3}+\frac{(c+dx)^2\text{ArcTan}\left(e^{a+bx}\right)}{b}+\frac{id^2\text{Li}_3\left(-ie^{a+bx}\right)}{b^3}-\frac{id^2\text{Li}_3\left(ie^{a+bx}\right)}{b^3}-\frac{id(c+dx)\text{Li}_2\left(-ie^{a+bx}\right)}{b^2}+\frac{id(c+dx)\text{Li}_2\left(ie^{a+bx}\right)}{b^2}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*Sech[a + b*x]^3,x]
```

```
[Out] ((c + d*x)^2*ArcTan[E^(a + b*x)])/b - (d^2*ArcTan[Sinh[a + b*x]])/b^3 - (I*
d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)])/b^2 + (I*d*(c + d*x)*PolyLog[2, I
*E^(a + b*x)])/b^2 + (I*d^2*PolyLog[3, (-I)*E^(a + b*x)])/b^3 - (I*d^2*Poly
Log[3, I*E^(a + b*x)])/b^3 + (d*(c + d*x)*Sech[a + b*x])/b^2 + ((c + d*x)^2
*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2\operatorname{sech}^3(a+bx)\,dx &= \frac{d(c+dx)\operatorname{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}+\frac{1}{2}\int (c+dx)^2\\ &= \frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\operatorname{sech}(a+bx)}{b^2}+\\ &= \frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2}\\ &= \frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2}\\ &= \frac{(c+dx)^2\tan^{-1}\left(e^{a+bx}\right)}{b}-\frac{d^2\tan^{-1}(\sinh(a+bx))}{b^3}-\frac{id(c+dx)\mathrm{Li}_2\left(-ie^{a+bx}\right)}{b^2}\end{aligned}$$

Mathematica [A]

time = 3.99, size = 270, normalized size = 1.54

$$\frac{i\left(-2ib^2c^2\text{ArcTan}\left(e^{a+bx}\right)+4id^2\text{ArcTan}\left(e^{a+bx}\right)+2b^2cdx\log\left(1-ie^{a+bx}\right)+b^2d^2x^2\log\left(1-ie^{a+bx}\right)-2b^2cdx\log\left(1+ie^{a+bx}\right)-b^2d^2x^2\log\left(1+ie^{a+bx}\right)-2bd(c+dx)\text{PolyLog}\left(2,-ie^{a+bx}\right)+2bd(c+dx)\text{PolyLog}\left(2,ie^{a+bx}\right)+2d^2\text{PolyLog}\left(3,-ie^{a+bx}\right)-2d^2\text{PolyLog}\left(3,ie^{a+bx}\right)\right)+b^2(c+dx)^2\text{sech}(a)\text{sech}^2(a+bx)\sinh(bx)+b(c+dx)\text{sech}(a+bx)(2d+b(c+dx)\tanh(a))}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*Sech[a + b*x]^3,x]
```

```
[Out] (I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + (4*I)*d^2*ArcTan[E^(a + b*x)] + 2*
b^2*c*d*x*Log[1 - I*E^(a + b*x)] + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b
^2*c*d*x*Log[1 + I*E^(a + b*x)] - b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*
d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(
a + b*x)] + 2*d^2*PolyLog[3, (-I)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a +
b*x)]) + b^2*(c + d*x)^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + b*(c + d*x)*Se
ch[a + b*x]*(2*d + b*(c + d*x)*Tanh[a]))/(2*b^3)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^2\,\text{sech}(bx+a)^3\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*sech(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^2*sech(b*x+a)^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] b^2*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 2*b^2*c
*d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^2*(arctan(e^
(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) +
e^(-4*b*x - 4*a) + 1))) + ((b*d^2*x^2*e^(3*a) + 2*c*d*e^(3*a) + 2*(b*c*d +
d^2)*x*e^(3*a))*e^(3*b*x) - (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(b*c*d - d^2)*x*
e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - 2*d^2*a
rctan(e^(b*x + a))/b^3
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2651 vs. 2(150) = 300.
time = 0.43, size = 2651, normalized size = 15.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="fricas")

[Out] 1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*sinh(b*x + a)^3 - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2*c*d - b*d^2)*x)*cosh(b*x + a) - 2*((-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^4 + 4*(-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*sinh(b*x + a)^4 - I*b*d^2*x - I*b*c*d + 2*(-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^2 + 2*(-I*b*d^2*x - I*b*c*d + 3*(-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^3 + (-I*b*d^2*x - I*b*c*d)*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 2*((I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^4 + 4*(I*b*d^2*x + I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*sinh(b*x + a)^4 + I*b*d^2*x + I*b*c*d + 2*(I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^2 + 2*(I*b*d^2*x + I*b*c*d + 3*(I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 4*((I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + ((I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)^4 - 4*(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*sinh(b*x + a)^4 + I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2 - 2*(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)^2 - 2*(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2 + 3*(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*((-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)^3 + (-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) + I) + ((-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)^4 - 4*(I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*sinh(b*x + a)^4 - I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2 - 2*(I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)^2 - 2*(I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2 + 3*(I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*((I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)^3 + (I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a)^4 - 4*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*sinh(b*x + a)^4 - 2*I*a*b*c*d + I*a^2*d^2 - 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a)^2 - 2*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2 + 3*(I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*((I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a)^3 + (I*b^2*d^2*x^2 + 2*I*b

```
^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(I*cos
h(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + (I*b^2
*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*cosh(b*x + a)^4 - 4*(-I
*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a)*sinh(
b*x + a)^3 + (I*b^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*sinh
(b*x + a)^4 + 2*I*a*b*c*d - I*a^2*d^2 - 2*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x -
 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a)^2 - 2*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d
*x - 2*I*a*b*c*d + I*a^2*d^2 + 3*(-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x - 2*I*a*b*
c*d + I*a^2*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*((-I*b^2*d^2*x^2 - 2*
I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a)^3 + (-I*b^2*d^2*x^2 -
2*I*b^2*c*d*x - 2*I*a*b*c*d + I*a^2*d^2)*cosh(b*x + a))*sinh(b*x + a))*log(
-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*(I*d^2*cosh(b*x + a)^4 + 4*I*d^
2*cosh(b*x + a)*sinh(b*x + a)^3 + I*d^2*sinh(b*x + a)^4 + 2*I*d^2*cosh(b*x
+ a)^2 + 2*(3*I*d^2*cosh(b*x + a)^2 + I*d^2)*sinh(b*x + a)^2 + I*d^2 + 4*(I
*d^2*cosh(b*x + a)^3 + I*d^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, I*cos
h(b*x + a) + I*sinh(b*x + a)) - 2*(-I*d^2*cosh(b*x + a)^4 - 4*I*d^2*cosh(b*
x + a)*sinh(b*x + a)^3 - I*d^2*sinh(b*x + a)^4 - 2*I*d^2*cosh(b*x + a)^2 +
2*(-3*I*d^2*cosh(b*x + a)^2 - I*d^2)*sinh(b*x + a)^2 - I*d^2 + 4*(-I*d^2*co
sh(b*x + a)^3 - I*d^2*cosh(b*x + a))*sinh(b*x + a))*polylog(3, -I*cosh(b*x
+ a) - I*sinh(b*x + a)) - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d - 3*(b^2*d^2*x
^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 + 2*(b^2*c*
d - b*d^2)*x)*sinh(b*x + a))/(b^3*cosh(b*x + a)^4 + 4*b^3*cosh(b*x + a)*sin
h(b*x + a)^3 + b^3*sinh(b*x + a)^4 + 2*b^3*cosh(b*x + a)^2 + b^3 + 2*(3*b^3
*cosh(b*x + a)^2 + b^3)*sinh(b*x + a)^2 + 4*(b^3*cosh(b*x + a)^3 + b^3*cosh
(b*x + a))*sinh(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^2 \operatorname{sech}^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*sech(b*x+a)**3,x)
[Out] Integral((c + d*x)**2*sech(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="giac")
[Out] integrate((d*x + c)^2*sech(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^2}{\cosh\left(a+b\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/cosh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)^2/cosh(a + b*x)^3, x)
```

3.38 $\int (c+dx)\mathbf{sech}^3(a+bx)\,dx$

Optimal. Leaf size=102

$$\frac{(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{id\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2}+\frac{id\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2}+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\text{sech}(a+$$

```
[Out] (d*x+c)*arctan(exp(b*x+a))/b-1/2*I*d*polylog(2,-I*exp(b*x+a))/b^2+1/2*I*d*p
olylog(2,I*exp(b*x+a))/b^2+1/2*d*sech(b*x+a)/b^2+1/2*(d*x+c)*sech(b*x+a)*ta
nh(b*x+a)/b
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4270, 4265, 2317, 2438}

$$\frac{(c+dx)\text{ArcTan}\left(e^{a+bx}\right)}{b}-\frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2}+\frac{id\text{Li}_2\left(ie^{a+bx}\right)}{2b^2}+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*Sech[a + b*x]^3,x]
```

```
[Out] ((c + d*x)*ArcTan[E^(a + b*x)])/b - ((I/2)*d*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + ((I/2)*d*PolyLog[2, I*E^(a + b*x)])/b^2 + (d*Sech[a + b*x])/(2*b^2) +
 ((c + d*x)*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int (c+dx)\text{sech}^3(a+bx)\,dx = \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} + \frac{1}{2}\int (c+dx)\text{sech}(a+bx)\,dx$$

$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} +$$

$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b} +$$

$$= \frac{(c+dx)\tan^{-1}\left(e^{a+bx}\right)}{b} - \frac{id\text{Li}_2\left(-ie^{a+bx}\right)}{2b^2} + \frac{id\text{Li}_2\left(ie^{a+bx}\right)}{2b^2} + \frac{d\text{sech}(a+bx)}{2b^2} +$$

Mathematica [A]
time = 2.41, size = 178, normalized size = 1.75

$$\frac{bc\text{ArcTan}(\sinh(a+bx)) + \frac{1}{2}d\left(-\left((-2ia+\pi-2ibx)\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)\right)+(-2ia+\pi)\log\left(\cot\left(\frac{1}{4}(2ia+\pi+2ibx)\right)\right)-2i\left(\text{PolyLog}\left(2,-ie^{a+bx}\right)-\text{PolyLog}\left(2,ie^{a+bx}\right)\right)\right)+bdx\text{sech}(a)\text{sech}^2(a+bx)\sinh(bx)+d\text{sech}(a+bx)(1+bx\tanh(a))+bc\text{sech}(a+bx)\tanh(a+bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*Sech[a + b*x]^3,x]
```

```
[Out] (b*c*ArcTan[Sinh[a + b*x]] + (d*(-(((-2*I)*a + Pi - (2*I)*b*x)*(Log[1 - I*E
^(a + b*x)] - Log[1 + I*E^(a + b*x)])) + ((-2*I)*a + Pi)*Log[Cot[((2*I)*a +
 Pi + (2*I)*b*x)/4]] - (2*I)*(PolyLog[2, (-I)*E^(a + b*x)] - PolyLog[2, I*E
^(a + b*x)])))/2 + b*d*x*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + d*Sech[a + b*x
]*(1 + b*x*Tanh[a]) + b*c*Sech[a + b*x]*Tanh[a + b*x])/(2*b^2)
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. 2(87) = 174.
time = 1.19, size = 216, normalized size = 2.12

method	result
risch	$\frac{e^{bx+a}\left(bdx\,e^{2bx+2a}+bc\,e^{2bx+2a}-bdx+e^{2bx+2a}d-bc+d\right)}{b^2\left(e^{2bx+2a}+1\right)^2}+\frac{c\arctan\left(e^{bx+a}\right)}{b}-\frac{id\ln\left(1+ie^{bx+a}\right)x}{2b}-\frac{id\ln\left(1+ie^{bx+a}\right)a}{2b^2}+\frac{id\ln(1-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)-b*d*x+exp(2*b*x+2*a)*d-
b*c+d)/b^2/(exp(2*b*x+2*a)+1)^2+1/b*c*arctan(exp(b*x+a))-1/2*I/b*d*ln(1+I*e
xp(b*x+a))*x-1/2*I/b^2*d*ln(1+I*exp(b*x+a))*a+1/2*I/b*d*ln(1-I*exp(b*x+a))*
x+1/2*I/b^2*d*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*d*dilog(1+I*exp(b*x+a))+1/2*I/
b^2*d*dilog(1-I*exp(b*x+a))-1/b^2*d*a*arctan(exp(b*x+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4*
b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/8*x*e^(b*x + a)/(
e^(2*b*x + 2*a) + 1), x)) - c*(arctan(e^(-b*x - a))/b - (e^(-b*x - a) - e^(
-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1)))
```

Fricas [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1267 vs. 2(81) = 162.
time = 0.39, size = 1267, normalized size = 12.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x + a
)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 - 2*(b*d*x + b*c -
d)*cosh(b*x + a) + (I*d*cosh(b*x + a)^4 + 4*I*d*cosh(b*x + a)*sinh(b*x + a)
^3 + I*d*sinh(b*x + a)^4 + 2*I*d*cosh(b*x + a)^2 - 2*(-3*I*d*cosh(b*x + a)^
2 - I*d)*sinh(b*x + a)^2 - 4*(-I*d*cosh(b*x + a)^3 - I*d*cosh(b*x + a))*sin
h(b*x + a) + I*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-I*d*cosh(b*x
 + a)^4 - 4*I*d*cosh(b*x + a)*sinh(b*x + a)^3 - I*d*sinh(b*x + a)^4 - 2*I*d
*cosh(b*x + a)^2 - 2*(3*I*d*cosh(b*x + a)^2 + I*d)*sinh(b*x + a)^2 - 4*(I*d
*cosh(b*x + a)^3 + I*d*cosh(b*x + a))*sinh(b*x + a) - I*d)*dilog(-I*cosh(b*
x + a) - I*sinh(b*x + a)) + ((I*b*c - I*a*d)*cosh(b*x + a)^4 - 4*(-I*b*c +
I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*c - I*a*d)*sinh(b*x + a)^4 - 2*
(-I*b*c + I*a*d)*cosh(b*x + a)^2 - 2*(3*(-I*b*c + I*a*d)*cosh(b*x + a)^2 -
I*b*c + I*a*d)*sinh(b*x + a)^2 + I*b*c - I*a*d - 4*((-I*b*c + I*a*d)*cosh(b
*x + a)^3 + (-I*b*c + I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a
) + sinh(b*x + a) + I) + ((-I*b*c + I*a*d)*cosh(b*x + a)^4 - 4*(I*b*c - I*a
*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*c + I*a*d)*sinh(b*x + a)^4 - 2*(I
*b*c - I*a*d)*cosh(b*x + a)^2 - 2*(3*(I*b*c - I*a*d)*cosh(b*x + a)^2 + I*b*
c - I*a*d)*sinh(b*x + a)^2 - I*b*c + I*a*d - 4*((I*b*c - I*a*d)*cosh(b*x +
```

```
a)^3 + (I*b*c - I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(cosh(b*x + a) + si
nh(b*x + a) - I) + ((-I*b*d*x - I*a*d)*cosh(b*x + a)^4 - 4*(I*b*d*x + I*a*d
)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*d*x - I*a*d)*sinh(b*x + a)^4 - I*b*
d*x - 2*(I*b*d*x + I*a*d)*cosh(b*x + a)^2 - 2*(I*b*d*x + 3*(I*b*d*x + I*a*d
)*cosh(b*x + a)^2 + I*a*d)*sinh(b*x + a)^2 - I*a*d - 4*((I*b*d*x + I*a*d)*c
osh(b*x + a)^3 + (I*b*d*x + I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log(I*cosh
(b*x + a) + I*sinh(b*x + a) + 1) + ((I*b*d*x + I*a*d)*cosh(b*x + a)^4 - 4*(
-I*b*d*x - I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*d*x + I*a*d)*sinh(b*
x + a)^4 + I*b*d*x - 2*(-I*b*d*x - I*a*d)*cosh(b*x + a)^2 - 2*(-I*b*d*x + 3
*(-I*b*d*x - I*a*d)*cosh(b*x + a)^2 - I*a*d)*sinh(b*x + a)^2 + I*a*d - 4*((
-I*b*d*x - I*a*d)*cosh(b*x + a)^3 + (-I*b*d*x - I*a*d)*cosh(b*x + a))*sinh(
b*x + a))*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 2*(b*d*x - 3*(b*d*x
 + b*c + d)*cosh(b*x + a)^2 + b*c - d)*sinh(b*x + a))/(b^2*cosh(b*x + a)^4
+ 4*b^2*cosh(b*x + a)*sinh(b*x + a)^3 + b^2*sinh(b*x + a)^4 + 2*b^2*cosh(b*
x + a)^2 + 2*(3*b^2*cosh(b*x + a)^2 + b^2)*sinh(b*x + a)^2 + b^2 + 4*(b^2*c
osh(b*x + a)^3 + b^2*cosh(b*x + a))*sinh(b*x + a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx) \operatorname{sech}^3(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)*sech(a + b*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)*sech(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{c + d\,x}{{\cosh\left(a + b\,x\right)}^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/cosh(a + b*x)^3,x)
```

```
[Out] int((c + d*x)/cosh(a + b*x)^3, x)
```

3.39 $\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{c+dx},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)^3/(d*x+c),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]^3/(c + d*x),x]
[Out] Defer[Int][Sech[a + b*x]^3/(c + d*x), x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx$$

Mathematica [F]
time = 180.02, size = 0, normalized size = 0.00

$$\$\text{Aborted}$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^3/(c + d*x),x]
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3}{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3/(d*x+c),x)
```

```
[Out] int(sech(b*x+a)^3/(d*x+c),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

```
[Out] ((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + d)*e^a
)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) + 2*
b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) + 2*(b^2*d^2*x^2*e^(2*a) + 2
*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^2*d^2
*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 2*d^2)*e^a)*e^(b*x)/(b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^(2*a) + 3*b^2*c*
d^2*x^2*e^(2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3/(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^{3}\left(a + bx\right)}{c + dx}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3/(d*x+c),x)
```

```
[Out] Integral(sech(a + b*x)**3/(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3/(d*x + c), x)
```

Mupad [A]

time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh\left(a+b\,x\right)^3\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*(c + d*x)),x)
```

```
[Out] int(1/(cosh(a + b*x)^3*(c + d*x)), x)
```

3.40 $\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} \, dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)^3/(d*x+c)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] Defer[Int][Sech[a + b*x]^3/(c + d*x)^2, x]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} \, dx = \int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} \, dx$$

Mathematica [F]
time = 180.01, size = 0, normalized size = 0.00

$$\$Aborted$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]^3/(c + d*x)^2,x]
```

```
[Out] $Aborted
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}(bx+a)^3}{(dx+c)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)^3/(d*x+c)^2,x)
```

```
[Out] int(sech(b*x+a)^3/(d*x+c)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] ((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + 2*d)
*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (
b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) + b^2
*c^3*e^(4*a))*e^(4*b*x) + 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2*a)
+ 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^
2*d^2*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 6*d^2)*e^a)*e^(b*x)/(b^2*d^4*x
^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 + (b^2*d
^4*x^4*e^(2*a) + 4*b^2*c*d^3*x^3*e^(2*a) + 6*b^2*c^2*d^2*x^2*e^(2*a) + 4*b^
2*c^3*d*x*e^(2*a) + b^2*c^4*e^(2*a))*e^(2*b*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(sech(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)**3/(d*x+c)**2,x)
```

```
[Out] Integral(sech(a + b*x)**3/(c + d*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate(sech(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\cosh\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)^3*(c + d*x)^2),x)
```

```
[Out] int(1/(cosh(a + b*x)^3*(c + d*x)^2), x)
```

3.41 $\int (c+dx)^{5/2}\cosh(a+bx)\,dx$

Optimal. Leaf size=171

$$-\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2}+\frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}$$

```
[Out] -5/2*d*(d*x+c)^(3/2)*cosh(b*x+a)/b^2+(d*x+c)^(5/2)*sinh(b*x+a)/b+15/16*d^(5
/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-15/16
*d^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)+
15/4*d^2*sinh(b*x+a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3389, 2211, 2235, 2236}

$$\frac{15\sqrt{\pi}\,d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}-\frac{15\sqrt{\pi}\,d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sinh(a+bx)}{4b^3}-\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x],x]
```

```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(2*b^2) + (15*d^(5/2)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)) - (15*d^(5/2)*
E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(7/2)
) + (15*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b^3) + ((c + d*x)^(5/2)*Sinh[a
+ b*x])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx = \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} - \frac{(5d)\int (c+dx)^{3/2}\sinh(a+bx)\,dx}{2b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b} + \frac{(15d^2)\int \sqrt{c+dx}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\,\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin}{b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\,\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin}{b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^2\sqrt{c+dx}\,\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sin}{b}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} + \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - 15d$$

Mathematica [A]
time = 0.04, size = 107, normalized size = 0.63

$$-\frac{d^3e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x],x]
[Out] -1/2*(d^3*E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -((
b*(c + d*x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c + d
*x))/d]))/(b^4*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}} \cosh(bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(131) = 262.
time = 0.27, size = 308, normalized size = 1.80

$$\frac{32\,(dx+c)^{\frac{7}{2}}\cosh(bx+a) - \frac{\left(\frac{105\sqrt{\pi}\,d^4\,\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}} - \frac{105\sqrt{\pi}\,d^4\,\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}} + \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^{\left(\frac{bc}{d}\right)} + 28\,(dx+c)^{\frac{5}{2}}b^2d^2e^{\left(\frac{bc}{d}\right)} + 70\,(dx+c)^{\frac{3}{2}}bd^3e^{\left(\frac{bc}{d}\right)} + 105\sqrt{dx+c}\,d^4e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^4} + \frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3de^a - 28\,(dx+c)^{\frac{5}{2}}b^2d^2e^a + 70\,(dx+c)^{\frac{3}{2}}bd^3e^a - 105\sqrt{dx+c}\,d^4e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^4}\right)b}{d}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/112*(32*(d*x + c)^(7/2)*cosh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) - 105*sqrt(pi)*d^4*erf(sqrt(d
*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) + 2*(8*(d*x + c)^(7/2)*b^
3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/2)*b
*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^
4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a + 70*(d
*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/d - b*c
/d)/b^4)*b/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(131) = 262.
time = 0.37, size = 523, normalized size = 3.06

$$\frac{15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) - d^3\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right) + 15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) + d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) + d^3\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right) - 2\left(4b^3d^2x^2 + 4b^3c^2 + 10b^2cd + 15bd^2 - \left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\cosh(bx+a)^2 - 2\left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\cosh(bx+a)\sinh(bx+a) - \left(4b^3d^2x^2 + 4b^3c^2 - 10b^2cd + 15bd^2 + 2\left(4b^3cd - 5b^2d^2\right)x\right)\sinh(bx+a)^2 + 2\left(4b^3cd + 5b^2d^2\right)x\right)\sqrt{dx+c}}{16\left(b^4\cosh(bx+a) + b^4\sinh(bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x +
a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a*d)
/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 15*sqrt(pi)*(d
^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*d)
/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*
sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 1
```

```
0*b^2*c*d + 15*b*d^2 - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 +
 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 - 2*(4*b^3*d^2*x^2 + 4*b^3*c^
2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)*sinh
(b*x + a) - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c
*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*sqrt(d*x
+ c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}}\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(5/2)*cosh(a + b*x), x)
```

Giac [A]
time = 0.47, size = 232, normalized size = 1.36

$$-\frac{\frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}\,b^3}-\frac{15\sqrt{\pi}d^4\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}\,b^3}-\frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d-10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+c}\,d^3\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}+\frac{2\left(4(dx+c)^{\frac{5}{2}}b^2d+10(dx+c)^{\frac{3}{2}}bd^2+15\sqrt{dx+c}\,d^3\right)e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(s
qrt(b*d)*b^3) - 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c -
 a*d)/d)/(sqrt(-b*d)*b^3) - 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3/2)
*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*(4*(
d*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(
-((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^(5/2),x)
```

```
[Out] int(cosh(a + b*x)*(c + d*x)^(5/2), x)
```

3.42 $\int (c+dx)^{3/2}\cosh(a+bx)\,dx$

Optimal. Leaf size=146

$$-\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2}+\frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+($$

[Out] (d*x+c)^(3/2)*sinh(b*x+a)/b+3/8*d^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+3/8*d^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-3/2*d*cosh(b*x+a)*(d*x+c)^(1/2)/b^2

Rubi [A]
time = 0.17, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,d^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}+\frac{3\sqrt{\pi}\,d^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}}-\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x],x]

[Out] (-3*d*Sqrt[c + d*x]*Cosh[a + b*x])/(2*b^2) + (3*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + (3*d^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*b^(5/2)) + ((c + d*x)^(3/2)*Sinh[a + b*x])/b

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} - \frac{(3d)\int \sqrt{c+dx}\,\sinh(a+bx)\,dx}{2b}$$

$$= -\frac{3d\sqrt{c+dx}\,\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{(3d^2)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,\cdots}{4b^2}$$

$$= -\frac{3d\sqrt{c+dx}\,\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{(3d^2)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,\cdots}{8b^2}$$

$$= -\frac{3d\sqrt{c+dx}\,\cosh(a+bx)}{2b^2} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b} + \frac{(3d)\text{Subst}\left(\int e^{i(ia\cdots}\right.}{\cdots}$$

$$= -\frac{3d\sqrt{c+dx}\,\cosh(a+bx)}{2b^2} + \frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3d^{3/2}e\cdots}{\cdots}$$

Mathematica [A]
time = 0.08, size = 107, normalized size = 0.73

$$\frac{de^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(-\frac{e^{2a}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}}-\frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x],x]
```

```
[Out] (d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d)
])/Sqrt[-((b*(c + d*x))/d)]) - (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d])/
Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}} \cosh (bx+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.
time = 0.29, size = 268, normalized size = 1.84

$$\frac{16\,(dx+c)^{\frac{5}{2}}\cosh (bx+a)+\frac{\left(\frac{15\,\sqrt{\pi}\,d^3\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}+\frac{15\,\sqrt{\pi}\,d^3\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10\,(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\,\sqrt{dx+c}\,d^3e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^3}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^a-10\,(dx+c)^{\frac{3}{2}}bd^2e^a+15\,\sqrt{dx+c}\,d^3e^a\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^3}\right)b}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x + c)^(5/2)*cosh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x
+ c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) - 2*(4*(d*x + c)^(5/2)*b^2*d
*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*e^(b
*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 10*(d*
x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d - b*c/d
)/b^3)*b/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(110) = 220$.
time = 0.37, size = 387, normalized size = 2.65

$$\frac{3\,\sqrt{\pi}\left(d^2\cosh (bx+a)\cosh \left(-\frac{bc-ad}{d}\right)-d^2\cosh (bx+a)\sinh \left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh \left(-\frac{bc-ad}{d}\right)-d^2\sinh \left(-\frac{bc-ad}{d}\right)\right)\sinh (bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)-3\,\sqrt{\pi}\left(d^2\cosh (bx+a)\cosh \left(-\frac{bc-ad}{d}\right)+d^2\cosh (bx+a)\sinh \left(-\frac{bc-ad}{d}\right)+\left(d^2\cosh \left(-\frac{bc-ad}{d}\right)+d^2\sinh \left(-\frac{bc-ad}{d}\right)\right)\sinh (bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)-2\left(2\,b^2dx+2\,b^2c-(2\,b^2dx+2\,b^2c-3\,bd)\cosh (bx+a)^2-2\,(2\,b^2dx+2\,b^2c-3\,bd)\cosh (bx+a)\sinh (bx+a)-(2\,b^2dx+2\,b^2c-3\,bd)\sinh (bx+a)^2+3\,bd\right)\sqrt{dx+c}}{8\left(b^3\cosh (bx+a)+b^3\sinh (bx+a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)
*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 3*sqrt(pi)*(d^2*
cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d)
 + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c - (2*b^2*d*x
 + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 - 2*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(
```

```
b*x + a)*sinh(b*x + a) - (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x + a)^2 + 3*
b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cosh(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a),x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x), x)
```

Giac [A]
time = 0.45, size = 202, normalized size = 1.38

$$-\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2}+\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2}-\frac{2\left(2(dx+c)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}+\frac{2\left(2(dx+c)^{\frac{3}{2}}bd+3\sqrt{dx+c}d^2\right)e^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] -1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqr
t(b*d)*b^2) + 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*
d)/d)/(sqrt(-b*d)*b^2) - 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)*e^
(((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d*x +
 c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)\,{\left(c+d\,x\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)*(c + d*x)^(3/2), x)
```

3.43 $\int \sqrt{c+dx} \cosh(a+bx)\, dx$

Optimal. Leaf size=123

$$\frac{\sqrt{d}\, e^{-a+\frac{bc}{d}} \sqrt{\pi}\, \text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}\, e^{a-\frac{bc}{d}} \sqrt{\pi}\, \text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\, \sinh(a+bx)}{b}$$

```
[Out] 1/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/
2)-1/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^
(3/2)+sinh(b*x+a)*(d*x+c)^(1/2)/b
```

Rubi [A]

time = 0.13, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3377, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\sqrt{d}\, e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{\pi}\sqrt{d}\, e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\, \sinh(a+bx)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

```
[Out] (Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4
*b^(3/2)) - (Sqrt[d]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(4*b^(3/2)) + (Sqrt[c + d*x]*Sinh[a + b*x])/b
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{c+dx}\cosh(a+bx)\,dx &= \frac{\sqrt{c+dx}\,\sinh(a+bx)}{b} - \frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{2b}\\ &= \frac{\sqrt{c+dx}\,\sinh(a+bx)}{b} - \frac{d\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b} + \frac{d\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4b}\\ &= \frac{\sqrt{c+dx}\,\sinh(a+bx)}{b} + \frac{\operatorname{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2b} - \frac{\operatorname{Subst}\Big(}{}\\ &= \frac{\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \end{aligned}$$

Mathematica [A]
time = 0.09, size = 105, normalized size = 0.85

$$\frac{e^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(\frac{e^{2a}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/S
qrt[-((b*(c + d*x))/d)] - (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqrt[
(b*(c + d*x))/d]))/(2*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \cosh(bx+a)\sqrt{dx+c}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.
time = 0.26, size = 230, normalized size = 1.87

$$\frac{8\,(dx+c)^{\frac{3}{2}}\cosh\left(bx+a\right)-\frac{\left(\frac{3\,\sqrt{\pi}\,d^{2}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^{2}\sqrt{-\frac{b}{d}}}-\frac{3\,\sqrt{\pi}\,d^{2}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^{2}\sqrt{\frac{b}{d}}}+\frac{2\left(2\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\,\sqrt{dx+c}\,d^{2}e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^{2}}+\frac{2\left(2\,(dx+c)^{\frac{3}{2}}bde^{a}-3\,\sqrt{dx+c}\,d^{2}e^{a}\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^{2}}\right)b}{d}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x + c)^(3/2)*cosh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*s
qrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) + 2*(2*(d*x + c)^(3/2)*b*d*e^(b
*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(2*(d
*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/
b^2)*b/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(91) = 182$.
time = 0.37, size = 302, normalized size = 2.46

$$\frac{\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)-d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)+\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)+d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(d\cosh\left(-\frac{bc-ad}{d}\right)+d\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{-\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)+2\left(b\cosh\left(bx+a\right)^{2}+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^{2}-b\right)\sqrt{dx+c}}{4\left(b^{2}\cosh\left(bx+a\right)+b^{2}\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(
-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*
x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*(d*cosh(b*x + a)*
cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c
 - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x
 + c)*sqrt(-b/d)) + 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a)
+ b*sinh(b*x + a)^2 - b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*sinh(b*x +
 a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\cosh\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x), x)
```

Giac [A]
time = 0.44, size = 169, normalized size = 1.37

$$-\frac{\frac{\sqrt{\pi}\, d^{2} \operatorname{erf}\left(-\frac{\sqrt{bd}\,\sqrt{dx+c}}{d}\right) e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}\, b} - \frac{\sqrt{\pi}\, d^{2} \operatorname{erf}\left(-\frac{\sqrt{-bd}\,\sqrt{dx+c}}{d}\right) e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}\, b} - \frac{2\sqrt{dx+c}\, de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}\, de^{\left(-\frac{(dx+c)b-bc+ad}{d}\right)}}{b}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt(
b*d)*b) - sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d)/
(sqrt(-b*d)*b) - 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2*sq
rt(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^(1/2),x)
```

```
[Out] int(cosh(a + b*x)*(c + d*x)^(1/2), x)
```

3.44 $\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=104

$$\frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}+\frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}$$

```
[Out] 1/2*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/
2)+1/2*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^
(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}+\frac{\sqrt{\pi}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\,\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]
)/(2*Sqrt[b]*Sqrt[d])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{1}{2}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx + \frac{1}{2}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx$$
$$= \frac{\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d} + \frac{\text{Subst}\left(\int e^{-i\left(ia-\frac{ibc}{d}\right)+\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d}$$
$$= \frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

Mathematica [A]
time = 0.03, size = 105, normalized size = 1.01

$$\frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*
x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]))
/(2*b*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(74) = 148$.

time = 0.28, size = 180, normalized size = 1.73

$$\frac{4\sqrt{dx+c}\cosh(bx+a) + \frac{\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} + \frac{\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c}\,de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b} - \frac{2\sqrt{dx+c}\,de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}\right)b}{d}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x + c)*cosh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b/
d))*e^(a - b*c/d)/(b*sqrt(-b/d)) + sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d))*
e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c
/d)/b - 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Fricas [A]

time = 0.36, size = 123, normalized size = 1.18

$$\frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(s
qrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sinh(
-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cosh(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]

time = 0.44, size = 89, normalized size = 0.86

$$-\frac{\left(\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-2ad}{d}\right)}}{\sqrt{-bd}}\right)e^{(-a)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^(b*c/d)/sqrt(b*d) + sqrt
(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - 2*a*d)/d)/sqrt(-b*d))*e^
(-a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^(1/2),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x)^(1/2), x)
```

3.45 $\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=119

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)+
exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3/2)-2
*cosh(b*x+a)/d/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3378, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\,\sqrt{b}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\,\sqrt{b}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[Pi]*E
rf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(a - (b*c)/d)*Sqr
t[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}} + \frac{(2b)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}} + \frac{b\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d} - \frac{b\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d} \\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}} - \frac{(2b)\text{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{d^2} + \frac{(2b)\text{Subst}\left(\int e^{-i\left(ia\right.}\right.}{} \\
&= -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}} - \frac{\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+\right.}{\sqrt{d}}}{d^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 118, normalized size = 0.99

$$\frac{e^{-a}\left(-e^{-bx}\left(1+e^{2(a+bx)}\right)+e^{\frac{bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},b\left(\frac{c}{d}+x\right)\right)+e^{2a-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

```
[Out] (-((1 + E^(2*(a + b*x)))/E^(b*x)) + E^((b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma
[1/2, b*(c/d + x)] + E^(2*a - (b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
-((b*(c + d*x))/d)])/(d*E^a*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh (bx + a)}{(dx + c)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.29, size = 104, normalized size = 0.87

$$\frac{\frac{\left(\frac{\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{dx + c}\sqrt{-\frac{b}{d}}\right) e^{\left(a - \frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} - \frac{\sqrt{\pi}\, \operatorname{erf}\left(\sqrt{dx + c}\sqrt{\frac{b}{d}}\right) e^{\left(-a + \frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right) b}{d} - \frac{2 \cosh(bx+a)}{\sqrt{dx + c}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] ((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) - sqrt(pi
)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*cosh(b*x +
 a)/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(91) = 182$.
time = 0.35, size = 338, normalized size = 2.84

$$-\frac{\sqrt{\pi}\left((dx + c)\cosh (bx + a)\cosh \left(-\frac{bc - ad}{d}\right) - (dx + c)\cosh (bx + a)\sinh \left(-\frac{bc - ad}{d}\right) + \left((dx + c)\cosh \left(-\frac{bc - ad}{d}\right) - (dx + c)\sinh \left(-\frac{bc - ad}{d}\right)\right)\sinh (bx + a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx + c}\sqrt{\frac{b}{d}}\right) + \sqrt{\pi}\left((dx + c)\cosh (bx + a)\cosh \left(-\frac{bc - ad}{d}\right) + (dx + c)\cosh (bx + a)\sinh \left(-\frac{bc - ad}{d}\right) + \left((dx + c)\cosh \left(-\frac{bc - ad}{d}\right) + (dx + c)\sinh \left(-\frac{bc - ad}{d}\right)\right)\sinh (bx + a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{dx + c}\sqrt{-\frac{b}{d}}\right) + \sqrt{dx + c}\left(\cosh (bx + a)^2 + 2\cosh (bx + a)\sinh (bx + a) + \sinh (bx + a)^2 + 1\right)}{(d^2x + cd)\cosh (bx + a) + (d^2x + cd)\sinh (bx + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b
*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*
sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d))
 + sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(
b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x + c)
*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/
d)) + sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh
```

```
(b*x + a)^2 + 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh(b*x + a
))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{(a + bx)}}{(c + dx)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a + b\,x)}{{(c + d\,x)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)/(c + d*x)^(3/2), x)
```

3.46 $\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=149

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\sinh(a+}{3d^2\sqrt{c+d}}$$

```
[Out] -2/3*cosh(b*x+a)/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x
+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)*(
d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)-4/3*b*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3378, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{\pi}\,b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}\,b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2*b^(3/2)*E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) - (4*b
*Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx &= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{(2b)\int\frac{\sinh(a+bx)}{(c+dx)^{3/2}}\,dx}{3d}\\
&= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{(4b^2)\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{(2b^2)\int\frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}+\frac{(2b^2)\int\frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{3d^2}\\
&= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}+\frac{(4b^2)\operatorname{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{3d^3}+\cdots\\
&= -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c\cdots}}{\sqrt{d}}\right.}{3d^{5/2}}\cdots
\end{aligned}
$$

Mathematica [A]
time = 0.51, size = 150, normalized size = 1.01

$$
\frac{e^{-a}\left(-e^{-bx}\left(d\left(1+e^{2(a+bx)}\right)+2b\left(-1+e^{2(a+bx)}\right)(c+dx)+2de^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},b\left(\frac{c}{d}+x\right)\right)\right)-2de^{2a-\frac{bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)}{3d^2(c+dx)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(5/2),x]

[Out] (-((d*(1 + E^(2*(a + b*x))) + 2*b*(-1 + E^(2*(a + b*x)))*(c + d*x) + 2*d*E^
(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, b*(c/d + x)])/E^(b*x)) - 2
```

```
*d*E^(2*a - (b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/
d)])/(3*d^2*E^a*(c + d*x)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh \left(bx+a\right)}{\left(dx+c\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.33, size = 115, normalized size = 0.77

$$\frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}\, e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2}, \frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}} - \frac{\sqrt{-\frac{(dx+c)b}{d}}\, e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2}, -\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d} - \frac{2\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*
x + c) - sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqr
t(d*x + c))*b/d - 2*cosh(b*x + a)/(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(111) = 222.
time = 0.37, size = 534, normalized size = 3.58

$$\frac{2\sqrt{\pi}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left(bd^2x^2+2\,bcdx+bc^2\right)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-2\sqrt{\pi}\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left(\left(bd^2x^2+2\,bcdx+bc^2\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(bd^2x^2+2\,bcdx+bc^2\right)\sinh\left(-\frac{bc-ad}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{-\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+\left(2\,bdx-\left(2\,bdx+2\,bc+d\right)\cosh\left(bx+a\right)^2-2\left(2\,bdx+2\,bc+d\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-\left(2\,bdx+2\,bc+d\right)\sinh\left(bx+a\right)^2+2\,bc-d\right)\sqrt{dx+c}}{3\left(\left(d^4x^2+2\,cd^3x+c^2d^2\right)\cosh\left(bx+a\right)+\left(d^4x^2+2\,cd^3x+c^2d^2\right)\sinh\left(bx+a\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c -
 a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/
d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2
*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d
*x + c)*sqrt(b/d)) - 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*si
```

```
nh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt
(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x - (2*b*d*x + 2*b*c + d)*cos
h(b*x + a)^2 - 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a) - (2*b*d
*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^4*x^2 + 2*c
*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x
+ a))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(5/2),x)
[Out] Integral(cosh(a + b*x)/(c + d*x)**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^(5/2),x)
[Out] int(cosh(a + b*x)/(c + d*x)^(5/2), x)
```

3.47 $\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}$$

```
[Out] -2/5*cosh(b*x+a)/d/(d*x+c)^(5/2)-4/15*b*sinh(b*x+a)/d^2/(d*x+c)^(3/2)-4/15*
b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(7/2)+4
/15*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(7/
2)-8/15*b^2*cosh(b*x+a)/d^3/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {3378, 3389, 2211, 2235, 2236}

$$-\frac{4\sqrt{\pi}\,b^{5/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{4\sqrt{\pi}\,b^{5/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}}-\frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (-2*Cosh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (8*b^2*Cosh[a + b*x])/(15*d^3*Sq
rt[c + d*x]) - (4*b^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]])/(15*d^(7/2)) + (4*b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (4*b*Sinh[a + b*x])/(15*d^2*(c
 + d*x)^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
```

```
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx &= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} + \frac{(2b)\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}\,dx}{5d} \\
&= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(4b^2)\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{15d^2} \\
&= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(8b^3)\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{15d^3} \\
&= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} + \frac{(4b^3)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{15d^3} - \frac{(4b^3)}{\cdots} \\
&= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(8b^3)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}} \cdots\right.}{15d^4} \\
&= -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.27, size = 191, normalized size = 1.10

$$
\frac{e^{-a}\left(2e^{2a}\left(-3d^2e^{bx}-2be^{-\frac{bc}{d}}(c+dx)\left(e^{b\left(\frac{c}{d}+x\right)}(d+2b(c+dx))+2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)\right)\right)+e^{-bx}\left(-6d^2+4bd(c+dx)-8b^2(c+dx)^2+8d^2e^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{5/2}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)\right)}{30d^3(c+dx)^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```

```
[Out] (2*E^(2*a)*(-3*d^2*E^(b*x) - (2*b*(c + d*x)*(E^(b*(c/d + x))*(d + 2*b*(c +
d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]))/E^(
(b*c)/d)) + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(c/d
 + x))*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (b*(c + d*x))/d])/E^(b*x))/(30*d^
3*E^a*(c + d*x)^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.32, size = 115, normalized size = 0.66

$$\frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}} - \frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}} e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d} - \frac{2\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] 1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x
+ c)^(3/2) - (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/
d)/(d*x + c)^(3/2))*b/d - 2*cosh(b*x + a)/(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 853 vs. $2(132) = 264$.
time = 0.41, size = 853, normalized size = 4.90

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 +
```

```
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh
(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt(pi)*((b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a
*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
 + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*
x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*
c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt
(d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + (4*b^2*d^2*x
^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2
 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)*sinh(b*x + a) + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^
2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2
)*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*cosh(b
*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)*sinh(b*x + a))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh{\left(a+bx \right)}}{\left(c+dx\right)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)**(7/2),x)
[Out] Integral(cosh(a + b*x)/(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\mathrm{cosh}\left(a+b\,x\right)}{{\left(c+d\,x\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)/(c + d*x)^(7/2),x)
[Out] int(cosh(a + b*x)/(c + d*x)^(7/2), x)
```

3.48 $\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=239

$$\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\cdots$$

```
[Out] 5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d-5/8*d*(d*x+c)^(3/2)*cosh(b*x+a
)^2/b^2+1/2*(d*x+c)^(5/2)*cosh(b*x+a)*sinh(b*x+a)/b+15/512*d^(5/2)*exp(-2*a
+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(7/
2)-15/512*d^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2))*2^(1/2)*Pi^(1/2)/b^(7/2)+15/64*d^2*sinh(2*b*x+2*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.29, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3392, 32, 3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}-\frac{15\sqrt{\frac{\pi}{2}}\,d^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}}+\frac{15d^2\sqrt{c+dx}\,\sinh(2a+2bx)}{64b^3}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{5d(c+dx)^{3/2}}{16b^2}+\frac{(c+dx)^{7/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (5*d*(c + d*x)^(3/2))/(16*b^2) + (c + d*x)^(7/2)/(7*d) - (5*d*(c + d*x)^(3/
2)*Cosh[a + b*x]^2)/(8*b^2) + (15*d^(5/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*E
rf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(256*b^(7/2)) - (15*d^(5/2)*E^
(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(256*b^(7/2)) + ((c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (15*
d^2*Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(64*b^3)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} +$$

$$= \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh}{2b}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}}{}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}}{}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}}{}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{(c+dx)^{5/2}}{}$$

$$= \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{15d^{5/2}e^{-2a+}}{}$$

Mathematica [A]
time = 1.19, size = 189, normalized size = 0.79

$$\frac{\sqrt{c+dx}\left(-7\sqrt{2}\,d^4\sqrt{-\frac{b^2(c+dx)^2}{d^2}}\,\Gamma\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)+b(c+dx)\left(64b^3(c+dx)^3\sqrt{\frac{b(c+dx)}{d}}+7\sqrt{2}\,d^3\Gamma\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)\left(-\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)\right)}{448b^3d^2\left(\frac{b(c+dx)}{d}\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*(-7*Sqrt[2]*d^4*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*Gamma[7/2, (-
2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + b*(c +
d*x)*(64*b^3*(c + d*x)^3*Sqrt[(b*(c + d*x))/d] + 7*Sqrt[2]*d^3*Gamma[7/2, (
2*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))))/(448*
b^3*d^2*((b*(c + d*x))/d)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}} \left(\cosh^2\left(bx+a\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

[Out] int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)

Maxima [A]
time = 0.48, size = 281, normalized size = 1.18

$$\frac{512\,(dx+c)^{\frac{7}{2}} - \frac{105\,\sqrt{2}\,\sqrt{\pi}\,d^3\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{2}\,\sqrt{\pi}\,d^3\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{2\,bc}{d}\right)}+20\,(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{2\,bc}{d}\right)}+15\,\sqrt{dx+c}\,d^3e^{\left(\frac{2\,bc}{d}\right)}\right)e^{\left(-2\,a-\frac{2\,(dx+c)b}{d}\right)}}{b^3} + \frac{28\left(16\,(dx+c)^{\frac{5}{2}}b^2de^{(2\,a)}-20\,(dx+c)^{\frac{3}{2}}bd^2e^{(2\,a)}+15\,\sqrt{dx+c}\,d^3e^{(2\,a)}\right)e^{\left(\frac{2\,(dx+c)b}{d}-\frac{2\,bc}{d}\right)}}{b^3}}{3584\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/3584*(512*(d*x + c)^(7/2) - 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*x
 + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) + 105*sqrt(2)*sqrt(pi)
*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b/d)
) - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2*e^(
2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d)/b^3
 + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^(2*a)
+ 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. 2(183) = 366.
time = 0.39, size = 1001, normalized size = 4.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) -
d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*
(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(d^4*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(b*c -
a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*
x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)
*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2 - 7*(16*b
^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^
2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^
2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*
d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 - 128*(b^4*d^3*x^3 + 3*b^4*
c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 - 2*(64*b^4*d^3*x^3 +
192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 + 21*(16*b^3*d^3*x^2 + 16*
b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b
```

```
*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3)*x - 4*(7*(16*b^3*
d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d
^3)*x)*cosh(b*x + a)^3 + 64*(b^4*d^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x
+ b^4*c^3)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4*d*cosh(b*x + a
)^2 + 2*b^4*d*cosh(b*x + a)*sinh(b*x + a) + b^4*d*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{5}{2}}\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a)**2,x)
[Out] Integral((c + d*x)**(5/2)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^(5/2)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cosh(a+b\,x)}^2\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^(5/2),x)
[Out] int(cosh(a + b*x)^2*(c + d*x)^(5/2), x)
```

3.49 $\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=211

$$\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3d^{3/2}e^{2a}\cdots}{\cdots}$$

```
[Out] 1/5*(d*x+c)^(5/2)/d+1/2*(d*x+c)^(3/2)*cosh(b*x+a)*sinh(b*x+a)/b+3/128*d^(3/
2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^
(1/2)/b^(5/2)+3/128*d^(3/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(5/2)+3/16*d*(d*x+c)^(1/2)/b^2-3/8*d*cosh(
b*x+a)^2*(d*x+c)^(1/2)/b^2
```

Rubi [A]

time = 0.23, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3\sqrt{\frac{\pi}{2}}\,d^{3/2}e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (3*d*Sqrt[c + d*x])/(16*b^2) + (c + d*x)^(5/2)/(5*d) - (3*d*Sqrt[c + d*x]*C
osh[a + b*x]^2)/(8*b^2) + (3*d^(3/2)*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(S
qrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(5/2)) + (3*d^(3/2)*E^(2*a -
(2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^
(5/2)) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx = -\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b}+\frac{1}{2}\int$$

$$=\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a}{2b}$$

$$=\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\cosh}{}$$

$$=\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\cosh}{}$$

$$=\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\cosh}{}$$

$$=\frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\,\cosh^2(a+bx)}{8b^2}+\frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{}}{}$$

Mathematica [A]
time = 0.54, size = 163, normalized size = 0.77

$$\frac{32b^3(c+dx)^3+5\sqrt{2}\,d^3\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)\left(-\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)+5\sqrt{2}\,d^3\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)}{160b^3d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

```
[Out] (32*b^3*(c + d*x)^3 + 5*Sqrt[2]*d^3*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (2*b*(
c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]) + 5*Sqrt[2]*d
^3*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-2*b*(c + d*x))/d]*(Cosh[2*a - (2*b
*c)/d] + Sinh[2*a - (2*b*c)/d]))/(160*b^3*d*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}}\left(\cosh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.47, size = 239, normalized size = 1.13

$$\frac{128\,(dx+c)^{\frac{5}{2}} + \frac{15\,\sqrt{2}\,\sqrt{\pi}\,d^2\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{15\,\sqrt{2}\,\sqrt{\pi}\,d^2\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{20\left(4\,(dx+c)^{\frac{3}{2}}\,bde^{\left(\frac{2\,bc}{d}\right)}+3\,\sqrt{dx+c}\,d^2e^{\left(\frac{2\,bc}{d}\right)}\right)e^{\left(-2\,a-\frac{2\,(dx+c)b}{d}\right)}}{b^2} + \frac{20\left(4\,(dx+c)^{\frac{3}{2}}\,bde^{(2\,a)}-3\,\sqrt{dx+c}\,d^2e^{(2\,a)}\right)e^{\left(\frac{2\,(dx+c)b}{d}-\frac{2\,bc}{d}\right)}}{b^2}}{640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] 1/640*(128*(d*x + c)^(5/2) + 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x +
 c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) + 15*sqrt(2)*sqrt(pi)*d^
2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d)) -
 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d))*e
^(-2*a - 2*(d*x + c)*b/d)/b^2 + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*sqrt(
d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. 2(159) = 318.
time = 0.44, size = 755, normalized size = 3.58

$$\frac{15\,\sqrt{2}\,\sqrt{\pi}\left(d^3\cosh(bx+a)^2\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d^3\cosh(bx+a)^2\sinh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(d^3\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d^3\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sinh(bx+a)^2+2\left(d^3\cosh(bx+a)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d^3\cosh(bx+a)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)-15\,\sqrt{2}\,\sqrt{\pi}\left(d^3\cosh(bx+a)^2\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+d^3\cosh(bx+a)^2\sinh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(d^3\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+d^3\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sinh(bx+a)^2+2\left(d^3\cosh(bx+a)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+d^3\cosh(bx+a)\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\,\mathrm{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)-4\left(20\,b^2d^2x-5\left(4\,b^2d^2x+4\,b^2cd-3\,bd^2\right)\cosh(bx+a)^4-20\left(4\,b^2d^2x+4\,b^2cd-3\,bd^2\right)\cosh(bx+a)\sinh(bx+a)^3-5\left(4\,b^2d^2x+4\,b^2cd-3\,bd^2\right)\sinh(bx+a)^4+20\,b^2cd+15\,bd^2-32\left(b^3d^2x^2+2\,b^3cdx+b^3c^2\right)\cosh(bx+a)^2-2\left(16\,b^3d^2x^2+32\,b^3cdx+16\,b^3c^2+15\left(4\,b^2d^2x+4\,b^2cd-3\,bd^2\right)\cosh(bx+a)^2\right)\sinh(bx+a)^2-4\left(5\left(4\,b^2d^2x+4\,b^2cd-3\,bd^2\right)\cosh(bx+a)^3+16\left(b^3d^2x^2+2\,b^3cdx+b^3c^2\right)\cosh(bx+a)\right)\sinh(bx+a)\right)\sqrt{dx+c}}{640\left(b^3d\cosh(bx+a)^2+2\,b^3d\cosh(bx+a)\sinh(bx+a)+b^3d\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^
3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) - d^
3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b
*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqr
t(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3*cosh
(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d
)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
 a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sin
h(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*
x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*sinh(b*x +
a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 + 20*b^2*c*d +
 15*b*d^2 - 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x + a)^2 - 2*(1
6*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 + 15*(4*b^2*d^2*x + 4*b^2*c*d - 3
*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x + 4*b^2*c*d -
3*b*d^2)*cosh(b*x + a)^3 + 16*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*
x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)^2 + 2*b^3*d*cosh
(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)^2*(c + d*x)^(3/2), x)
```

3.50 $\int \sqrt{c+dx}\, \cosh^2(a+bx)\, dx$

Optimal. Leaf size=166

$$\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\, e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \text{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\, e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \text{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+}}{}$$

```
[Out] 1/3*(d*x+c)^(3/2)/d+1/32*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2
)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)-1/32*exp(2*a-2*b*c/d)*erfi(2^(1
/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*2^(1/2)*Pi^(1/2)/b^(3/2)+1/4*sin
h(2*b*x+2*a)*(d*x+c)^(1/2)/b
```

Rubi [A]

time = 0.21, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\, e^{\frac{2bc}{d}-2a}\text{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{d}\, e^{2a-\frac{2bc}{d}}\text{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*c
)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2))
 + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\,\cosh^2(a+bx)\,dx &= \int \left(\frac{1}{2}\sqrt{c+dx}+\frac{1}{2}\sqrt{c+dx}\,\cosh(2a+2bx)\right)dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{1}{2}\int \sqrt{c+dx}\,\cosh(2a+2bx)\,dx \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}-\frac{d\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{8b} \\
&= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}-\frac{d\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{16b}+\frac{d\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}
\end{aligned}$$

$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{c+dx}\,\sinh(2a+2bx)}{4b}+\frac{\operatorname{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{}\right.}{8b}$$

$$= \frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}\,e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\operatorname{erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}\,e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}}{}$$

Mathematica [A]
time = 0.33, size = 129, normalized size = 0.78

$$\frac{1}{48}\sqrt{c+dx}\left(\frac{16(c+dx)}{d}+\frac{3\sqrt{2}\,e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}\,e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```

```
[Out] (Sqrt[c + d*x]*((16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3/2
, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a +
(2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/48
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\cosh^2\left(bx+a\right)\right)\sqrt{dx+c}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2*(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^2*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.49, size = 189, normalized size = 1.14

$$-\frac{\frac{3\sqrt{2}\sqrt{\pi}\, d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{3\sqrt{2}\sqrt{\pi}\, d\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-32\left(dx+c\right)^{\frac{3}{2}}-\frac{12\sqrt{dx+c}\, de^{\left(2a+\frac{2(dx+c)b}{d}-\frac{2bc}{d}\right)}}{b}+\frac{12\sqrt{dx+c}\, de^{\left(-2a-\frac{2(dx+c)b}{d}+\frac{2bc}{d}\right)}}{b}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) - 32*(d*x + c)^(3/2) - 12*sqrt(d*
x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(-2*a
 - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(122) = 244.
time = 0.39, size = 590, normalized size = 3.55

$$\frac{3\sqrt{2}\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)^2\cosh\left(-\frac{2(bc-ad)}{d}\right)-d^2\cosh\left(bx+a\right)^2\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left(d^2\cosh\left(-\frac{2(bc-ad)}{d}\right)-d^2\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)^2+2\left(d^2\cosh\left(bx+a\right)\cosh\left(-\frac{2(bc-ad)}{d}\right)-d^2\cosh\left(bx+a\right)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+3\sqrt{2}\sqrt{\pi}\left(d^2\cosh\left(bx+a\right)^2\cosh\left(-\frac{2(bc-ad)}{d}\right)+d^2\cosh\left(bx+a\right)^2\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left(d^2\cosh\left(-\frac{2(bc-ad)}{d}\right)+d^2\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)^2+2\left(d^2\cosh\left(bx+a\right)\cosh\left(-\frac{2(bc-ad)}{d}\right)+d^2\cosh\left(bx+a\right)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh\left(bx+a\right)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+4\left(3bd\cosh\left(bx+a\right)^4+12bd\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+3bd\sinh\left(bx+a\right)^4+8\left(b^2dx+b^2c\right)\cosh\left(bx+a\right)^2+2\left(4b^2dx+9bd\cosh\left(bx+a\right)^2+4b^2c\right)\sinh\left(bx+a\right)^2-3bd+4\left(3bd\cosh\left(bx+a\right)^3+4\left(b^2dx+b^2c\right)\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)\right)\sqrt{dx+c}}{96\left(b^2d\cosh\left(bx+a\right)^2+2b^2d\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b^2d\sinh\left(bx+a\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^2*
sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c
```

```
 - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*
x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x + a)^3 +
3*b*d*sinh(b*x + a)^4 + 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 + 2*(4*b^2*d*x
+ 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*(3*b*d*cosh(
b*x + a)^3 + 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c
))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x + a) + b^2*d*sin
h(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2*(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(d*x + c)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^(1/2),x)
```

```
[Out] int(cosh(a + b*x)^2*(c + d*x)^(1/2), x)
```

3.51 $\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}} \, dx$

Optimal. Leaf size=138

$$\frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}$$

```
[Out] 1/8*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi
^(1/2)/b^(1/2)/d^(1/2)+1/8*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3393, 3388, 2211, 2235, 2236}

$$\frac{\sqrt{\frac{\pi}{2}}\,e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}\,e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]
*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{1}{2\sqrt{c+dx}}+\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}\right)dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{1}{2}\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{1}{4}\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{4}\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx\\
&= \frac{\sqrt{c+dx}}{d}+\frac{\operatorname{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{2d}+\frac{\operatorname{Subst}\left(\int e^{-i\left(2ia-\frac{2ibc}{d}\right)+\frac{2bx^2}{d}}\right.}{2d}\\
&= \frac{\sqrt{c+dx}}{d}+\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+}\right.}{4\sqrt{b}\sqrt{d}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 141, normalized size = 1.02

$$\frac{\sqrt{c+dx}}{d}+\frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}\,b\sqrt{c+dx}}-\frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}\,b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

```
[Out] Sqrt[c + d*x]/d + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
(-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/d)*Sq
rt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*
x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.47, size = 107, normalized size = 0.78

$$\frac{\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{2}\,\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}}+8\,\sqrt{dx+c}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/
d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2
*a + 2*b*c/d)/sqrt(b/d) + 8*sqrt(d*x + c))/d
```

Fricas [A]
time = 0.39, size = 155, normalized size = 1.12

$$\frac{\sqrt{2}\,\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)-d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{\frac{b}{d}}\right)-\sqrt{2}\,\sqrt{\pi}\left(d\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+d\sinh\left(-\frac{2\,(bc-ad)}{d}\right)\right)\sqrt{-\frac{b}{d}}\,\operatorname{erf}\left(\sqrt{2}\,\sqrt{dx+c}\,\sqrt{-\frac{b}{d}}\right)+8\,\sqrt{dx+c}\,b}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cosh(
-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d
*x + c)*sqrt(-b/d)) + 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(1/2),x)
```

```
[Out] Integral(cosh(a + b*x)**2/sqrt(c + d*x), x)
```

Giac [A]
time = 0.41, size = 115, normalized size = 0.83

$$-\frac{\left(\frac{\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{2\,bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{2}\sqrt{\pi}\,d\,\mathrm{erf}\left(-\frac{\sqrt{2}\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{2\,(bc-2\,ad)}{d}\right)}}{\sqrt{-bd}}-8\,\sqrt{dx+c}\,e^{(2\,a)}\right)e^{(-2\,a)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)/d)*e^(2*b*c/d
)/sqrt(b*d) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(-b*d)*sqrt(d*x + c)/d)*e
^(-2*(b*c - 2*a*d)/d)/sqrt(-b*d) - 8*sqrt(d*x + c)*e^(2*a))*e^(-2*a)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\cosh\left(a+b\,x\right)}^2}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^(1/2),x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^(1/2), x)
```

3.52 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\, dx$

Optimal. Leaf size=142

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{\sqrt{b}\, e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\, e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\, \mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -1/2*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*2
^(1/2)*Pi^(1/2)/d^(3/2)+1/2*exp(2*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*b^(1/2)*2^(1/2)*Pi^(1/2)/d^(3/2)-2*cosh(b*x+a)^2/d/(d*x+c)^(1
/2)
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3394, 12, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{b}\, e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\frac{\pi}{2}}\,\sqrt{b}\, e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^2)/(d*Sqrt[c + d*x]) - (Sqrt[b]*E^(-2*a + (2*b*c)/d)*Sqrt
[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/d^(3/2) + (Sqrt[b]*E^(
2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
d^(3/2)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{(4ib)\int -\frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{(2b)\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{b\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}-\frac{b\int \frac{e^{i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{d}\\ &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{(2b)\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{d^2}+\frac{(2b)\text{Subst}\left(\int e\right.}{}\\ &= -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{\sqrt{b}\,e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{b}\,e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\,\text{erfi}}{d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 570 vs. 2(142) = 284.

time = 2.48, size = 570, normalized size = 4.01

$$\frac{e^{-\frac{2b(c+dx)}{d}}\left(-2\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}-\sqrt{d}\cosh(2a)\cosh\left(\frac{2bc}{d}\right)-\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh(2a)\cosh\left(\frac{2bc}{d}\right)+\sqrt{d}\cosh\left(\frac{2bc}{d}\right)\sinh(2a)-\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh\left(\frac{2bc}{d}\right)\sinh(2a)+\sqrt{2}\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\cosh\left(\frac{2bc}{d}\right)\sinh(2a)\right)-\sqrt{d}\cosh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\cosh(2a)\sinh\left(\frac{2bc}{d}\right)-\sqrt{b}\,e^{\frac{2b(c+dx)}{d}}\sqrt{2\pi}\sqrt{c+dx}\cosh(2a)\operatorname{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\sinh\left(\frac{2bc}{d}\right)-\sqrt{b}\,e^{\frac{2b(c+dx)}{d}}\sqrt{2\pi}\sqrt{c+dx}\cosh(2a)\operatorname{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\sinh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{d}\,e^{\frac{4b(c+dx)}{d}}\sinh(2a)\sinh\left(\frac{2bc}{d}\right)+\sqrt{2}\sqrt{d}\,e^{\frac{2b(c+dx)}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)\left(\cosh(2a)\cosh\left(\frac{2bc}{d}\right)-\sinh(2a)\left(\cosh\left(\frac{2bc}{d}\right)+\sinh\left(\frac{2bc}{d}\right)\right)\right)\right)}{2d^{3/2}\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Sqrt[d]*E^((2*b*(c + d*x))/d) - Sqrt[d]*Cosh[2*a]*Cosh[(2*b*c)/d] - Sqr
t[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Cosh[(2*b*c)/d] + Sqrt[d]*Cosh[(2*b*c)
/d]*Sinh[2*a] - Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[(2*b*c)/d]*Sinh[2*a] + S
qrt[2]*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-
2*b*(c + d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Cosh[(2*b*c)/d]*Sinh[2*a]) - Sqr
t[d]*Cosh[2*a]*Sinh[(2*b*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Cosh[2*a]*Si
nh[(2*b*c)/d] - Sqrt[b]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh
[2*a]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] - Sqrt[b
]*E^((2*b*(c + d*x))/d)*Sqrt[2*Pi]*Sqrt[c + d*x]*Cosh[2*a]*Erfi[(Sqrt[2]*Sq
rt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[(2*b*c)/d] + Sqrt[d]*Sinh[2*a]*Sinh[(2*b
*c)/d] + Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[2*a]*Sinh[(2*b*c)/d] + Sqrt[2]*
Sqrt[d]*E^((2*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*
x))/d]*(Cosh[2*a]*Cosh[(2*b*c)/d] - Sinh[2*a]*(Cosh[(2*b*c)/d] + Sinh[(2*b*
c)/d])))/(2*d^(3/2)*E^((2*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.33, size = 116, normalized size = 0.82

$$-\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{4}{\sqrt{dx+c}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)*g
amma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) + 4/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(109) = 218$.
time = 0.42, size = 569, normalized size = 4.01

$$-\frac{\sqrt{2}\sqrt{\pi}\left((dx+c)\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left((dx+c)\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)^2+2\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{2(bc-ad)}{d}\right)-(dx+c)\cosh(bx+a)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)+\sqrt{2}\sqrt{\pi}\left((dx+c)\cosh(bx+a)^2\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\cosh(bx+a)^2\sinh\left(-\frac{2(bc-ad)}{d}\right)+\left((dx+c)\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)^2+2\left((dx+c)\cosh(bx+a)\cosh\left(-\frac{2(bc-ad)}{d}\right)+(dx+c)\cosh(bx+a)\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sinh(bx+a)\right)\sqrt{-\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+\left(\cosh(bx+a)^4+4\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+1\right)\sinh(bx+a)^2+2\cosh(bx+a)^2+4\left(\cosh(bx+a)^3+\cosh(bx+a)\right)\sinh(bx+a)+1\right)\sqrt{dx+c}}{2\left((d^2x+cd)\cosh(bx+a)^2+2(d^2x+cd)\cosh(bx+a)\sinh(bx+a)+(d^2x+cd)\sinh(bx+a)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c
- a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c
)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))
 + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (d*
x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*c - a
*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x + c)*c
osh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
+ (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)^4 + 2*
(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4*(cosh(b*x +
 a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cos
h(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) + (d^2*x + c*d)*
sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^(3/2),x)
[Out] int(cosh(a + b*x)^2/(c + d*x)^(3/2), x)
```

3.53 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=174

$$-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-$$

```
[Out] -2/3*cosh(b*x+a)^2/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2
)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(5/2)+2/3*b^(3/2)*exp(2
*a-2*b*c/d)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^
(5/2)-8/3*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{2\pi}\,b^{3/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{2\pi}\,b^{3/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)) + (2*b^(3/2)*E^(-2*a + (2*b*c)/d
)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(3*d^(5/2)) + (2
*b^(3/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(3*d^(5/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*Sqrt[c +
 d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} - \frac{(8b^2)\int \frac{1}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{(16b^2)\int \frac{c}{}}{3}$$

$$= -\frac{16b^2\sqrt{c+dx}}{3d^3} - \frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{(16b^2)\int \left(\frac{1}{2\sqrt{c-}}\right.}{}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{(8b^2)\int \frac{\cosh(2a+2bx)}{\sqrt{c+dx}}\,dx}{3d^2}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{(4b^2)\int \frac{e^{-i(2ia+2ibx)}}{\sqrt{c+dx}}\,dx}{3d^2} + \frac{(4b^2)\int \frac{e^{}}{\sqrt{}}}{3}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}} + \frac{(8b^2)\,\text{Subst}\left(\int e^{i\left(2ia-\frac{2ibc}{d}\right)-\frac{2bx^2}{d}}\,dx\right.}{3d^3}$$

$$= -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\text{er}}{3}$$

Mathematica [A]
time = 0.95, size = 156, normalized size = 0.90

$$-\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}\,de^{4a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}\,de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}\left(d\cosh^2(a+bx)+2b(c+dx)\sinh(2(a+bx))\right)\right)}{3d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

```
[Out] (-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*x
))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(c
+ d*x))/d] + E^(2*(a + (b*c)/d))*(d*Cosh[a + b*x]^2 + 2*b*(c + d*x)*Sinh[2*
(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.34, size = 118, normalized size = 0.68

$$-\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{2}{(dx+c)^{\frac{3}{2}}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d*
x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) + 2/(d*x + c)^(3/2))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 861 vs. 2(134) = 268.
time = 0.39, size = 861, normalized size = 4.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*co
sh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh
(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d
) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2
 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d)
- (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(2)*sqrt
(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d
) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d)
+ ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*
b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*
c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c + d)*cosh(b*x
 + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a)^3 + (4*b*d*x
+ 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x + 2*d*cosh(b*x + a)^2 + 2*(3*(4*b*d*
x + 4*b*c + d)*cosh(b*x + a)^2 + d)*sinh(b*x + a)^2 - 4*b*c + 4*((4*b*d*x +
 4*b*c + d)*cosh(b*x + a)^3 + d*cosh(b*x + a))*sinh(b*x + a) + d)*sqrt(d*x
+ c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 + 2*(d^4*x^2 + 2*c*d
```

```
^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^
2)*sinh(b*x + a)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(5/2),x)
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^(5/2),x)
[Out] int(cosh(a + b*x)^2/(c + d*x)^(5/2), x)
```

3.54 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=220

$$\frac{16b^2}{15d^3\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}-\frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8b^{5/2}e^{2}}{}$$

```
[Out] -2/5*cosh(b*x+a)^2/d/(d*x+c)^(5/2)-8/15*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+
c)^(3/2)-8/15*b^(5/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d
^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+8/15*b^(5/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)
*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(7/2)+16/15*b^2/d^3/(d*x
+c)^(1/2)-32/15*b^2*cosh(b*x+a)^2/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3395, 32, 3394, 12, 3389, 2211, 2235, 2236}

$$-\frac{8\sqrt{2\pi}\,b^{5/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}+\frac{8\sqrt{2\pi}\,b^{5/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}}-\frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^2)/(5*d*(c + d*x)^(5/2))
 - (32*b^2*Cosh[a + b*x]^2)/(15*d^3*Sqrt[c + d*x]) - (8*b^(5/2)*E^(-2*a + (
2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(15*d^(7
/2)) + (8*b^(5/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]])/(15*d^(7/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(15*d
^2*(c + d*x)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
```

```
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}} - \frac{(8b^2)\int \frac{1}{(c+dx)^{3/2}}\,dx}{15d^2} + \frac{(16b^2)\int \frac{c\cdots}{}}{1\cdots}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+b\cdots}{15d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+b\cdots}{15d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+b\cdots}{15d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b\cosh(a+bx)\sinh(a+b\cdots}{15d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\cdots}{}\right.}{15d^{7/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 825 vs. 2(220) = 440.
time = 2.51, size = 825, normalized size = 3.75

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```

```
[Out] (-6*d^2*E^((2*b*(c + d*x))/d) - 16*b^2*c^2*Cosh[2*a - (2*b*c)/d] + 4*b*c*d*
Cosh[2*a - (2*b*c)/d] - 3*d^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*c^2*E^((4*b*(c
 + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/d)*Cosh[2*a
- (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Cosh[2*a - (2*b*c)/d] - 32*b^2*c
*d*x*Cosh[2*a - (2*b*c)/d] + 4*b*d^2*x*Cosh[2*a - (2*b*c)/d] - 32*b^2*c*d*E
^((4*b*(c + d*x))/d)*x*Cosh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*(c + d*x))/d
)*x*Cosh[2*a - (2*b*c)/d] - 16*b^2*d^2*x^2*Cosh[2*a - (2*b*c)/d] - 16*b^2*d
^2*E^((4*b*(c + d*x))/d)*x^2*Cosh[2*a - (2*b*c)/d] + 16*Sqrt[2]*d^2*E^((2*b
*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (2*b*(c + d*x))/d]*(Cosh[
2*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d]) + 16*b^2*c^2*Sinh[2*a - (2*b*c)/d
] - 4*b*c*d*Sinh[2*a - (2*b*c)/d] + 3*d^2*Sinh[2*a - (2*b*c)/d] - 16*b^2*c^
2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d] - 4*b*c*d*E^((4*b*(c + d*x))/
d)*Sinh[2*a - (2*b*c)/d] - 3*d^2*E^((4*b*(c + d*x))/d)*Sinh[2*a - (2*b*c)/d
] + 32*b^2*c*d*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*x*Sinh[2*a - (2*b*c)/d] -
```

```
32*b^2*c*d*E^((4*b*(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] - 4*b*d^2*E^((4*b*
(c + d*x))/d)*x*Sinh[2*a - (2*b*c)/d] + 16*b^2*d^2*x^2*Sinh[2*a - (2*b*c)/d
] - 16*b^2*d^2*E^((4*b*(c + d*x))/d)*x^2*Sinh[2*a - (2*b*c)/d] + 16*Sqrt[2]
*d^2*E^((2*b*(c + d*x))/d)*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-2*b*(c +
 d*x))/d]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d]))/(30*d^3*E^((2*b*
(c + d*x))/d)*(c + d*x)^(5/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
[Out] int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.34, size = 116, normalized size = 0.53

$$-\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{1}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
[Out] -1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c -
 a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) + 1/(d*x + c)^(5/2))
/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. 2(172) = 344.
time = 0.43, size = 1350, normalized size = 6.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
[Out] -1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
```

```
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2
*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d
)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 16*sq
rt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*c
osh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*
b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c - a*d)/d) + (b
^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-2*(b*c - a*d)
/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*
x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) + (16*b^2*d^2*x^2
 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x
)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^3 + (16*b^2*d^2*x^2 + 16*b
^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 + 16*b^
2*c^2 + 6*d^2*cosh(b*x + a)^2 - 4*b*c*d + 6*((16*b^2*d^2*x^2 + 16*b^2*c^2 +
 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^2 + d^2)*sinh(b*x
 + a)^2 + 3*d^2 + 4*(8*b^2*c*d - b*d^2)*x + 4*((16*b^2*d^2*x^2 + 16*b^2*c^2
 + 4*b*c*d + 3*d^2 + 4*(8*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + 3*d^2*cosh(
b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4
*x + c^3*d^3)*cosh(b*x + a)^2 + 2*(d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^
3*d^3)*cosh(b*x + a)*sinh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x +
 c^3*d^3)*sinh(b*x + a)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(7/2),x)
[Out] Integral(cosh(a + b*x)**2/(c + d*x)**(7/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^(7/2),x)
```

```
[Out] int(cosh(a + b*x)^2/(c + d*x)^(7/2), x)
```

3.55 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal. Leaf size=251

$$\frac{16b^2}{105d^3(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}-\frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}}+\frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+32b$$

```
[Out] 16/105*b^2/d^3/(d*x+c)^(3/2)-2/7*cosh(b*x+a)^2/d/(d*x+c)^(7/2)-32/105*b^2*c
osh(b*x+a)^2/d^3/(d*x+c)^(3/2)-8/35*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(
5/2)+32/105*b^(7/2)*exp(-2*a+2*b*c/d)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(
1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)+32/105*b^(7/2)*exp(2*a-2*b*c/d)*erfi(2^(1/2)
*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*2^(1/2)*Pi^(1/2)/d^(9/2)-128/105*b^3*cosh(b
*x+a)*sinh(b*x+a)/d^4/(d*x+c)^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 32, 3393, 3388, 2211, 2235, 2236}

$$\frac{32\sqrt{2\pi}\,b^{7/2}e^{\frac{2bc}{d}-2a}\mathrm{Erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}+\frac{32\sqrt{2\pi}\,b^{7/2}e^{2a-\frac{2bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}}-\frac{128b^3\sinh(a+bx)\cosh(a+bx)}{105d^4\sqrt{c+dx}}-\frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cosh[a + b*x]^2)/(7*d*(c + d*x)^(7/
2)) - (32*b^2*Cosh[a + b*x]^2)/(105*d^3*(c + d*x)^(3/2)) + (32*b^(7/2)*E^(-
2*a + (2*b*c)/d)*Sqrt[2*Pi]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
105*d^(9/2)) + (32*b^(7/2)*E^(2*a - (2*b*c)/d)*Sqrt[2*Pi]*Erfi[(Sqrt[2]*Sqr
t[b]*Sqrt[c + d*x])/Sqrt[d]])/(105*d^(9/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b
*x])/(35*d^2*(c + d*x)^(5/2)) - (128*b^3*Cosh[a + b*x]*Sinh[a + b*x])/(105*
d^4*Sqrt[c + d*x])
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{(8b^2)\int \frac{1}{(c+dx)^{5/2}}\,dx}{35d^2} + \frac{(16b^2)\int \frac{c}{}}{3}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a}{35d^2(c+dx)^{5/2}}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{256b^4\sqrt{c+dx}}{105d^5} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\,\mathrm{co}}{}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a}{35d^2(c+dx)^{5/2}}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a}{35d^2(c+dx)^{5/2}}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} - \frac{8b\cosh(a+bx)\sinh(a}{35d^2(c+dx)^{5/2}}$$

$$= \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}(}{105d}$$

Mathematica [A]
time = 0.55, size = 222, normalized size = 0.88

$$\frac{2\left(8b^2d(c+dx)^2-15d^3\cosh^2(a+bx)-16b^2d(c+dx)^2\cosh^2(a+bx)+16\sqrt{2}\,b^3e^{2a-\frac{2bc}{d}}(c+dx)^3\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)-16\sqrt{2}\,b^3e^{-2a+\frac{2bc}{d}}(c+dx)^3\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)-6bd^2(c+dx)\sinh(2(a+bx))-32b^3(c+dx)^3\sinh(2(a+bx))\right)}{105d^4(c+dx)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

```
[Out] (2*(8*b^2*d*(c + d*x)^2 - 15*d^3*Cosh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Cos
h[a + b*x]^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*Sqrt[-((b*(c
+ d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E^(-2*a + (2*b*
c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d] - 6*b
*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x)]))/(
105*d^4*(c + d*x)^(7/2))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^2(bx+a)}{(dx+c)^{\frac{9}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

```
[Out] int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Maxima [A]
time = 0.32, size = 116, normalized size = 0.46

$$-\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}+\frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}}+\frac{1}{(dx+c)^{\frac{7}{2}}}}{7d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

```
[Out] -1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(d
*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) + 1/(d*x + c)^(7/2
))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1825 vs. $2(199) = 398$.
time = 0.41, size = 1825, normalized size = 7.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

```
[Out] 1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*
cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) - (b^3*
d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c -
a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*
x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)
*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 +
 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x +
a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^
2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*
```

```
x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*
((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c -
a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) +
(64*b^3*d^3*x^3 + 64*b^3*c^3 - 16*b^2*c^2*d - 30*d^3*cosh(b*x + a)^2 - (64*
b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*
c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x
 + a)^4 - 4*(64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d
^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*
d^3)*x)*cosh(b*x + a)*sinh(b*x + a)^3 - (64*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b
^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^
3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*sinh(b*x + a)^4 + 12*b*c*d^2 - 15*d^3 +
 16*(12*b^3*c*d^2 - b^2*d^3)*x^2 - 6*(5*d^3 + (64*b^3*d^3*x^3 + 64*b^3*c^3
+ 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^3*c*d^2 + b^2*d^3)*x^2 + 4*
(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2
+ 4*(48*b^3*c^2*d - 8*b^2*c*d^2 + 3*b*d^3)*x - 4*(15*d^3*cosh(b*x + a) + (6
4*b^3*d^3*x^3 + 64*b^3*c^3 + 16*b^2*c^2*d + 12*b*c*d^2 + 15*d^3 + 16*(12*b^
3*c*d^2 + b^2*d^3)*x^2 + 4*(48*b^3*c^2*d + 8*b^2*c*d^2 + 3*b*d^3)*x)*cosh(b
*x + a)^3)*sinh(b*x + a))*sqrt(d*x + c))/((d^8*x^4 + 4*c*d^7*x^3 + 6*c^2*d^
6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)^2 + 2*(d^8*x^4 + 4*c*d^7*x^3 +
 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*cosh(b*x + a)*sinh(b*x + a) + (d^8*
x^4 + 4*c*d^7*x^3 + 6*c^2*d^6*x^2 + 4*c^3*d^5*x + c^4*d^4)*sinh(b*x + a)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**2/(d*x+c)**(9/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3877 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2/(c + d*x)^(9/2),x)
[Out] int(cosh(a + b*x)^2/(c + d*x)^(9/2), x)
```

3.56 $\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx$

Optimal. Leaf size=381

$$-\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}+\frac{5d^{5/2}e}{}$$

```
[Out] -5/3*d*(d*x+c)^(3/2)*cosh(b*x+a)/b^2-5/18*d*(d*x+c)^(3/2)*cosh(b*x+a)^3/b^2
+2/3*(d*x+c)^(5/2)*sinh(b*x+a)/b+1/3*(d*x+c)^(5/2)*cosh(b*x+a)^2*sinh(b*x+a
)/b+5/1728*d^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1
/2))*3^(1/2)*Pi^(1/2)/b^(7/2)-5/1728*d^(5/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)*
b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(7/2)+45/64*d^(5/2)*exp(-
a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)-45/64*d^(5/2)*
exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(7/2)+45/16*d^2
*sinh(b*x+a)*(d*x+c)^(1/2)/b^3+5/144*d^2*sinh(3*b*x+3*a)*(d*x+c)^(1/2)/b^3
```

Rubi [A]
time = 0.69, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 3377, 3389, 2211, 2235, 2236, 3393}

$$\frac{45\sqrt{\pi}\,d^{5/2}e^{\frac{bc}{d}-a}\text{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}+\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}e^{\frac{3bc}{d}-3a}\text{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}-\frac{45\sqrt{\pi}\,d^{5/2}e^{a-\frac{bc}{d}}\text{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}}-\frac{5\sqrt{\frac{\pi}{3}}\,d^{5/2}e^{3a-\frac{3bc}{d}}\text{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}}+\frac{45d^2\sqrt{c+dx}\sinh(a+bx)}{16b^3}+\frac{5d^2\sqrt{c+dx}\sinh(3a+3bx)}{144b^3}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}-\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2}+\frac{2(c+dx)^{5/2}\sinh(a+bx)}{3b}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```

```
[Out] (-5*d*(c + d*x)^(3/2)*Cosh[a + b*x])/(3*b^2) - (5*d*(c + d*x)^(3/2)*Cosh[a
 + b*x]^3)/(18*b^2) + (45*d^(5/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(64*b^(7/2)) + (5*d^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[P
i/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(576*b^(7/2)) - (45*d^(5
/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(64*b^(
7/2)) - (5*d^(5/2)*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(576*b^(7/2)) + (45*d^2*Sqrt[c + d*x]*Sinh[a + b*x])/
(16*b^3) + (2*(c + d*x)^(5/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(5/2)*Cosh[
a + b*x]^2*Sinh[a + b*x])/(3*b) + (5*d^2*Sqrt[c + d*x]*Sinh[3*a + 3*b*x])/(
144*b^3)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx = -\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{(c+dx)^{5/2}\cosh^2(a+bx)\sinh(a+bx)}{3b} + \cdots$$

$$= -\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5/2}\sinh(a+bx)}{3b} + \frac{(c+dx)^{5/2}\,\mathrm{c}\cdots}{\cdots}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{2(c+dx)^{5\cdots}}{\cdots}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{45d^2\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{45d^2\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{45d^2\sqrt{c+\cdots}}{\cdots}$$

$$= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{45d^{5/2}e^{-a+\cdots}}{\cdots}$$

Mathematica [**A**]
time = 3.22, size = 243, normalized size = 0.64

$$-\frac{d^3\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)\left(\cosh\left(3a-\frac{3bc}{d}\right)+\sinh\left(3a-\frac{3bc}{d}\right)\right)+\left(\sqrt{\frac{b(c+dx)}{d}}\left(243\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,\Gamma\left(\frac{7}{2},\frac{3b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)-\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)+243\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)\left(\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\right)\right)}{648b^4\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```

```
[Out] -1/648*(d^3*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, (-3*b*(c + d*x))/d
]*(Cosh[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (Sqrt[(b*(c + d*x))/d]*
(243*Gamma[7/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma[7/2, (3*b*(c + d*x))/d]*(C
osh[2*a - (2*b*c)/d] - Sinh[2*a - (2*b*c)/d])) + 243*Sqrt[-((b*(c + d*x))/d
)]*Gamma[7/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*
c)/d]))*(Cosh[a - (b*c)/d] - Sinh[a - (b*c)/d])))/(b^4*Sqrt[c + d*x])
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{5}{2}}\left(\cosh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

Maxima [A]
time = 0.48, size = 513, normalized size = 1.35

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{162\left(4(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{bc}{d}\right)}+15\sqrt{dx+c}\,d^3e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^3}+\frac{6\left(12(dx+c)^{\frac{5}{2}}b^2de^{\left(\frac{3bc}{d}\right)}+10(dx+c)^{\frac{3}{2}}bd^2e^{\left(\frac{3bc}{d}\right)}+5\sqrt{dx+c}\,d^3e^{\left(\frac{3bc}{d}\right)}\right)e^{\left(-3a-\frac{3(dx+c)b}{d}\right)}}{b^3}-\frac{6\left(12(dx+c)^{\frac{5}{2}}b^2de^{(3a)}-10(dx+c)^{\frac{3}{2}}bd^2e^{(3a)}+5\sqrt{dx+c}\,d^3e^{(3a)}\right)e^{\left(\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b^3}-\frac{162\left(4(dx+c)^{\frac{5}{2}}b^2de^{a}-10(dx+c)^{\frac{3}{2}}bd^2e^{a}+15\sqrt{dx+c}\,d^3e^{a}\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^3}}{1728\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*
a - 3*b*c/d)/(b^3*sqrt(-b/d)) - 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) + 1215*sqrt(pi)*d^3*erf
(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(pi)*d
^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(4*(d*
x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt
(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 + 6*(12*(d*x + c)^(5/2)
*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(d*x + c)
*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)^(5/2)*b^
2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*d^3*e^(3*a
))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 - 162*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
b*c/d)/b^3)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2092 vs. $2(291) = 582$.
time = 0.43, size = 2092, normalized size = 5.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^
3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) - d^
3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b
*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/
d)) + 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^3*
cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) + d^3*
sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3*(b*c
 - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*
(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-3*(
```

```
b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/
d)) + 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^3*cosh(b*
x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c
- a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^
3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a
)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b
*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*sqrt(pi)*(d^3*cosh(b
*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) +
 (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*
(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c - a*
d)/d))*sinh(b*x + a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^3*
cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*
x + c)*sqrt(-b/d)) - 6*(12*b^3*d^2*x^2 - (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*
b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^6 - 6*(12*b
^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)
*x)*cosh(b*x + a)*sinh(b*x + a)^5 - (12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c
*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^6 + 12*b^3*c^2 -
 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b
^2*d^2)*x)*cosh(b*x + a)^4 - 3*(36*b^3*d^2*x^2 + 36*b^3*c^2 - 90*b^2*c*d +
135*b*d^2 + 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^2 + 2*(12*b
^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d - 5*b^2*d^2)*x)*sinh
(b*x + a)^4 + 10*b^2*c*d - 4*(5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d +
 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^3 + 27*(4*b^3*d^2*x^
2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b
*x + a))*sinh(b*x + a)^3 + 5*b*d^2 + 27*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2
*c*d + 15*b*d^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a)^2 + 3*(36*b^3*
d^2*x^2 + 36*b^3*c^2 - 5*(12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*c*d + 5*b*d^
2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^4 + 90*b^2*c*d + 135*b*d^2
- 54*(4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*
b^2*d^2)*x)*cosh(b*x + a)^2 + 18*(4*b^3*c*d + 5*b^2*d^2)*x)*sinh(b*x + a)^2
 + 2*(12*b^3*c*d + 5*b^2*d^2)*x - 6*((12*b^3*d^2*x^2 + 12*b^3*c^2 - 10*b^2*
c*d + 5*b*d^2 + 2*(12*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^5 + 18*(4*b^3*d
^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*c
osh(b*x + a)^3 - 9*(4*b^3*d^2*x^2 + 4*b^3*c^2 + 10*b^2*c*d + 15*b*d^2 + 2*(
4*b^3*c*d + 5*b^2*d^2)*x)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^4
*cosh(b*x + a)^3 + 3*b^4*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^4*cosh(b*x + a
)*sinh(b*x + a)^2 + b^4*sinh(b*x + a)^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(5/2)*cosh(b*x+a)**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(5/2)*cosh(b*x + a)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a+b\,x)^3\,{(c+d\,x)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^(5/2),x)
```

```
[Out] int(cosh(a + b*x)^3*(c + d*x)^(5/2), x)
```

3.57 $\int (c+dx)^{3/2} \cosh^3(a+bx) \, dx$

Optimal. Leaf size=326

$$-\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{d^{3/2}e^{-3a+\frac{3bc}{d}}}{}$$

```
[Out] 2/3*(d*x+c)^(3/2)*sinh(b*x+a)/b+1/3*(d*x+c)^(3/2)*cosh(b*x+a)^2*sinh(b*x+a)
/b+1/288*d^(3/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2
))*3^(1/2)*Pi^(1/2)/b^(5/2)+1/288*d^(3/2)*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(
1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(5/2)+9/32*d^(3/2)*exp(-a+b*
c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)+9/32*d^(3/2)*exp(a
-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(5/2)-d*cosh(b*x+a)*
(d*x+c)^(1/2)/b^2-1/6*d*cosh(b*x+a)^3*(d*x+c)^(1/2)/b^2
```

Rubi [A]
time = 0.53, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 20, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3392, 3377, 3388, 2211, 2235, 2236, 3393}

$$\frac{9\sqrt{\pi}\,d^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}+\frac{9\sqrt{\pi}\,d^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}}+\frac{\sqrt{\frac{\pi}{3}}\,d^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}-\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2}+\frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

```
[Out] -((d*Sqrt[c + d*x]*Cosh[a + b*x])/b^2) - (d*Sqrt[c + d*x]*Cosh[a + b*x]^3)/
(6*b^2) + (9*d^(3/2)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt
[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(96*b^(5/2)) + (9*d^(3/2)*E^(a - (b*c)
/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(32*b^(5/2)) + (d^(3/2)
*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(96*b^(5/2)) + (2*(c + d*x)^(3/2)*Sinh[a + b*x])/(3*b) + ((c + d*x)^(3/
2)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = -\frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{(c+dx)^{3/2}\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{2}{3}$$

$$= -\frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cos}{}$$

$$= -\frac{d\sqrt{c+dx}\,\cosh(a+bx)}{b^2} - \frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin}{3b}$$

$$= -\frac{d\sqrt{c+dx}\,\cosh(a+bx)}{b^2} - \frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin}{3b}$$

$$= -\frac{d\sqrt{c+dx}\,\cosh(a+bx)}{b^2} - \frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{2(c+dx)^{3/2}\sin}{3b}$$

$$= -\frac{d\sqrt{c+dx}\,\cosh(a+bx)}{b^2} - \frac{d\sqrt{c+dx}\,\cosh^3(a+bx)}{6b^2} + \frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}}{3}$$

Mathematica [A]
time = 1.38, size = 243, normalized size = 0.75

$$\frac{d^2\left(\sqrt{3}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)\left(\cosh\left(3a-\frac{3bc}{d}\right)+\sinh\left(3a-\frac{3bc}{d}\right)\right)+\left(81\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)\left(\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)+\sqrt{\frac{b(c+dx)}{d}}\left(-81\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,\Gamma\left(\frac{5}{2},\frac{3b(c+dx)}{d}\right)\left(-\cosh\left(2a-\frac{2bc}{d}\right)+\sinh\left(2a-\frac{2bc}{d}\right)\right)\right)\right)\left(\cosh\left(a-\frac{bc}{d}\right)-\sinh\left(a-\frac{bc}{d}\right)\right)\right)}{216b^3\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

```
[Out] (d^2*(Sqrt[3]*Sqrt[-((b*(c + d*x))/d)]*Gamma[5/2, (-3*b*(c + d*x))/d]*(Cosh
[3*a - (3*b*c)/d] + Sinh[3*a - (3*b*c)/d]) + (81*Sqrt[-((b*(c + d*x))/d)]*G
amma[5/2, -((b*(c + d*x))/d)]*(Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d
]) + Sqrt[(b*(c + d*x))/d]*(-81*Gamma[5/2, (b*(c + d*x))/d] + Sqrt[3]*Gamma
[5/2, (3*b*(c + d*x))/d]*(-Cosh[2*a - (2*b*c)/d] + Sinh[2*a - (2*b*c)/d])))
*(Cosh[a - (b*c)/d] - Sinh[a - (b*c)/d])))/(216*b^3*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^{\frac{3}{2}}\left(\cosh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

Maxima [A]
time = 0.50, size = 429, normalized size = 1.32

$$\frac{\frac{\sqrt{3}\sqrt{\pi}\,d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}\,d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}+\frac{81\sqrt{\pi}\,d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}+\frac{81\sqrt{\pi}\,d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}}-\frac{54\left(2\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}\,d^2e^{\left(\frac{bc}{d}\right)}\right)e^{\left(-a-\frac{(dx+c)b}{d}\right)}}{b^2}-\frac{6\left(2\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{3bc}{d}\right)}+\sqrt{dx+c}\,d^2e^{\left(\frac{3bc}{d}\right)}\right)e^{\left(-3a-\frac{3(dx+c)b}{d}\right)}}{b^2}+\frac{6\left(2\,(dx+c)^{\frac{3}{2}}bde^{(3a)}-\sqrt{dx+c}\,d^2e^{(3a)}\right)e^{\left(\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b^2}+\frac{54\left(2\,(dx+c)^{\frac{3}{2}}bde^{a}-3\sqrt{dx+c}\,d^2e^{a}\right)e^{\left(\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b^2}}{288\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b^2*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*
sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) + 81*sqrt(pi)*d^2*erf(sqrt(d*
x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf(sqr
t(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)^(3/2
)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2
 - 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c/d))*e^
(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - sqrt(d*x
+ c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 + 54*(2*(d*x + c)^(3/2)
*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. $2(246) = 492$.
time = 0.38, size = 1545, normalized size = 4.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] 1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*c
osh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2*s
inh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c
- a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(
d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))
 - sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^2*cosh(
b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d^2*sinh(
-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b*c - a*
d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*
cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-3*(b*c -
 a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) +
 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^
3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)/
d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(
```

```
b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cos
h(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
)*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d^2*cosh(b*x + a)^3
*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cos
h(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh
(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*si
nh(b*x + a)^2 + 3*(d^2*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x
+ a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sq
rt(-b/d)) + 6*((2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^6 + 6*(2*b^2*d*x +
 2*b^2*c - b*d)*cosh(b*x + a)*sinh(b*x + a)^5 + (2*b^2*d*x + 2*b^2*c - b*d)
*sinh(b*x + a)^6 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^4 + 3*(6*b
^2*d*x + 6*b^2*c + 5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)^2 - 9*b*d)*s
inh(b*x + a)^4 - 2*b^2*d*x + 4*(5*(2*b^2*d*x + 2*b^2*c - b*d)*cosh(b*x + a)
^3 + 9*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b^2
*c - 9*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a)^2 + 3*(5*(2*b^2*d*x + 2*
b^2*c - b*d)*cosh(b*x + a)^4 - 6*b^2*d*x - 6*b^2*c + 18*(2*b^2*d*x + 2*b^2*
c - 3*b*d)*cosh(b*x + a)^2 - 9*b*d)*sinh(b*x + a)^2 - b*d + 6*((2*b^2*d*x +
 2*b^2*c - b*d)*cosh(b*x + a)^5 + 6*(2*b^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x
+ a)^3 - 3*(2*b^2*d*x + 2*b^2*c + 3*b*d)*cosh(b*x + a))*sinh(b*x + a))*sqrt
(d*x + c))/(b^3*cosh(b*x + a)^3 + 3*b^3*cosh(b*x + a)^2*sinh(b*x + a) + 3*b
^3*cosh(b*x + a)*sinh(b*x + a)^2 + b^3*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^{\frac{3}{2}}\cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**(3/2)*cosh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**(3/2)*cosh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^(3/2)*cosh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^(3/2),x)
[Out] int(cosh(a + b*x)^3*(c + d*x)^(3/2), x)
```

3.58 $\int \sqrt{c+dx} \cosh^3(a+bx)\,dx$

Optimal. Leaf size=275

$$\frac{3\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{d}\,e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{d}\,e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\sqrt{\cdots}\right.}{16b^{3/2}}$$

```
[Out] 1/144*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*
3^(1/2)*Pi^(1/2)/b^(3/2)-1/144*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c
)^(1/2)/d^(1/2))*d^(1/2)*3^(1/2)*Pi^(1/2)/b^(3/2)+3/16*exp(-a+b*c/d)*erf(b^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)-3/16*exp(a-b*c/d)*erf
i(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/b^(3/2)+3/4*sinh(b*x+a)*(
d*x+c)^(1/2)/b+1/12*sinh(3*b*x+3*a)*(d*x+c)^(1/2)/b
```

Rubi [A]

time = 0.34, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3393, 3377, 3389, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,\sqrt{d}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\,\sqrt{d}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{\pi}\,\sqrt{d}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\,\sqrt{d}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{c+dx}\,\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\,\sinh(3a+3bx)}{12b}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

```
[Out] (3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
(16*b^(3/2)) + (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (3*Sqrt[d]*E^(a - (b*c)/d)*Sqrt[P
i]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(3*a -
(3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^
(3/2)) + (3*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sinh[3*a +
3*b*x])/(12*b)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \sqrt{c+dx}\cosh^3(a+bx)\,dx &= \int \left(\frac{3}{4}\sqrt{c+dx}\cosh(a+bx)+\frac{1}{4}\sqrt{c+dx}\cosh(3a+3bx)\right)dx \\
&= \frac{1}{4}\int \sqrt{c+dx}\cosh(3a+3bx)\,dx+\frac{3}{4}\int \sqrt{c+dx}\cosh(a+bx)\,dx \\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}-\frac{d\int \frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx}{24b}- \\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}-\frac{d\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx}{48b}+ \\
&= \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}+\frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-}\right.}{2} \\
&= \frac{3\sqrt{d}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{d}\,e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+}}{\sqrt{d}}\right.}{48b^{3/2}}
\end{aligned}$$

Mathematica [A]

time = 0.21, size = 210, normalized size = 0.76

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(\sqrt{3}\,e^{6a}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)+27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\left(27e^{2a}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{3b(c+dx)}{d}\right)\right)\right)}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

```
[Out] (Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c +
 d*x))/d] + 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*(c
 + d*x))/d)] - E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(27*E^(2*a)*Gamma[3/2
, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/d])))
/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(\cosh^3\left(bx+a\right)\right)\sqrt{dx+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3*(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^3*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.49, size = 334, normalized size = 1.21

$$-\frac{\frac{\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{\sqrt{3}\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}+\frac{27\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}}-\frac{27\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}}-\frac{6\sqrt{dx+c}\,de^{\left(3a+\frac{3(dx+c)b}{d}-\frac{3bc}{d}\right)}}{b}-\frac{54\sqrt{dx+c}\,de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b}+\frac{54\sqrt{dx+c}\,de^{\left(-a-\frac{(dx+c)b}{d}+\frac{bc}{d}\right)}}{b}+\frac{6\sqrt{dx+c}\,de^{\left(-3a-\frac{3(dx+c)b}{d}+\frac{3bc}{d}\right)}}{b}}{144\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3
*b*c/d)/(b*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(
b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) + 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c)*sq
rt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(d*x +
 c)*b/d - 3*b*c/d)/b - 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/d)/b +
 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b + 6*sqrt(d*x + c)*d*e^
(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(201) = 402$.
time = 0.37, size = 1217, normalized size = 4.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cosh(
b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d)
- d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x +
 a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(
pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-3*(
b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d))*sinh(
b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-3*(b*c -
 a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b
/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*
cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b
*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)
*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 27*sq
rt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)^3*sinh(-(b
*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x +
 a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*
c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) +
d*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(
d*x + c)*sqrt(-b/d)) + 6*(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x +
a)^5 + b*sinh(b*x + a)^6 + 9*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + 3
*b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3 + 9*b*cosh(b*x + a))*sinh(b*x
+ a)^3 - 9*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 18*b*cosh(b*x + a)^
2 - 3*b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^5 + 6*b*cosh(b*x + a)^3 - 3*b
*cosh(b*x + a))*sinh(b*x + a) - b)*sqrt(d*x + c))/(b^2*cosh(b*x + a)^3 + 3*
b^2*cosh(b*x + a)^2*sinh(b*x + a) + 3*b^2*cosh(b*x + a)*sinh(b*x + a)^2 + b
^2*sinh(b*x + a)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \cosh^3(a + bx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3*(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x)*cosh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3*(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)*cosh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a+b\,x)^3\,\sqrt{c+d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^(1/2),x)
[Out] int(cosh(a + b*x)^3*(c + d*x)^(1/2), x)
```

3.59 $\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=228

$$\frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+$$

```
[Out] 1/24*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*P
i^(1/2)/b^(1/2)/d^(1/2)+1/24*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^
(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8*exp(-a+b*c/d)*erf(b^(1/
2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)+3/8*exp(a-b*c/d)*erfi(b^
(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/b^(1/2)/d^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$,
Rules used = {3393, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[
b]*Sqrt[d]) + (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
+ d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*
Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d
])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx &= \int \left(\frac{3\cosh(a+bx)}{4\sqrt{c+dx}}+\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx \\
&= \frac{1}{4}\int \frac{\cosh(3a+3bx)}{\sqrt{c+dx}}\,dx+\frac{3}{4}\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx \\
&= \frac{1}{8}\int \frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx+\frac{1}{8}\int \frac{e^{i(3ia+3ibx)}}{\sqrt{c+dx}}\,dx+\frac{3}{8}\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx+\frac{3}{8}\int \frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx \\
&= \frac{\text{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx,x,\sqrt{c+dx}\right)}{4d}+\frac{\text{Subst}\left(\int e^{-i\left(3ia-\frac{3ibc}{d}\right)+\frac{3bx^2}{d}}\,dx,x,\sqrt{c+\right.}{4d} \\
&= \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3e^{a-\frac{bc}{d}}\sqrt{}}{}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 192, normalized size = 0.84

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}\,e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(9e^{2a}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)+\sqrt{3}\,e^{\frac{2bc}{d}}\Gamma\left(\frac{1}{2},\frac{3b(c+dx)}{d}\right)\right)\right)}{24b\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```

```
[Out] (Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] +
9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))/d
)] - E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(9*E^(2*a)*Gamma[1/2, (b*(c + d*x)
)/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(3*(a
+ (b*c)/d))*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx+a)}{\sqrt{dx+c}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.48, size = 177, normalized size = 0.78

$$\frac{\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{\sqrt{\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*c
/d)/sqrt(-b/d) + sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^(-
3*a + 3*b*c/d)/sqrt(b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a -
b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/
sqrt(b/d))/d
```

Fricas [A]
time = 0.38, size = 253, normalized size = 1.11

$$\frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)-\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)+\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)+9\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-9\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)}{24\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c - a
*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*sqrt(-b/d)*
(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)
*sqrt(-b/d)) + 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*
```

```
d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a
*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(1/2),x)
[Out] Integral(cosh(a + b*x)**3/sqrt(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3/sqrt(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cosh(a+b\,x)}^3}{\sqrt{c+d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^(1/2),x)
[Out] int(cosh(a + b*x)^3/(c + d*x)^(1/2), x)
```

3.60 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal. Leaf size=246

$$-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}-\frac{3\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{\sqrt{b}\,e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{3\sqrt{b}}{}$$

```
[Out] -3/4*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d^(3
/2)+3/4*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(1/2)*Pi^(1/2)/d
^(3/2)-1/4*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*b^(
1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)+1/4*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))*b^(1/2)*3^(1/2)*Pi^(1/2)/d^(3/2)-2*cosh(b*x+a)^3/d/(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3394, 3389, 2211, 2235, 2236}

$$-\frac{3\sqrt{\pi}\,\sqrt{b}\,e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{\sqrt{3\pi}\,\sqrt{b}\,e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{3\sqrt{\pi}\,\sqrt{b}\,e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}+\frac{\sqrt{3\pi}\,\sqrt{b}\,e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^3)/(d*Sqrt[c + d*x]) - (3*Sqrt[b]*E^(-a + (b*c)/d)*Sqrt[P
i]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2)) - (Sqrt[b]*E^(-3*a + (
3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/
2)) + (3*Sqrt[b]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(4*d^(3/2)) + (Sqrt[b]*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*d^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx &= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(6ib)\int\left(-\frac{i\sinh(a+bx)}{4\sqrt{c+dx}} - \frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d} \\
&= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}\,dx}{2d} + \frac{(3b)\int\frac{\sinh(3a+3bx)}{\sqrt{c+dx}}\,dx}{2d} \\
&= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} + \frac{(3b)\int\frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} - \frac{(3b)\int\frac{e^{i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{4d} + \frac{(3b)\int\frac{e^{-i(3ia+3ibx)}}{\sqrt{c+dx}}\,d\cdots}{4d} \\
&= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} - \frac{(3b)\mathrm{Subst}\left(\int e^{i\left(3ia-\frac{3ibc}{d}\right)-\frac{3bx^2}{d}}\,dx, x, \sqrt{c+dx}\right)}{2d^2} - \frac{(3b)\mathrm{Subst}\left(\int\cdots\right.}{\cdots} \\
&= -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}\,e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{\sqrt{b}\,e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\cdots\right.}{4d^{3/2}}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 717 vs. 2(246) = 492.
time = 2.39, size = 717, normalized size = 2.91

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

```
[Out] (-(Sqrt[d]*Cosh[3*a - (3*b*c)/d]) - Sqrt[d]*E^((6*b*(c + d*x))/d)*Cosh[3*a
- (3*b*c)/d] - 3*Sqrt[d]*E^((2*b*(c + d*x))/d)*Cosh[a - (b*c)/d] - 3*Sqrt[d
]*E^((4*b*(c + d*x))/d)*Cosh[a - (b*c)/d] + Sqrt[3]*Sqrt[d]*E^((3*b*(c + d*
x))/d)*Sqrt[-((b*(c + d*x))/d)]*Cosh[3*a - (3*b*c)/d]*Gamma[1/2, (-3*b*(c +
 d*x))/d] + 3*Sqrt[d]*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Cosh[a -
(b*c)/d]*Gamma[1/2, (b*(c + d*x))/d] + Sqrt[3]*Sqrt[d]*E^((3*b*(c + d*x))/d
)*Sqrt[(b*(c + d*x))/d]*Cosh[3*a - (3*b*c)/d]*Gamma[1/2, (3*b*(c + d*x))/d]
 + Sqrt[d]*Sinh[3*a - (3*b*c)/d] - Sqrt[d]*E^((6*b*(c + d*x))/d)*Sinh[3*a -
 (3*b*c)/d] + Sqrt[b]*E^((3*b*(c + d*x))/d)*Sqrt[3*Pi]*Sqrt[c + d*x]*Erf[(S
qrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]]*Sinh[3*a - (3*b*c)/d] + Sqrt[b]*E^((
3*b*(c + d*x))/d)*Sqrt[3*Pi]*Sqrt[c + d*x]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d
*x])/Sqrt[d]]*Sinh[3*a - (3*b*c)/d] + 3*Sqrt[d]*E^((2*b*(c + d*x))/d)*Sinh[
a - (b*c)/d] - 3*Sqrt[d]*E^((4*b*(c + d*x))/d)*Sinh[a - (b*c)/d] - 3*Sqrt[d
]*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]*S
inh[a - (b*c)/d] + 3*Sqrt[d]*E^((3*b*(c + d*x))/d)*Sqrt[-((b*(c + d*x))/d)]
*Gamma[1/2, -((b*(c + d*x))/d)]*(Cosh[a - (b*c)/d] + Sinh[a - (b*c)/d]))/(4
*d^(3/2)*E^((3*b*(c + d*x))/d)*Sqrt[c + d*x])
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Maxima [A]
time = 0.37, size = 196, normalized size = 0.80

$$-\frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{\frac{(dx+c)b}{d}}\,e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{-\frac{(dx+c)b}{d}}\,e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x +
c)*b/d)/sqrt(d*x + c) + sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)*g
amma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt((d*x + c)*b/d)*e^(-a +
b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d)*e^
(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(182) = 364$.
time = 0.47, size = 1344, normalized size = 5.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
(d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c
- a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c
)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c -
 a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*((d*x +
c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(
-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*sinh(-3*(
b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-3*(b*c -
a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
 a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*
x + c)*sqrt(-b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-(b*c - a*d
)/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b
*c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x +
c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-(b*c
- a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-(b*c - a*d)
/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b
/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*co
sh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*x
 + c)*cosh(-(b*c - a*d)/d) + (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x +
a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cos
h(-(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*
x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (cosh(b*x + a)^6 + 6*cos
h(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2 + 1)*si
nh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(b*x + a))
*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*sinh(b*x +
 a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)^3 + cosh(b
*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2*x + c*d)*cosh(b*x + a)^3 +
 3*(d^2*x + c*d)*cosh(b*x + a)^2*sinh(b*x + a) + 3*(d^2*x + c*d)*cosh(b*x +
 a)*sinh(b*x + a)^2 + (d^2*x + c*d)*sinh(b*x + a)^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(3/2),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^(3/2),x)
```

```
[Out] int(cosh(a + b*x)^3/(c + d*x)^(3/2), x)
```

3.61 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal. Leaf size=277

$$-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}+\frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{b^{3/2}e^{a}}{}$$

```
[Out] -2/3*cosh(b*x+a)^3/d/(d*x+c)^(3/2)+1/2*b^(3/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d
*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)+1/2*b^(3/2)*exp(a-b*c/d)*erfi(b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5/2)+1/2*b^(3/2)*exp(-3*a+3*b*c/d)*erf(
3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(5/2)+1/2*b^(3/2)
*exp(3*a-3*b*c/d)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1
/2)/d^(5/2)-4*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3395, 3388, 2211, 2235, 2236, 3393}

$$\frac{\sqrt{\pi}\,b^{3/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}\,b^{3/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{\pi}\,b^{3/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}+\frac{\sqrt{3\pi}\,b^{3/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\,\sqrt{b}\,\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) + (b^(3/2)*E^(-a + (b*c)/d)*Sqrt
[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) + (b^(3/2)*E^(-3*a +
 (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(
5/2)) + (b^(3/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt
[d]])/(2*d^(5/2)) + (b^(3/2)*E^(3*a - (3*b*c)/d)*Sqrt[3*Pi]*Erfi[(Sqrt[3]*S
qrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*d^(5/2)) - (4*b*Cosh[a + b*x]^2*Sinh[a +
 b*x])/(d^2*Sqrt[c + d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{(8b^2)\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx}{d^2} + \frac{(12b^2)\ \cdots}{\cdots}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{(4b^2)\int \frac{e^{-i(ia+ibx)}}{\sqrt{c+dx}}\,dx}{d^2} - \frac{(4b^2)\int \cdots}{\cdots}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}} - \frac{(8b^2)\,\mathrm{Subst}\left(\int e^{i\left(ia-\frac{ibc}{d}\right)-\frac{bx^2}{d}}\,dx \cdots\right.}{d^3}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\ \cdots}{\cdots}\right.}{d^{5/2}}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} - \frac{4b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{5/2}} - \frac{4b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\ \cdots}{\cdots}\right.}{d^{5/2}}$$

$$= -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\ \cdots}{\cdots}\right.}{2d^{5/2}}$$

Mathematica [**A**]
time = 2.31, size = 253, normalized size = 0.91

$$\frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}\,de^{6a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-3de^{2a+\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)-3\sqrt{3}\,de^{\frac{6bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{3b(c+dx)}{d}\right)-4e^{3\left(a+\frac{bc}{d}\right)}\cosh^2(a+bx)(d\cosh(a+bx)+6b(c+dx)\sinh(a+bx))\right)}{6d^2(c+dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

```
[Out] (-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x)
)/d] - 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(
c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2,
(b*(c + d*x))/d] - 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[
1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Cosh[a + b*x]^2*(d*Cosh[a +
 b*x] + 6*b*(c + d*x)*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c + d*x)^
(3/2))
```

Maple [**F**]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3\left(bx+a\right)}{\left(dx+c\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Maxima [A]
time = 0.37, size = 194, normalized size = 0.70

$$-\frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

```
[Out] -3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x
+ c)*b/d)/(d*x + c)^(3/2) + sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a*d
)/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) + ((d*x + c)*b/d)^(3/2)*
e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)*b/d
)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. $2(209) = 418$.
time = 0.40, size = 2058, normalized size = 7.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*c
osh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sin
h(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/
d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d)
 - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*
(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))
 - 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh
(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-
3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*d)/d)
+ (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 +
 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) +
(b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(
b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-3*(b*
```

```
c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-3*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))
+ 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c -
a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)
/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 +
2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 +
2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*
d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b*d^2*x^
2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*
x + b*c^2)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x +
b*c^2)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c
^2)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d
)/d))*sinh(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*co
sh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b
*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x
 + a)^2*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a
)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(
-b/d)) - ((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^6 + 6*(6*b*d*x + 6*b*c + d)*c
osh(b*x + a)*sinh(b*x + a)^5 + (6*b*d*x + 6*b*c + d)*sinh(b*x + a)^6 + 3*(2
*b*d*x + 2*b*c + d)*cosh(b*x + a)^4 + 3*(2*b*d*x + 5*(6*b*d*x + 6*b*c + d)*
cosh(b*x + a)^2 + 2*b*c + d)*sinh(b*x + a)^4 + 4*(5*(6*b*d*x + 6*b*c + d)*c
osh(b*x + a)^3 + 3*(2*b*d*x + 2*b*c + d)*cosh(b*x + a))*sinh(b*x + a)^3 - 6
*b*d*x - 3*(2*b*d*x + 2*b*c - d)*cosh(b*x + a)^2 + 3*(5*(6*b*d*x + 6*b*c +
d)*cosh(b*x + a)^4 - 2*b*d*x + 6*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^2 - 2*
b*c + d)*sinh(b*x + a)^2 - 6*b*c + 6*((6*b*d*x + 6*b*c + d)*cosh(b*x + a)^5
 + 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)^3 - (2*b*d*x + 2*b*c - d)*cosh(b*x
 + a))*sinh(b*x + a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*c
osh(b*x + a)^3 + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2*sinh(b*x
 + a) + 3*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a)^2 + (
d^4*x^2 + 2*c*d^3*x + c^2*d^2)*sinh(b*x + a)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(5/2),x)
```

```
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Giac [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^(5/2),x)
```

```
[Out] int(cosh(a + b*x)^3/(c + d*x)^(5/2), x)
```

3.62 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal. Leaf size=331

$$\frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}-\frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}}-\frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{3b^{5/2}e^{-3a+\frac{3}{}}}{}$$

```
[Out] -2/5*cosh(b*x+a)^3/d/(d*x+c)^(5/2)-4/5*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x
+c)^(3/2)-1/5*b^(5/2)*exp(-a+b*c/d)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*Pi^(
1/2)/d^(7/2)+1/5*b^(5/2)*exp(a-b*c/d)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*P
i^(1/2)/d^(7/2)-3/5*b^(5/2)*exp(-3*a+3*b*c/d)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)+3/5*b^(5/2)*exp(3*a-3*b*c/d)*erfi(3^
(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))*3^(1/2)*Pi^(1/2)/d^(7/2)+16/5*b^2*cosh
(b*x+a)/d^3/(d*x+c)^(1/2)-24/5*b^2*cosh(b*x+a)^3/d^3/(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 7, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3395, 3378, 3389, 2211, 2235, 2236, 3394}

$$-\frac{\sqrt{\pi}\,b^{5/2}e^{\frac{bc}{d}-a}\mathrm{Erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{3\sqrt{3\pi}\,b^{5/2}e^{\frac{3bc}{d}-3a}\mathrm{Erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{\sqrt{\pi}\,b^{5/2}e^{a-\frac{bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}+\frac{3\sqrt{3\pi}\,b^{5/2}e^{3a-\frac{3bc}{d}}\mathrm{Erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}}-\frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}}+\frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (16*b^2*Cosh[a + b*x])/(5*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^3)/(5*d*(c
+ d*x)^(5/2)) - (24*b^2*Cosh[a + b*x]^3)/(5*d^3*Sqrt[c + d*x]) - (b^(5/2)*E
^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2))
- (3*b^(5/2)*E^(-3*a + (3*b*c)/d)*Sqrt[3*Pi]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c +
d*x])/Sqrt[d]])/(5*d^(7/2)) + (b^(5/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) + (3*b^(5/2)*E^(3*a - (3*b*c)/d)*Sq
rt[3*Pi]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(5*d^(7/2)) - (4*b*
Cosh[a + b*x]^2*Sinh[a + b*x])/(5*d^2*(c + d*x)^(3/2))
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
```

```
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rubi steps

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}} - \frac{(8b^2)\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx}{5d^2} + \frac{(12b^2)\int}{}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(}{5d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(}{5d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{4b\cosh^2(a+bx)\sinh(}{5d^2(c+dx)^{3/2}}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} + \frac{8b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\Big(\sqrt{}}{5d^{7/2}}$$

$$= \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\Big(\sqrt{b}}{5d^{7/2}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 3211 vs. $2(331) = 662$.
time = 6.29, size = 3211, normalized size = 9.70

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```

```
[Out] (3*(Sinh[a]*(-1/30*((-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c + d*x) + 4*b^
2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -((b*(c + d*x)
)/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*E^((b*(c + d*
x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])/E^((b
*(c + d*x))/d))*Sinh[(b*c)/d])/(d^3*(c + d*x)^(5/2)) + (2*Cosh[(b*c)/d]*(-1
/2*(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c +
 d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) + 2
*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d])
)/E^((b*(c + d*x))/d))) - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)^(
5/2))) + Cosh[a]*((Cosh[(b*c)/d]*(-2*E^((b*(c + d*x))/d)*(3*d^2 + 2*b*d*(c
+ d*x) + 4*b^2*(c + d*x)^2) + 8*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamma[1/2, -
((b*(c + d*x))/d)] + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*b*d*
E^((b*(c + d*x))/d)*(c + d*x)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*
```

```
x))/d])/E^((b*(c + d*x))/d)))/(30*d^3*(c + d*x)^(5/2)) - (2*Sinh[(b*c)/d]*(
-1/2*(b*(c + d*x)*(2*E^((b*(c + d*x))/d)*(d + 2*b*(c + d*x)) + 4*d*(-((b*(c
 + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)] + (2*(d - 2*b*(c + d*x) +
 2*d*E^((b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (b*(c + d*x))/d
]))/E^((b*(c + d*x))/d))) - 3*d^2*Sinh[(b*(c + d*x))/d]))/(15*d^3*(c + d*x)
^(5/2)))))/4 + (Sinh[3*a]*(-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c +
d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((
b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c
+ d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c +
 d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[
(b*c)/d])/(d^3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])
*(-6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])
/Sqrt[d]] - 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt
[c + d*x])/Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d
^2 + 12*b^2*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5
/2))) + Cosh[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c +
 d*x))/d)*(d^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-(
(b*(c + d*x))/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c
 + d*x) - 24*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c
+ d*x))/d)^(5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10
*d^3*(c + d*x)^(5/2)) + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2
)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] -
 6*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2
*(c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/4 +
(-(Cosh[3*a]*(-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2
 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/
d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b
^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/
2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(d^
3*(c + d*x)^(5/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*
Sqrt[3*Pi]*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6
*b^(5/2)*Sqrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]] + Sqrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(
c + d*x)^2)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2)))) - Sinh
[3*a]*((Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d
^2 + 2*b*d*(c + d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x)
)/d))^(5/2)*Gamma[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24
*b^2*(c + d*x)^2 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(
5/2)*Gamma[1/2, (3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d)))/(10*d^3*(c + d*
x)^(5/2)) + (2*(1 + 2*Cosh[(2*b*c)/d])*Sinh[(b*c)/d]*(-6*b^(5/2)*Sqrt[3*Pi]
*(c + d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*S
qrt[3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + S
qrt[d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (d^2 + 12*b^2*(c + d*x)^2
)*Sinh[(3*b*(c + d*x))/d])))/(5*d^(7/2)*(c + d*x)^(5/2))))/8 + (Cosh[3*a]*(
```

```
-1/10*((1 + 2*Cosh[(2*b*c)/d])*(-2*E^((3*b*(c + d*x))/d)*(d^2 + 2*b*d*(c +
d*x) + 12*b^2*(c + d*x)^2) + 24*Sqrt[3]*d^2*(-((b*(c + d*x))/d))^(5/2)*Gamm
a[1/2, (-3*b*(c + d*x))/d] + (-2*d^2 + 4*b*d*(c + d*x) - 24*b^2*(c + d*x)^2
 + 24*Sqrt[3]*d^2*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(5/2)*Gamma[1/2,
(3*b*(c + d*x))/d])/E^((3*b*(c + d*x))/d))*Sinh[(b*c)/d])/(d^3*(c + d*x)^(5
/2)) - (2*Cosh[(b*c)/d]*(-1 + 2*Cosh[(2*b*c)/d])*(-6*b^(5/2)*Sqrt[3*Pi]*(c
+ d*x)^(5/2)*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] - 6*b^(5/2)*Sqrt[
3*Pi]*(c + d*x)^(5/2)*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]] + Sqrt[
d]*(2*b*d*(c + d*x)*Cosh[(3*b*(c + d*x))/d] + (...
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

```
[Out] int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Maxima [A]
time = 0.36, size = 196, normalized size = 0.59

$$-\frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

```
[Out] -3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d*
x + c)*b/d)/(d*x + c)^(5/2) + 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c -
 a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) + ((d*x + c)*b/d)^(5
/2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x + c)
*b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3280 vs. 2(253) = 506.
time = 0.43, size = 3280, normalized size = 9.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

```
[Out] -1/20*(12*sqrt(3)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*
d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c -
a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^
2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d
)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
 b^2*c^3)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*s
inh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + 12*sqrt(3)*s
qrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
 + a)^3*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2
*d*x + b^2*c^3)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*
x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-3*(b*c - a*d)/d))*si
nh(b*x + a)^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3
)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)
^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)^2*cosh(-3*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*
d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(
-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 4*sqrt(pi)*((b^2*d^3*x^3 + 3*
b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^3*cosh(-(b*c - a*d)/
d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^
2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((b^2*d^3*x^3 +
3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(b*c - a*d)/
d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a
)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-(b*
c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt
(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x +
a)^3*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b
^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
^3 + 3*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x
+ a)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
+ b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((b^2*d^
3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*cosh(-(b
*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*co
```

```
sh(b*x + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x
+ c)*sqrt(-b/d)) + ((12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^
2*c*d + b*d^2)*x)*cosh(b*x + a)^6 + 6*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*
d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a)^5 + (12*b^2
*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^2)*x)*sinh(b*x
+ a)^6 + 12*b^2*d^2*x^2 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*
(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c
*d + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*c*d + b*d^
2)*x)*cosh(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^4 +
12*b^2*c^2 + 4*(5*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 + 2*(12*b^2*
c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*
d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a))*sinh(b*x + a)^3 - 2*b*c*d + (
4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh
(b*x + a)^2 + (4*b^2*d^2*x^2 + 15*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d +
d^2 + 2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*b^2*c^2 - 2*b*c*d + 6*(
4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh
(b*x + a)^2 + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*sinh(b*x + a)^2 + d^2 + 2*(1
2*b^2*c*d - b*d^2)*x + 2*(3*(12*b^2*d^2*x^2 + 12*b^2*c^2 + 2*b*c*d + d^2 +
2*(12*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^5 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 +
2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^3 + (4*b^2*d^2*x^2
 + 4*b^2*c^2 - 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d - b*d^2)*x)*cosh(b*x + a))*si
nh(b*x + a))*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^3 (a + bx)}{(c + dx)^{\frac{7}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)**3/(d*x+c)**(7/2),x)
[Out] Integral(cosh(a + b*x)**3/(c + d*x)**(7/2), x)
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
[Out] integrate(cosh(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cosh(a + b\,x)^3}{(c + d\,x)^{7/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3/(c + d*x)^(7/2),x)
```

```
[Out] int(cosh(a + b*x)^3/(c + d*x)^(7/2), x)
```

3.63 $\int (dx)^{3/2} \cosh(fx)\, dx$

Optimal. Leaf size=111

$$-\frac{3d\sqrt{dx}\cosh(fx)}{2f^2}+\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3d^{3/2}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{(dx)^{3/2}\sinh(fx)}{f}$$

```
[Out] (d*x)^(3/2)*sinh(f*x)/f+3/8*d^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/
2)/f^(5/2)+3/8*d^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/f^(5/2)-3
/2*d*cosh(f*x)*(d*x)^(1/2)/f^2
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3377, 3388, 2211, 2235, 2236}

$$\frac{3\sqrt{\pi}\,d^{3/2}\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}+\frac{3\sqrt{\pi}\,d^{3/2}\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}}-\frac{3d\sqrt{dx}\cosh(fx)}{2f^2}+\frac{(dx)^{3/2}\sinh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*x)^(3/2)*Cosh[f*x],x]
```

```
[Out] (-3*d*Sqrt[d*x]*Cosh[f*x])/(2*f^2) + (3*d^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[
d*x])/Sqrt[d]])/(8*f^(5/2)) + (3*d^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/
Sqrt[d]])/(8*f^(5/2)) + ((d*x)^(3/2)*Sinh[f*x])/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{(dx)^{3/2}\sinh(fx)}{f} - \frac{(3d)\int \sqrt{dx}\,\sinh(fx)\,dx}{2f}$$

$$= -\frac{3d\sqrt{dx}\,\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{(3d^2)\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{4f^2}$$

$$= -\frac{3d\sqrt{dx}\,\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{(3d^2)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{8f^2} + \frac{(3d^2)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{8f^2}$$

$$= -\frac{3d\sqrt{dx}\,\cosh(fx)}{2f^2} + \frac{(dx)^{3/2}\sinh(fx)}{f} + \frac{(3d)\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{4f^2} + \frac{(3d)\mathrm{S}}{}$$

$$= -\frac{3d\sqrt{dx}\,\cosh(fx)}{2f^2} + \frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{(}{}$$

Mathematica [**A**]
time = 0.01, size = 51, normalized size = 0.46

$$\frac{d^2\left(\sqrt{-fx}\,\Gamma\left(\frac{5}{2},-fx\right) - \sqrt{fx}\,\Gamma\left(\frac{5}{2},fx\right)\right)}{2f^3\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d*x)^(3/2)*Cosh[f*x],x]
```

```
[Out] (d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] - Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3*
Sqrt[d*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.72, size = 133, normalized size = 1.20

method	result
meijerg	$-\frac{2i(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\,\left(-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}(10fx+15)\mathrm{e}^{-fx}}{80\sqrt{\pi}\,f^{2}}-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}(-10fx+15)\mathrm{e}^{fx}}{80\sqrt{\pi}\,f^{2}}+\frac{3(if)^{\frac{5}{2}}\sqrt{2}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{32f^{\frac{5}{2}}}+\frac{3(i\ldots}{}\right.}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x)^(3/2)*cosh(f*x),x,method=_RETURNVERBOSE)
```

```
[Out] -2*I*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/80/Pi^(1/2)*x^(
1/2)*2^(1/2)*(I*f)^(5/2)*(10*f*x+15)/f^2*exp(-f*x)-1/80/Pi^(1/2)*x^(1/2)*2^
(1/2)*(I*f)^(5/2)*(-10*f*x+15)/f^2*exp(f*x)+3/32*(I*f)^(5/2)*2^(1/2)/f^(5/2
)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^(1/2
)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(77) = 154.
time = 0.27, size = 174, normalized size = 1.57

$$\frac{16\,(dx)^{\frac{5}{2}}\cosh(fx)+\frac{f\left(\frac{15\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}}+\frac{15\,\sqrt{\pi}\,d^3\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}}-\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2-10\,(dx)^{\frac{3}{2}}d^2f+15\,\sqrt{dx}\,d^3\right)e^{(fx)}}{f^3}-\frac{2\left(4\,(dx)^{\frac{5}{2}}df^2+10\,(dx)^{\frac{3}{2}}d^2f+15\,\sqrt{dx}\,d^3\right)e^{(-fx)}}{f^3}\right)}{d}}{40\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="maxima")
```

```
[Out] 1/40*(16*(d*x)^(5/2)*cosh(f*x) + f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d)
)/(f^3*sqrt(f/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-f/
d)) - 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*e^(
f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3
)*e^(-f*x)/f^3)/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(77) = 154.
time = 0.37, size = 191, normalized size = 1.72

$$\frac{3\,\sqrt{\pi}\,(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)-3\,\sqrt{\pi}\,(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)-2\,(2\,df^2x-(2\,df^2x-3\,df)\cosh(fx)^2-2\,(2\,df^2x-3\,df)\cosh(fx)\sinh(fx)-(2\,df^2x-3\,df)\sinh(fx)^2+3\,df)\sqrt{dx}}{8\,(f^3\cosh(fx)+f^3\sinh(fx))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="fricas")
```

```
[Out] 1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqr
t(f/d)) - 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*
x)*sqrt(-f/d)) - 2*(2*d*f^2*x - (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 - 2*(2*d*f^
2*x - 3*d*f)*cosh(f*x)*sinh(f*x) - (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3*d*f)
*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 14.47, size = 131, normalized size = 1.18

$$\frac{5d^{\frac{3}{2}}x^{\frac{3}{2}}\sinh{(fx)}\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} - \frac{15d^{\frac{3}{2}}\sqrt{x}\cosh{(fx)}\Gamma\left(\frac{5}{4}\right)}{8f^2\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{2}\sqrt{\pi}\,d^{\frac{3}{2}}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)**(3/2)*cosh(f*x),x)
```

```
[Out] 5*d**(3/2)*x**(3/2)*sinh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 15*d**(3/2)*sqr
t(x)*cosh(f*x)*gamma(5/4)/(8*f**2*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*d**(3/2
)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*gamma
(5/4)/(16*f**(5/2)*gamma(9/4))
```

Giac [A]
time = 0.42, size = 145, normalized size = 1.31

$$-\frac{1}{8}d\left(\frac{\frac{3\sqrt{\pi}\,d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}\,f^2}+\frac{2\left(2\sqrt{dx}\,d^2fx+3\sqrt{dx}\,d^2\right)e^{(-fx)}}{f^2}}{d^2}+\frac{\frac{3\sqrt{\pi}\,d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}\,f^2}-\frac{2\left(2\sqrt{dx}\,d^2fx-3\sqrt{dx}\,d^2\right)e^{(fx)}}{f^2}}{d^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="giac")
```

```
[Out] -1/8*d*((3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2*
sqrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d^2 + (3*sqrt(pi)*d^3*er
f(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3*sq
rt(d*x)*d^2)*e^(f*x)/f^2)/d^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(f\,x)\,\left(d\,x\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)*(d*x)^(3/2),x)
```

```
[Out] int(cosh(f*x)*(d*x)^(3/2), x)
```

3.64 $\int \sqrt{dx} \cosh(fx)\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{d}\,\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{d}\,\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\,\sinh(fx)}{f}$$

```
[Out] 1/4*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)-1/4*erfi(f^(1
/2)*(d*x)^(1/2)/d^(1/2))*d^(1/2)*Pi^(1/2)/f^(3/2)+sinh(f*x)*(d*x)^(1/2)/f
```

Rubi [A]
time = 0.07, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3377, 3389, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,\sqrt{d}\,\mathrm{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{\pi}\,\sqrt{d}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\,\sinh(fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d*x]*Cosh[f*x],x]
```

```
[Out] (Sqrt[d]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) - (Sqrt[d]*
Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(4*f^(3/2)) + (Sqrt[d*x]*Sinh[f
*x])/f
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{dx}\cosh(fx)\,dx &= \frac{\sqrt{dx}\sinh(fx)}{f} - \frac{d\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{2f} \\
&= \frac{\sqrt{dx}\sinh(fx)}{f} + \frac{d\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{4f} - \frac{d\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{4f} \\
&= \frac{\sqrt{dx}\sinh(fx)}{f} + \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} - \frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{2f} \\
&= \frac{\sqrt{d}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} - \frac{\sqrt{d}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}} + \frac{\sqrt{dx}\sinh(fx)}{f}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 48, normalized size = 0.52

$$-\frac{d\left(\sqrt{-fx}\,\Gamma\left(\frac{3}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{3}{2},fx\right)\right)}{2f^2\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d*x]*Cosh[f*x],x]
```

```
[Out] -1/2*(d*(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)] + Sqrt[f*x]*Gamma[3/2, f*x]))/(f^2
*Sqrt[d*x])
```

Maple [C] Result contains complex when optimal does not.
time = 0.72, size = 121, normalized size = 1.32

method	result

meijerg	$-\frac{i\sqrt{\pi}\,\sqrt{dx}\,\sqrt{2}\left(\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{3}{2}}\mathrm{e}^{fx}}{4\sqrt{\pi}\,f}-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{3}{2}}\mathrm{e}^{-fx}}{4\sqrt{\pi}\,f}+\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{3}{2}}}-\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{8f^{\frac{3}{2}}}\right)}{\sqrt{x}\,\sqrt{if}\,f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)*(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -I*Pi^(1/2)*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2)
*2^(1/2)*(I*f)^(3/2)/f*exp(f*x)-1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(3/2)/f*
exp(-f*x)+1/8*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*f)^(3
/2)*2^(1/2)/f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(62) = 124$.
time = 0.28, size = 148, normalized size = 1.61

$$\frac{8\,(dx)^{\frac{3}{2}}\cosh(fx)+\frac{f\left(\frac{3\sqrt{\pi}\,d^{2}\operatorname{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{f^{2}\sqrt{\frac{f}{d}}}-\frac{3\sqrt{\pi}\,d^{2}\operatorname{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{f^{2}\sqrt{-\frac{f}{d}}}-\frac{2\left(2\,(dx)^{\frac{3}{2}}df-3\sqrt{dx}\,d^{2}\right)e^{(fx)}}{f^{2}}-\frac{2\left(2\,(dx)^{\frac{3}{2}}df+3\sqrt{dx}\,d^{2}\right)e^{(-fx)}}{f^{2}}\right)}{d}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/12*(8*(d*x)^(3/2)*cosh(f*x) + f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))/
(f^2*sqrt(f/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d))
 - 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2)*d
*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.
time = 0.38, size = 138, normalized size = 1.50

$$\frac{\sqrt{\pi}\,(d\cosh(fx)+d\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\,(d\cosh(fx)+d\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)+2\left(f\cosh(fx)^{2}+2\,f\cosh(fx)\sinh(fx)+f\sinh(fx)^{2}-f\right)\sqrt{dx}}{4\left(f^{2}\cosh(fx)+f^{2}\sinh(fx)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)
) + sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f/d
)) + 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 - f)*sqrt(d
*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 0.86, size = 100, normalized size = 1.09

$$\frac{3\sqrt{d}\,\sqrt{x}\,\sinh\left(fx\right)\Gamma\left(\frac{3}{4}\right)}{4f\Gamma\left(\frac{7}{4}\right)}-\frac{3\sqrt{2}\,\sqrt{\pi}\,\sqrt{d}\,e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)**(1/2),x)
```

```
[Out] 3*sqrt(d)*sqrt(x)*sinh(f*x)*gamma(3/4)/(4*f*gamma(7/4)) - 3*sqrt(2)*sqrt(pi
)*sqrt(d)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(
pi))*gamma(3/4)/(8*f**(3/2)*gamma(7/4))
```

Giac [A]
time = 0.43, size = 108, normalized size = 1.17

$$-\frac{\frac{\sqrt{\pi}\,d^{2}\operatorname{erf}\left(-\frac{\sqrt{df}\,\sqrt{dx}}{d}\right)}{\sqrt{df}\,f}+\frac{2\sqrt{dx}\,de^{\left(-fx\right)}}{f}}{4\,d}+\frac{\frac{\sqrt{\pi}\,d^{2}\operatorname{erf}\left(-\frac{\sqrt{-df}\,\sqrt{dx}}{d}\right)}{\sqrt{-df}\,f}+\frac{2\sqrt{dx}\,de^{\left(fx\right)}}{f}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)*(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + 2*sqrt(d*x)*
d*e^(-f*x)/f)/d + 1/4*(sqrt(pi)*d^2*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f
)*f) + 2*sqrt(d*x)*d*e^(f*x)/f)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(f\,x)\,\sqrt{d\,x}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)*(d*x)^(1/2),x)
```

```
[Out] int(cosh(f*x)*(d*x)^(1/2), x)
```

3.65 $\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx$

Optimal. Leaf size=77

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

```
[Out] 1/2*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)+1/2*erfi(f^(1
/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(1/2)/f^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3388, 2211, 2235, 2236}

$$\frac{\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/Sqrt[d*x],x]
```

```
[Out] (Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]
*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx &= \frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}\,dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}\,dx\\ &= \frac{\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}+\frac{\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx,x,\sqrt{dx}\right)}{d}\\ &= \frac{\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\end{aligned}$$

Mathematica [**A**]
time = 0.01, size = 48, normalized size = 0.62

$$\frac{\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)-\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)}{2f\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/Sqrt[d*x],x]
[Out] (Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*x
])
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.71, size = 72, normalized size = 0.94

method	result	size
meijerg	$-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{if}\left(\frac{\sqrt{if}\,\sqrt{2}\,\text{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{2\sqrt{f}}+\frac{\sqrt{if}\,\sqrt{2}\,\text{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{2\sqrt{f}}\right)}{2\sqrt{dx}\,f}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*I*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(1/2*(I*f)^(1/2)*
2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erfi(x
^(1/2)*f^(1/2)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(49) = 98$.
time = 0.27, size = 117, normalized size = 1.52

$$\frac{4\sqrt{dx}\cosh(fx) - \frac{\left(\frac{2\sqrt{dx}\,de^{(fx)}}{f} + \frac{2\sqrt{dx}\,de^{(-fx)}}{f} - \frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}\,d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(4*sqrt(d*x)*cosh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f + 2*sqrt(d*x)*d*e^(-f
*x)/f - sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*erf(
sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Fricas [A]
time = 0.43, size = 59, normalized size = 0.77

$$\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - \sqrt{\pi}\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) - sqrt(pi)*sqrt(-f/d)*erf(
sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] Result contains complex when optimal does not.
time = 0.57, size = 66, normalized size = 0.86

$$\frac{\sqrt{2}\sqrt{\pi}\,e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\sqrt{d}\sqrt{f}\,\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(1/2),x)
```

```
[Out] sqrt(2)*sqrt(pi)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/
sqrt(pi))*gamma(1/4)/(4*sqrt(d)*sqrt(f)*gamma(5/4))
```

Giac [A]
time = 0.40, size = 60, normalized size = 0.78

$$-\frac{\frac{\sqrt{\pi}\, d\operatorname{erf}\left(-\frac{\sqrt{df}\,\sqrt{dx}}{d}\right)}{\sqrt{df}}+\frac{\sqrt{\pi}\, d\operatorname{erf}\left(-\frac{\sqrt{-df}\,\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) + sqrt(pi)*d*erf(-sq
rt(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(f\,x)}{\sqrt{d\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(1/2),x)
```

```
[Out] int(cosh(f*x)/(d*x)^(1/2), x)
```

3.66 $\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx$

Optimal. Leaf size=88

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{\sqrt{f}\,\sqrt{\pi}\,\text{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{f}\,\sqrt{\pi}\,\text{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

```
[Out] -erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)+erfi(f^(1/2)*(d*
x)^(1/2)/d^(1/2))*f^(1/2)*Pi^(1/2)/d^(3/2)-2*cosh(f*x)/d/(d*x)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3378, 3389, 2211, 2235, 2236}

$$-\frac{\sqrt{\pi}\,\sqrt{f}\,\text{Erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}+\frac{\sqrt{\pi}\,\sqrt{f}\,\text{Erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}-\frac{2\cosh(fx)}{d\sqrt{dx}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (-2*Cosh[f*x])/(d*Sqrt[d*x]) - (Sqrt[f]*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/d^(3/2) + (Sqrt[f]*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/d^(3
/2)
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3389

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Dist[I
/2, Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Dist[I/2, Int[(c + d*x)^m*E^(
I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx &= -\frac{2\cosh(fx)}{d\sqrt{dx}} + \frac{(2f)\int \frac{\sinh(fx)}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\cosh(fx)}{d\sqrt{dx}} - \frac{f\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{d} + \frac{f\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{d}\\ &= -\frac{2\cosh(fx)}{d\sqrt{dx}} - \frac{(2f)\text{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2} + \frac{(2f)\text{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{d^2}\\ &= -\frac{2\cosh(fx)}{d\sqrt{dx}} - \frac{\sqrt{f}\,\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f}\,\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\,\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.76

$$\frac{e^{-fx}x\left(-1-e^{2fx}+e^{fx}\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)+e^{fx}\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)\right)}{(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/(d*x)^(3/2),x]
```

```
[Out] (x*(-1 - E^(2*f*x) + E^(f*x)*Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + E^(f*x)*Sqrt
[f*x]*Gamma[1/2, f*x]))/(E^(f*x)*(d*x)^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.71, size = 115, normalized size = 1.31

method	result
meijerg	$-\frac{i\sqrt{\pi}\, x^{\frac{3}{2}}\sqrt{2}\,(if)^{\frac{3}{2}}\left(-\frac{2\sqrt{2}\,\mathrm{e}^{fx}}{\sqrt{\pi}\,\sqrt{x}\,\sqrt{if}}-\frac{2\sqrt{2}\,\mathrm{e}^{-fx}}{\sqrt{\pi}\,\sqrt{x}\,\sqrt{if}}-\frac{2\sqrt{2}\,\sqrt{f}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{\sqrt{if}}+\frac{2\sqrt{2}\,\sqrt{f}\,\mathrm{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{\sqrt{if}}\right)}{4(dx)^{\frac{3}{2}} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*I*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(-2/Pi^(1/2)/x^(1
/2)*2^(1/2)/(I*f)^(1/2)*exp(f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)*exp
(-f*x)-2/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^(1/2)*f^(1/2))+2/(I*f)^(1/2)*2^(
1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A]
time = 0.26, size = 76, normalized size = 0.86

$$-\frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}+\frac{2\cosh(fx)}{\sqrt{dx}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

```
[Out] -(f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) - sqrt(pi)*erf(sqrt(d*x)*s
qrt(-f/d))/sqrt(-f/d))/d + 2*cosh(f*x)/sqrt(d*x))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(62) = 124$.
time = 0.37, size = 136, normalized size = 1.55

$$-\frac{\sqrt{\pi}\,(dx\cosh(fx)+dx\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}\,(dx\cosh(fx)+dx\sinh(fx))\sqrt{-\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\,\sqrt{-\frac{f}{d}}\right)+\sqrt{dx}\,\left(\cosh(fx)^2+2\cosh(fx)\sinh(fx)+\sinh(fx)^2+1\right)}{d^2x\cosh(fx)+d^2x\sinh(fx)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

```
[Out] -(sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) + sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt
(-f/d)) + sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 + 1)
)/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 99, normalized size = 1.12

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,\sqrt{f}\,e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)}+\frac{\cosh\left(fx\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\,\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(3/2),x)
```

```
[Out] -sqrt(2)*sqrt(pi)*sqrt(f)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*e
xp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(2*d**(3/2)*gamma(3/4)) + cosh(f*x)*gamma(
-1/4)/(2*d**(3/2)*sqrt(x)*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x)/(d*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(f\,x)}{{(d\,x)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(3/2),x)
```

```
[Out] int(cosh(f*x)/(d*x)^(3/2), x)
```

3.67 $\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx$

Optimal. Leaf size=114

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}+\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2f^{3/2}\sqrt{\pi}\,\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}$$

```
[Out] -2/3*cosh(f*x)/d/(d*x)^(3/2)+2/3*f^(3/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))*P
i^(1/2)/d^(5/2)+2/3*f^(3/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))*Pi^(1/2)/d^(5
/2)-4/3*f*sinh(f*x)/d^2/(d*x)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.417$, Rules used = {3378, 3388, 2211, 2235, 2236}

$$\frac{2\sqrt{\pi}\,f^{3/2}\mathrm{Erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}+\frac{2\sqrt{\pi}\,f^{3/2}\mathrm{Erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}}-\frac{4f\sinh(fx)}{3d^2\sqrt{dx}}-\frac{2\cosh(fx)}{3d(dx)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (-2*Cosh[f*x])/(3*d*(d*x)^(3/2)) + (2*f^(3/2)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*
x])/Sqrt[d]])/(3*d^(5/2)) + (2*f^(3/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sq
rt[d]])/(3*d^(5/2)) - (4*f*Sinh[f*x])/(3*d^2*Sqrt[d*x])
```

Rule 2211

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Dist[2/d, Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d*
x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx &= -\frac{2\cosh(fx)}{3d(dx)^{3/2}} + \frac{(2f)\int \frac{\sinh(fx)}{(dx)^{3/2}}\,dx}{3d} \\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}} - \frac{4f\sinh(fx)}{3d^2\sqrt{dx}} + \frac{(4f^2)\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx}{3d^2} \\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}} - \frac{4f\sinh(fx)}{3d^2\sqrt{dx}} + \frac{(2f^2)\int \frac{e^{-fx}}{\sqrt{dx}}\,dx}{3d^2} + \frac{(2f^2)\int \frac{e^{fx}}{\sqrt{dx}}\,dx}{3d^2} \\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}} - \frac{4f\sinh(fx)}{3d^2\sqrt{dx}} + \frac{(4f^2)\,\mathrm{Subst}\left(\int e^{-\frac{fx^2}{d}}\,dx, x, \sqrt{dx}\right)}{3d^3} + \frac{(4f^2)\,\mathrm{Subst}\left(\int e^{\frac{fx^2}{d}}\,dx\right.}{3d^3} \\
&= -\frac{2\cosh(fx)}{3d(dx)^{3/2}} + \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f\sinh(fx)}{3d^2\sqrt{dx}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 78, normalized size = 0.68

$$\frac{x\left(-2e^{fx}(1+2fx)-4(-fx)^{3/2}\Gamma\left(\frac{1}{2},-fx\right)+e^{-fx}\left(-2+4fx-4e^{fx}(fx)^{3/2}\Gamma\left(\frac{1}{2},fx\right)\right)\right)}{6(dx)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[f*x]/(d*x)^(5/2),x]
```

```
[Out] (x*(-2*E^(f*x)*(1 + 2*f*x) - 4*(-(f*x))^(3/2)*Gamma[1/2, -(f*x)] + (-2 + 4*
f*x - 4*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x])/E^(f*x)))/(6*(d*x)^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 0.71, size = 126, normalized size = 1.11

method	result
meijerg	$-\frac{i\sqrt{\pi}\, x^{\frac{5}{2}}\sqrt{2}\,(if)^{\frac{5}{2}}\left(-\frac{8\sqrt{2}\,\left(-fx+\frac{1}{2}\right)\mathrm{e}^{-fx}}{3\sqrt{\pi}\, x^{\frac{3}{2}}(if)^{\frac{3}{2}}}-\frac{8\sqrt{2}\,\left(fx+\frac{1}{2}\right)\mathrm{e}^{fx}}{3\sqrt{\pi}\, x^{\frac{3}{2}}(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}\, f^{\frac{3}{2}}\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{3(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}\, f^{\frac{3}{2}}\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{3(if)^{\frac{3}{2}}}\right.}{8(dx)^{\frac{5}{2}} f}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*I*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-8/3/Pi^(1/2)/x^
(3/2)*2^(1/2)/(I*f)^(3/2)*(-f*x+1/2)*exp(-f*x)-8/3/Pi^(1/2)/x^(3/2)*2^(1/2)
/(I*f)^(3/2)*(f*x+1/2)*exp(f*x)+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erf(x^(1/2)
*f^(1/2))+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Maxima [A]
time = 0.30, size = 58, normalized size = 0.51

$$\frac{\frac{f\left(\frac{\sqrt{fx}\,\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}}-\frac{\sqrt{-fx}\,\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d}-\frac{2\cosh(fx)}{(dx)^{\frac{3}{2}}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) - sqrt(-f*x)*gamma(-1/2, -f*x)
/sqrt(d*x))/d - 2*cosh(f*x)/(d*x)^(3/2))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(78) = 156.
time = 0.42, size = 179, normalized size = 1.57

$$\frac{2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-2\sqrt{\pi}\left(dfx^2\cosh\left(fx\right)+dfx^2\sinh\left(fx\right)\right)\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)-\left(\left(2\,fx+1\right)\cosh\left(fx\right)^2+2\left(2\,fx+1\right)\cosh\left(fx\right)\sinh\left(fx\right)+\left(2\,fx+1\right)\sinh\left(fx\right)^2-2\,fx+1\right)\sqrt{dx}}{3\left(d^3x^2\cosh\left(fx\right)+d^3x^2\sinh\left(fx\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(
d*x)*sqrt(f/d)) - 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(-
f/d)*erf(sqrt(d*x)*sqrt(-f/d)) - ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x + 1)*c
osh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 - 2*f*x + 1)*sqrt(d*x))/(d^3*x
^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [C] Result contains complex when optimal does not.
time = 16.03, size = 124, normalized size = 1.09

$$-\frac{\sqrt{2}\,\sqrt{\pi}\,f^{\frac{3}{2}}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\,\sqrt{f}\,\sqrt{x}\,e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)}+\frac{f\sinh\left(fx\right)\Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}}\sqrt{x}\,\Gamma\left(\frac{1}{4}\right)}+\frac{\cosh\left(fx\right)\Gamma\left(-\frac{3}{4}\right)}{2d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)**(5/2),x)
```

```
[Out] -sqrt(2)*sqrt(pi)*f**(3/2)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*ex
p(I*pi/4)/sqrt(pi))*gamma(-3/4)/(d**(5/2)*gamma(1/4)) + f*sinh(f*x)*gamma(-
3/4)/(d**(5/2)*sqrt(x)*gamma(1/4)) + cosh(f*x)*gamma(-3/4)/(2*d**(5/2)*x**(
3/2)*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x)/(d*x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cosh(f\,x)}{{(d\,x)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x)/(d*x)^(5/2),x)
```

```
[Out] int(cosh(f*x)/(d*x)^(5/2), x)
```

3.68 $\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$

Optimal. Leaf size=19

$$\mathrm{Int}\left(\sqrt{c+dx}\,\mathrm{sech}(a+bx),x\right)$$

```
[Out] Unintegrable(sech(b*x+a)*(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sqrt[c + d*x]*Sech[a + b*x],x]
[Out] Defer[Int][Sqrt[c + d*x]*Sech[a + b*x], x]
```

Rubi steps

$$\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$$

Mathematica [A]
time = 8.88, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sqrt[c + d*x]*Sech[a + b*x],x]
[Out] Integrate[Sqrt[c + d*x]*Sech[a + b*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \mathrm{sech}(bx+a)\sqrt{dx+c}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)*(d*x+c)^(1/2),x)
```

```
[Out] int(sech(b*x+a)*(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(d*x + c)*sech(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c + dx} \operatorname{sech}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)**(1/2),x)
[Out] Integral(sqrt(c + d*x)*sech(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)*(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{\sqrt{c + d\,x}}{\cosh\left(a + b\,x\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^(1/2)/cosh(a + b*x),x)
[Out] int((c + d*x)^(1/2)/cosh(a + b*x), x)
```

3.69 $\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal. Leaf size=19

$$\text{Int}\left(\frac{\text{sech}(a+bx)}{\sqrt{c+dx}},x\right)$$

```
[Out] Unintegrable(sech(b*x+a)/(d*x+c)^(1/2),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Sech[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Defer[Int][Sech[a + b*x]/Sqrt[c + d*x], x]
```

Rubi steps

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Mathematica [A]
time = 8.18, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Sech[a + b*x]/Sqrt[c + d*x],x]
```

```
[Out] Integrate[Sech[a + b*x]/Sqrt[c + d*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\text{sech}(bx+a)}{\sqrt{dx+c}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(b*x+a)/(d*x+c)^(1/2),x)
[Out] int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
[Out] integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
[Out] integral(sech(b*x + a)/sqrt(d*x + c), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\operatorname{sech}{\left(a + bx \right)}}{\sqrt{c + dx}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)**(1/2),x)
[Out] Integral(sech(a + b*x)/sqrt(c + d*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
[Out] integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{\mathrm{cosh}\left(a+b\,x\right)\,\sqrt{c+d\,x}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cosh(a + b*x)*(c + d*x)^(1/2)),x)
[Out] int(1/(cosh(a + b*x)*(c + d*x)^(1/2)), x)
```

3.70 $\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal. Leaf size=63

$$-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\cosh(x)}\,\sinh(x)}{4x}-\frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\cosh(x)}},x\right)+\frac{9}{8}\text{Int}\left(\frac{\cosh^{\frac{3}{2}}(x)}{x},x\right)$$

```
[Out] -1/2*cosh(x)^(3/2)/x^2-3/4*sinh(x)*cosh(x)^(1/2)/x+9/8*Unintegrable(cosh(x)
^(3/2)/x,x)-3/8*Unintegrable(1/x/cosh(x)^(1/2),x)
```

Rubi [A]
time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Int[Cosh[x]^(3/2)/x^3,x]
```

```
[Out] -1/2*Cosh[x]^(3/2)/x^2 - (3*Sqrt[Cosh[x]]*Sinh[x])/(4*x) - (3*Defer[Int][1/
(x*Sqrt[Cosh[x]]), x])/8 + (9*Defer[Int][Cosh[x]^(3/2)/x, x])/8
```

Rubi steps

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx=-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sqrt{\cosh(x)}\,\sinh(x)}{4x}-\frac{3}{8}\int\frac{1}{x\sqrt{\cosh(x)}}\,dx+\frac{9}{8}\int\frac{\cosh^{\frac{3}{2}}(x)}{x}\,dx$$

Mathematica [A]
time = 2.70, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[Cosh[x]^(3/2)/x^3,x]
```

```
[Out] Integrate[Cosh[x]^(3/2)/x^3, x]
```

Maple [A]
time = 0.21, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(3/2)/x^3,x)
```

```
[Out] int(cosh(x)^(3/2)/x^3,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(cosh(x)^(3/2)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**(3/2)/x**3,x)
```

```
[Out] Integral(cosh(x)**(3/2)/x**3, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(cosh(x)^(3/2)/x^3, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cosh(x)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^(3/2)/x^3,x)
[Out] int(cosh(x)^(3/2)/x^3, x)
```

3.71 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx$

Optimal. Leaf size=20

$$-4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}$$

```
[Out] 2*x*sinh(x)/cosh(x)^(1/2)-4*cosh(x)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}=0.059$, Rules used = {3396}

$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] -4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx &= \int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x\sqrt{\cosh(x)}\,dx\\ &= -4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

time = 0.27, size = 46, normalized size = 2.30

$$\frac{2\sinh(x)\left(x-\frac{2\cosh(x)\sinh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(-1+\cosh(x))^{3/2}\sqrt{1+\cosh(x)}}\right)}{\sqrt{\cosh(x)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

```
[Out] (2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2)
*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Maple [**F**]
time = 1.20, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh(x)^{\frac{3}{2}}}+x\left(\sqrt{\cosh}\,(x)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\left(\cosh^{2}\left(x\right)+1\right)}{\cosh^{\frac{3}{2}}\left(x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

```
[Out] Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Mupad [**B**]
time = 0.97, size = 39, normalized size = 1.95

$$-\frac{2\,\sqrt{\frac{{\mathrm{e}}^{-x}}{2}+\frac{{\mathrm{e}}^{x}}{2}}\,\left(x+2\,{\mathrm{e}}^{2\,x}-x\,{\mathrm{e}}^{2\,x}+2\right)}{{\mathrm{e}}^{2\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
```

```
[Out] -(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2*
x) + 1)
```

3.72 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx$

Optimal. Leaf size=24

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

```
[Out] 2/3*x*sinh(x)/cosh(x)^(3/2)+4/3/cosh(x)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {3396}

$$\frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] 4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}} \right) dx &= -\left(\frac{1}{3} \int \frac{x}{\sqrt{\cosh(x)}}\, dx \right) + \int \frac{x}{\cosh^{\frac{5}{2}}(x)}\, dx \\
&= \frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 16, normalized size = 0.67

$$\frac{2(2 + x\tanh(x))}{3\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

```
[Out] (2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Maple [F]
time = 1.26, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh \left(x\right)^{\frac{5}{2}}}-\frac{x}{3 \sqrt{\cosh \left(x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(16) = 32$.
time = 0.38, size = 109, normalized size = 4.54

$$\frac{4 \left(\left(x+2\right) \cosh \left(x\right)^{3}+3 \left(x+2\right) \cosh \left(x\right) \sinh \left(x\right)^{2}+\left(x+2\right) \sinh \left(x\right)^{3}-\left(x-2\right) \cosh \left(x\right)+\left(3 \left(x+2\right) \cosh \left(x\right)^{2}-x+2\right) \sinh \left(x\right)\right) \sqrt{\cosh \left(x\right)}}{3 \left(\cosh \left(x\right)^{4}+4 \cosh \left(x\right) \sinh \left(x\right)^{3}+\sinh \left(x\right)^{4}+2 \left(3 \cosh \left(x\right)^{2}+1\right) \sinh \left(x\right)^{2}+2 \cosh \left(x\right)^{2}+4 \left(\cosh \left(x\right)^{3}+\cosh \left(x\right)\right) \sinh \left(x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] 4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
(x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(cos
h(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 +
2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(-\frac{3x}{\cosh^{\frac{5}{2}}\left(x\right)}\right)\,dx+\int \frac{x}{\sqrt{\cosh \left(x\right)}}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Mupad [B]
time = 0.94, size = 42, normalized size = 1.75

$$\frac{4\,\mathrm{e}^{x}\,\sqrt{\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}}\,\left(2\,\mathrm{e}^{2\,x}-x+x\,\mathrm{e}^{2\,x}+2\right)}{3\,{\left(\mathrm{e}^{2\,x}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

```
[Out] (4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/(
3*(exp(2*x) + 1)^2)
```

3.73 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx$

Optimal. Leaf size=47

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

```
[Out] 4/15/cosh(x)^(3/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*sinh(x)/cosh(x)^(1/2)-
12/5*cosh(x)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {3396}

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] 4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/2
)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Rule 3396

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[(c + d*x)*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1))), x] +
 (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)*(b*Sin[e + f*x])^(n + 2), x], x
] - Simp[d*((b*Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; Fre
eQ[{b, c, d, e, f}, x] && LtQ[n, -1] && NeQ[n, -2]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx &= \frac{3}{5}\int x\sqrt{\cosh(x)}\,dx+\int \frac{x}{\cosh^{\frac{7}{2}}(x)}\,dx\\
&= \frac{4}{15\cosh^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int \frac{x}{\cosh^{\frac{3}{2}}(x)}\,dx+\frac{3}{5}\int x\sqrt{\cosh(x)}\,d\\
&= \frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.45, size = 64, normalized size = 1.36

$$\frac{1}{5}\sqrt{\cosh(x)}\left(-\frac{12\sinh^2(x)}{\sqrt{-1+\cosh(x)}\,(1+\cosh(x))^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}}+6x\tanh(x)+\operatorname{sech}^2(x)\left(\frac{4}{3}+2x\tanh(x)\right)\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

```
[Out] (Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sqr
t[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Maple [F]
time = 1.61, size = 0, normalized size = 0.00

$$\int \frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}+\frac{3x\left(\sqrt{\cosh}\left(x\right)\right)}{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

```
[Out] int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3880 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
[Out] integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Mupad [**B**]
time = 1.08, size = 110, normalized size = 2.34

$$\frac{e^{2x}\left(\frac{8x}{5}+\frac{16}{15}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{\left(e^{2x}+1\right)^2}-\left(\frac{6x}{5}+\frac{12}{5}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}+\frac{12\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\,\left(e^{2x}+1\right)}-\frac{16\,x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
[Out] (exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
- ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)/2
 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp(x)
/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

3.74 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$

Optimal. Leaf size=36

$$-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

```
[Out] -16*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticE(I*sinh(1/2*x),2^(1/2))+2*
x^2*sinh(x)/cosh(x)^(1/2)-8*x*cosh(x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {3397, 2719}

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[Co
sh[x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3397

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n + 1)/(b*f*(n + 1)
)), x] + (Dist[(n + 2)/(b^2*(n + 1)), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n +
 2), x], x] + Dist[d^2*m*((m - 1)/(b^2*f^2*(n + 1)*(n + 2))), Int[(c + d*x)
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] - Simp[d*m*(c + d*x)^(m - 1)*((b*
Sin[e + f*x])^(n + 2)/(b^2*f^2*(n + 1)*(n + 2))), x]) /; FreeQ[{b, c, d, e,
 f}, x] && LtQ[n, -1] && NeQ[n, -2] && GtQ[m, 1]
```

Rubi steps

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx = \int \frac{x^2}{\cosh^{\frac{3}{2}}(x)}\,dx+\int x^2\sqrt{\cosh(x)}\,dx$$
$$= -8x\sqrt{\cosh(x)}+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}+8\int \sqrt{\cosh(x)}\,dx$$
$$= -8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 0.71, size = 76, normalized size = 2.11

$$\frac{4\sqrt{\cosh(x)}\,(\cosh(x)+\sinh(x))\left(-4(-2+x)\cosh(x)+x^2\sinh(x)+8\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2x}\right)(-\cosh(x)+\sinh(x))\sqrt{1+\cosh(2x)+\sinh(2x)}\right)}{1+e^{2x}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

```
[Out] (4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] + 8
*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1 +
Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Maple [F]
time = 1.19, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\cosh(x)^{\frac{3}{2}}}+x^2\left(\sqrt{\cosh}\,(x)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

```
[Out] int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 (\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

```
[Out] Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.03

$$\int x^2 \sqrt{\cosh(x)} + \frac{x^2}{\cosh(x)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
```

```
[Out] int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

3.75 $\int (c+dx)^m (b\cosh(e+fx))^n \, dx$

Optimal. Leaf size=21

$$\text{Int}((c+dx)^m (b\cosh(e+fx))^n, x)$$

```
[Out] Unintegrable((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx = \int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Mathematica [A]
time = 1.99, size = 0, normalized size = 0.00

$$\int (c+dx)^m (b\cosh(e+fx))^n \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(b\cosh\left(fx+e\right)\right)^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

```
[Out] int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\cosh(e+fx))^n (c+dx)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(b*cosh(f*x+e))**n,x)
[Out] Integral((b*cosh(e + f*x))**n*(c + d*x)**m, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int (b\cosh(e+f\,x))^n\,(c+d\,x)^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*cosh(e + f*x))^n*(c + d*x)^m,x)
[Out] int((b*cosh(e + f*x))^n*(c + d*x)^m, x)
```

3.76 $\int (c+dx)^m \cosh^3(a+bx)\,dx$

Optimal. Leaf size=237

$$\frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c-}{}\right.}{8b}$$

```
[Out] 1/8*3^(-1-m)*exp(3*a-3*b*c/d)*(d*x+c)^m*GAMMA(1+m,-3*b*(d*x+c)/d)/b/((-b*(d
*x+c)/d)^m)+3/8*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+
c)/d)^m)-3/8*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d
)^m)-1/8*3^(-1-m)*exp(-3*a+3*b*c/d)*(d*x+c)^m*GAMMA(1+m,3*b*(d*x+c)/d)/b/((
b*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.21, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{8b}-\frac{3e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{\frac{3bc}{d}-3a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3b(c+dx)}{d}\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/d
])/(8*b*(-((b*(c + d*x))/d))^m) + (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 +
m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/d)*
(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) - (3^(
-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/d])/(
8*b*((b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m \cosh^3(a+bx)\,dx &= \int \left(\frac{3}{4}(c+dx)^m\cosh(a+bx)+\frac{1}{4}(c+dx)^m\cosh(3a+3bx)\right)\,dx\\ &= \frac{1}{4}\int (c+dx)^m\cosh(3a+3bx)\,dx+\frac{3}{4}\int (c+dx)^m\cosh(a+bx)\,dx\\ &= \frac{1}{8}\int e^{-i(3ia+3ibx)}(c+dx)^m\,dx+\frac{1}{8}\int e^{i(3ia+3ibx)}(c+dx)^m\,dx+\frac{3}{8}\int e^{-i(ia+ibx}\\ &= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx}{}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 205, normalized size = 0.86

$$\frac{3^{-1-m}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)+3^{2+m}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^m\left(3^{2+m}e^{2a}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\Gamma\left(1+m,\frac{3b(c+dx)}{d}\right)\right)\right)}{8b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

```
[Out] (3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d*
x))/d] + 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*(c
 + d*x))/d)] - E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(3^(2 + m)*E^(2*a)*Gamm
a[1 + m, (b*(c + d*x))/d] + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/d]))
)/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\left(\cosh^3(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a)^3,x)
```

```
[Out] int((d*x+c)^m*cosh(b*x+a)^3,x)
```

Maxima [A]
time = 0.09, size = 161, normalized size = 0.68

$$-\frac{(dx+c)^{m+1}e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_{-m}\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d}-\frac{3\,(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{3\,(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8\,d}-\frac{(dx+c)^{m+1}e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_{-m}\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/
d)/d - 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/
d)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d*x +
 c)*b/d)/d
```

Fricas [A]
time = 0.12, size = 340, normalized size = 1.43

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)+9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\cosh\left(\frac{dm\log\left(-\frac{3b}{d}\right)+3bc-3ad}{d}\right)\Gamma\left(m+1,-\frac{3(bdx+bc)}{d}\right)-\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)-9\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)+9\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)+\Gamma\left(m+1,-\frac{3(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{3b}{d}\right)+3bc-3ad}{d}\right)}{24b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="fricas")
```

```
[Out] -1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*c
)/d) + 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) -
 9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - cos
h((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d) - g
amma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d) - 9
*gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9*gamma
(m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) + gamma(m + 1
, -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cosh^3(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a)**3,x)
```

```
[Out] Integral((c + d*x)**m*cosh(a + b*x)**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cosh(b*x + a)^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cosh(a+b\,x)^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^3*(c + d*x)^m,x)
```

```
[Out] int(cosh(a + b*x)^3*(c + d*x)^m, x)
```

3.77 $\int (c+dx)^m \cosh^2(a+bx)\,dx$

Optimal. Leaf size=144

$$\frac{(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c+dx)^m\,(\frac{b(c+dx}{d}}{b}$$

```
[Out] 1/2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*exp(2*a-2*b*c/d)*(d*x+c)^m*GAMMA(1+m,-2*
b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^(-3-m)*exp(-2*a+2*b*c/d)*(d*x+c)^m*GAMM
A(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.14, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3393, 3388, 2212}

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2b(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

```
[Out] (c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x)
^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3 -
m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*((b
*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
```

```
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \int \left(\frac{1}{2}(c+dx)^m + \frac{1}{2}(c+dx)^m \cosh(2a+2bx)\right) dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{2}\int (c+dx)^m \cosh(2a+2bx)\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{1}{4}\int e^{-i(2ia+2ibx)}(c+dx)^m\,dx + \frac{1}{4}\int e^{i(2ia+2ibx)}(c+dx)^m\,dx$$
$$= \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} -$$

Mathematica [A]
time = 0.15, size = 132, normalized size = 0.92

$$\frac{1}{8}(c+dx)^m\left(\frac{4c+4dx}{d+dm}+\frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m}e^{-2a+\frac{2bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x]^2,x]
[Out] ((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m, (
-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/d)*
Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(\cosh^2(bx+a)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a)^2,x)
[Out] int((d*x+c)^m*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.06, size = 102, normalized size = 0.71

$$-\frac{(dx+c)^{m+1}e^{\left(-2a+\frac{2bc}{d}\right)}E_{-m}\left(\frac{2(dx+c)b}{d}\right)}{4d}-\frac{(dx+c)^{m+1}e^{\left(2a-\frac{2bc}{d}\right)}E_{-m}\left(-\frac{2(dx+c)b}{d}\right)}{4d}+\frac{(dx+c)^{m+1}}{2d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(d*x + c)^(m + 1)*e^(-2*a + 2*b*c/d)*exp_integral_e(-m, 2*(d*x + c)*b/
d)/d - 1/4*(d*x + c)^(m + 1)*e^(2*a - 2*b*c/d)*exp_integral_e(-m, -2*(d*x +
 c)*b/d)/d + 1/2*(d*x + c)^(m + 1)/(d*(m + 1))
```

Fricas [A]
time = 0.11, size = 241, normalized size = 1.67

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)-(dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)+(dm+d)\Gamma\left(m+1,-\frac{2(bdx+bc)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)-4(bdx+bc)\cosh\left(m\log\left(dx+c\right)\right)-4(bdx+bc)\sinh\left(m\log\left(dx+c\right)\right)}{8(bdm+bd)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] -1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b*
d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma(m
 + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*sinh(
(d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x + b*
c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) - 4*(b*d*x + b*c)*cosh(m*lo
g(d*x + c)) - 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \cosh^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a)**2,x)
```

```
[Out] Integral((c + d*x)**m*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)^2*(c + d*x)^m,x)
[Out] int(cosh(a + b*x)^2*(c + d*x)^m, x)
```

3.78 $\int (c+dx)^m \cosh(a+bx)\, dx$

Optimal. Leaf size=110

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{2b}$$

```
[Out] 1/2*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-1/2
*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.06, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3388, 2212}

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*Cosh[a + b*x],x]
```

```
[Out] (E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*(
c + d*x))/d))^m) - (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))
/d])/(2*b*((b*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\int (c+dx)^m \cosh(a+bx)\,dx=\frac{1}{2}\int e^{-i(ia+ibx)}(c+dx)^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}(c+dx)^m\,dx$$

$$=\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+}{d}\right.}{}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 0.93

$$\frac{e^{-a-\frac{bc}{d}}(c+dx)^m\left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)-e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*Cosh[a + b*x],x]
```

```
[Out] (E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/(
-((b*(c + d*x))/d))^m - (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*(c
/d + x))^m))/(2*b)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \cosh(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*cosh(b*x+a),x)
```

```
[Out] int((d*x+c)^m*cosh(b*x+a),x)
```

Maxima [A]
time = 0.06, size = 79, normalized size = 0.72

$$-\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d}-\frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d -
 1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Fricas [A]
time = 0.10, size = 168, normalized size = 1.53

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)+\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) - co
sh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(m
+ 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + gamma(m + 1, -(b
*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*cosh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*cosh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cosh(a+b\,x)\,\,{\left(c+d\,x\right)}^m \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(a + b*x)*(c + d*x)^m,x)
[Out] int(cosh(a + b*x)*(c + d*x)^m, x)
```

3.79 $\int (c+dx)^m \mathbf{sech}(a+bx)\,dx$

Optimal. Leaf size=17

$$\text{Int}((c+dx)^m\text{sech}(a+bx),x)$$

```
[Out] Unintegrable((d*x+c)^m*sech(b*x+a),x)
```

Rubi [A]
time = 0.02, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m\text{sech}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sech[a + b*x],x]
[Out] Defer[Int][(c + d*x)^m*Sech[a + b*x], x]
```

Rubi steps

$$\int (c+dx)^m\text{sech}(a+bx)\,dx=\int (c+dx)^m\text{sech}(a+bx)\,dx$$

Mathematica [A]
time = 4.13, size = 0, normalized size = 0.00

$$\int (c+dx)^m\text{sech}(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sech[a + b*x],x]
[Out] Integrate[(c + d*x)^m*Sech[a + b*x], x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,\text{sech}(bx+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sech(b*x+a),x)
```

```
[Out] int((d*x+c)^m*sech(b*x+a),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sech(b*x + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="fricas")
[Out] integral((d*x + c)^m*sech(b*x + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c + dx)^m \operatorname{sech}(a + bx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sech(b*x+a),x)
[Out] Integral((c + d*x)**m*sech(a + b*x), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a),x, algorithm="giac")
[Out] integrate((d*x + c)^m*sech(b*x + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{(c + d\,x)^m}{\cosh(a + b\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/cosh(a + b*x),x)
[Out] int((c + d*x)^m/cosh(a + b*x), x)
```

3.80 $\int (c+dx)^m \mathbf{sech}^2(a+bx)\,dx$

Optimal. Leaf size=19

$$\operatorname{Int}\left((c+dx)^m \operatorname{sech}^2(a+bx),x\right)$$

```
[Out] Unintegrable((d*x+c)^m*sech(b*x+a)^2,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*Sech[a + b*x]^2,x]
[Out] Defer[Int][(c + d*x)^m*Sech[a + b*x]^2, x]
```

Rubi steps

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

Mathematica [A]
time = 2.43, size = 0, normalized size = 0.00

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*Sech[a + b*x]^2,x]
[Out] Integrate[(c + d*x)^m*Sech[a + b*x]^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \operatorname{sech}(bx+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*sech(b*x+a)^2,x)
```

```
[Out] int((d*x+c)^m*sech(b*x+a)^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m*sech(b*x + a)^2, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*sech(b*x+a)**2,x)
[Out] Integral((c + d*x)**m*sech(a + b*x)**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(c+d\,x)^m}{\cosh\left(a+b\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/cosh(a + b*x)^2,x)
[Out] int((c + d*x)^m/cosh(a + b*x)^2, x)
```

3.81 $\int x^{3+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(4+m,-b*x)/b^4/((-b*x)^m)-1/2*x^m*GAMMA(4+m,b*x)/b^4/e
xp(a)/((b*x)^m)
```

Rubi [A]
time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3388, 2212}

$$-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+4,-bx)}{2b^4}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+4,bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[4 + m, -(b*x)])/(b^4*(-(b*x))^m) - (x^m*Gamma[4 + m, b*
x])/(2*b^4*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\cosh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{3+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{3+m}\,dx\\&=-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 54, normalized size = 0.92

$$-\frac{e^a x^m (-bx)^{-m} \Gamma(4+m,-bx) + e^{-a} x^m (bx)^{-m} \Gamma(4+m,bx)}{2b^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*((E^a*x^m*Gamma[4 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[4 + m, b*x])/(
E^a*(b*x)^m))/b^4
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.32, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{4+m} \text{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+3\right],\frac{b^2 x^2}{4}\right) \cosh(a)}{4+m}+\frac{b\, x^{m+5} \text{hypergeom}\left(\left[\frac{m}{2}+\frac{5}{2}\right],\left[\frac{3}{2},\frac{7}{2}+\frac{m}{2}\right],\frac{b^2 x^2}{4}\right) \sinh(a)}{m+5}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+3)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,1/2*m+3],1/4*b^2*x^2)*cosh(a)+b/(m
+5)*x^(m+5)*hypergeom([1/2*m+5/2],[3/2,7/2+1/2*m],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.07, size = 55, normalized size = 0.93

$$-\frac{1}{2} (bx)^{-m-4} x^{m+4} e^{(-a)} \Gamma(m+4,bx) - \frac{1}{2} (-bx)^{-m-4} x^{m+4} e^a \Gamma(m+4,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m - 4
)*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Fricas [A]
time = 0.10, size = 86, normalized size = 1.46

$$-\frac{\cosh\left((m+3)\log(b)+a\right)\Gamma(m+4,bx) - \cosh\left((m+3)\log(-b)-a\right)\Gamma(m+4,-bx) + \Gamma(m+4,-bx)\sinh\left((m+3)\log(-b)-a\right) - \Gamma(m+4,bx)\sinh\left((m+3)\log(b)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) - cosh((m + 3)*log(-b) - a
)*gamma(m + 4, -b*x) + gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gamma
(m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cosh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m + 3)*cosh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+3}\cosh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*cosh(a + b*x),x)
[Out] int(x^(m + 3)*cosh(a + b*x), x)
```

3.82 $\int x^{2+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(3+m,-b*x)/b^3/((-b*x)^m)-1/2*x^m*GAMMA(3+m,b*x)/b^3/ex
p(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3388, 2212}

$$\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+3,-bx)}{2b^3}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+3,bx)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cosh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) - (x^m*Gamma[3 + m, b*x])
/(2*b^3*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{2+m}\cosh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{2+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{2+m}\,dx\\&=\frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(3+m,-bx)-(bx)^{-m}\Gamma(3+m,bx)\right)}{2b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cosh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m - Gamma[3 + m, b*x]/(b*x)^m
))/(2*b^3*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.36, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{m+3}\,\text{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{m+3}+\frac{b\,x^{4+m}\,\text{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+3\right],\frac{b^2x^2}{4}\right)\sinh(a)}{4+m}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(m+3)*x^(m+3)*hypergeom([3/2+1/2*m],[1/2,1/2*m+5/2],1/4*b^2*x^2)*cosh(a)+
b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,1/2*m+3],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.07, size = 55, normalized size = 0.93

$$-\frac{1}{2}\left(bx\right)^{-m-3}x^{m+3}e^{(-a)}\Gamma(m+3,bx)-\frac{1}{2}\left(-bx\right)^{-m-3}x^{m+3}e^{a}\Gamma(m+3,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m - 3
)*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Fricas [A]
time = 0.09, size = 86, normalized size = 1.46

$$-\frac{\cosh\left((m+2)\log\left(b\right)+a\right)\Gamma(m+3,bx)-\cosh\left((m+2)\log\left(-b\right)-a\right)\Gamma(m+3,-bx)+\Gamma(m+3,-bx)\sinh\left((m+2)\log\left(-b\right)-a\right)-\Gamma(m+3,bx)\sinh\left((m+2)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) - cosh((m + 2)*log(-b) - a
)*gamma(m + 3, -b*x) + gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gamma
(m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cosh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+2}\cosh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*cosh(a + b*x),x)
```

```
[Out] int(x^(m + 2)*cosh(a + b*x), x)
```

3.83 $\int x^{1+m} \cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(2+m,-b*x)/b^2/((-b*x)^m)-1/2*x^m*GAMMA(2+m,b*x)/b^2/e
xp(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3388, 2212}

$$-\frac{e^a x^m(-bx)^{-m}\text{Gamma}(m+2,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\text{Gamma}(m+2,bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[2 + m, -(b*x)])/(b^2*(-(b*x))^m) - (x^m*Gamma[2 + m, b*
x])/(2*b^2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{1+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{1+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{1+m}\,dx\\ &= -\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 54, normalized size = 0.92

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)+e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*((E^a*x^m*Gamma[2 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[2 + m, b*x])/(
E^a*(b*x)^m))/b^2
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.35, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{1}{2},2+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{2+m}+\frac{b\,x^{m+3}\,\mathrm{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{m}{2}+\frac{5}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{m+3}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*b^2*x^2)*cosh(a)+b/(m
+3)*x^(m+3)*hypergeom([3/2+1/2*m],[3/2,1/2*m+5/2],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.08, size = 55, normalized size = 0.93

$$-\frac{1}{2}\,(bx)^{-m-2}\,x^{m+2}e^{(-a)}\Gamma(m+2,bx)-\frac{1}{2}\,(-bx)^{-m-2}\,x^{m+2}e^{a}\Gamma(m+2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m - 2
)*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Fricas [A]
time = 0.09, size = 86, normalized size = 1.46

$$-\frac{\cosh\left((m+1)\log\left(b\right)+a\right)\Gamma(m+2,bx)-\cosh\left((m+1)\log\left(-b\right)-a\right)\Gamma(m+2,-bx)+\Gamma(m+2,-bx)\sinh\left((m+1)\log\left(-b\right)-a\right)-\Gamma(m+2,bx)\sinh\left((m+1)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) - cosh((m + 1)*log(-b) - a
)*gamma(m + 2, -b*x) + gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gamma
(m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cosh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m+1}\cosh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*cosh(a + b*x),x)
```

```
[Out] int(x^(m + 1)*cosh(a + b*x), x)
```

3.84 $\int x^m \cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$\frac{e^a x^m (-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\Gamma(1+m,bx)}{2b}$$

```
[Out] 1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a)
/((b*x)^m)
```

Rubi [A]
time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3388, 2212}

$$\frac{e^a x^m (-bx)^{-m}\text{Gamma}(m+1,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\text{Gamma}(m+1,bx)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x],x]
```

```
[Out] (E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/(
2*b*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^m \cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^m\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^m\,dx\\ &= \frac{e^a x^m (-bx)^{-m}\Gamma(1+m,-bx)}{2b}-\frac{e^{-a} x^m (bx)^{-m}\Gamma(1+m,bx)}{2b}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.92

$$\frac{e^{-a}x^m(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx)-(bx)^{-m}\Gamma(1+m,bx))}{2b}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x],x]
```

```
[Out] (x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m - Gamma[1 + m, b*x]/(b*x)^m
))/(2*b*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.34, size = 73, normalized size = 1.24

method	result	size
meijerg	$\frac{x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{1+m}+\frac{b\,x^{2+m}\,\mathrm{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{3}{2},2+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{2+m}$	73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*b^2*x^2)*cosh(a)+
b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 0.93

$$-\frac{1}{2}\,(bx)^{-m-1}\,x^{m+1}e^{(-a)}\Gamma(m+1,bx)-\frac{1}{2}\,(-bx)^{-m-1}\,x^{m+1}e^{a}\Gamma(m+1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m - 1
)*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Fricas [A]
time = 0.09, size = 78, normalized size = 1.32

$$-\frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma\left(m+1,bx\right)-\cosh\left(m\log\left(-b\right)-a\right)\Gamma\left(m+1,-bx\right)+\Gamma\left(m+1,-bx\right)\sinh\left(m\log\left(-b\right)-a\right)-\Gamma\left(m+1,bx\right)\sinh\left(m\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) - cosh(m*log(-b) - a)*gamma(m +
1, -b*x) + gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sinh(
m*log(b) + a))/b
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(x^m*cosh(b*x + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^m\cosh(a+b\,x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x),x)
[Out] int(x^m*cosh(a + b*x), x)
```

3.85 $\int x^{-1+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=49

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

```
[Out] -1/2*exp(a)*x^m*GAMMA(m,-b*x)/((-b*x)^m)-1/2*x^m*GAMMA(m,b*x)/exp(a)/((b*x)
^m)
```

Rubi [A]
time = 0.04, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3388, 2212}

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\text{Gamma}(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\text{Gamma}(m,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m - (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-1+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-1+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-1+m}\,dx\\ &= -\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 49, normalized size = 1.00

$$-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m - (x^m*Gamma[m, b*x])/(2*E^a*(b*
x)^m)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.39, size = 67, normalized size = 1.37

method	result	size
meijerg	$\frac{x^m \,\mathrm{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{1}{2},1+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{m}+\frac{b\,x^{1+m}\,\mathrm{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{1+m}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*b^2*x^2)*cosh(a)+b/(1+m)*x^(1+m
)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.08, size = 43, normalized size = 0.88

$$-\frac{x^m e^{(-a)}\Gamma(m,bx)}{2\,(bx)^m}-\frac{x^m e^a\Gamma(m,-bx)}{2\,(-bx)^m}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^m
```

Fricas [A]
time = 0.11, size = 78, normalized size = 1.59

$$-\frac{\cosh\left((m-1)\log\left(b\right)+a\right)\Gamma(m,bx)-\cosh\left((m-1)\log\left(-b\right)-a\right)\Gamma(m,-bx)+\Gamma(m,-bx)\sinh\left((m-1)\log\left(-b\right)-a\right)-\Gamma(m,bx)\sinh\left((m-1)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) - cosh((m - 1)*log(-b) - a)*ga
mma(m, -b*x) + gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*sin
h((m - 1)*log(b) + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cosh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 1)*cosh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-1}\cosh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*cosh(a + b*x),x)
[Out] int(x^(m - 1)*cosh(a + b*x), x)
```

3.86 $\int x^{-2+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=55

$$\frac{1}{2}be^ax^m(-bx)^{-m}\Gamma(-1+m,-bx)-\frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)$$

```
[Out] 1/2*b*exp(a)*x^m*GAMMA(-1+m,-b*x)/((-b*x)^m)-1/2*b*x^m*GAMMA(-1+m,b*x)/exp(
a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}=0.167$, Rules used = {3388, 2212}

$$\frac{1}{2}e^abx^m(-bx)^{-m}\text{Gamma}(m-1,-bx)-\frac{1}{2}e^{-a}bx^m(bx)^{-m}\text{Gamma}(m-1,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cosh[a + b*x],x]
```

```
[Out] (b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) - (b*x^m*Gamma[-1 + m, b*x
])/(2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-2+m}\cosh(a+bx)\,dx&=\frac{1}{2}\int e^{-i(ia+ibx)}x^{-2+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-2+m}\,dx\\&=\frac{1}{2}be^ax^m(-bx)^{-m}\Gamma(-1+m,-bx)-\frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 52, normalized size = 0.95

$$\frac{1}{2}be^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(-1+m,-bx)-(bx)^{-m}\Gamma(-1+m,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cosh[a + b*x],x]
```

```
[Out] (b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m - Gamma[-1 + m, b*x]/(b*
x)^m))/(2*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.41, size = 67, normalized size = 1.22

method	result	size
meijerg	$\frac{x^{-1+m}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{-1+m}+\frac{b\,x^m\,\mathrm{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{3}{2},1+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{m}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*b^2*x^2)*cosh(
a)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 1.00

$$-\frac{1}{2}(bx)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx)-\frac{1}{2}(-bx)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m + 1
)*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

Fricas [A]
time = 0.10, size = 86, normalized size = 1.56

$$-\frac{\cosh\left((m-2)\log\left(b\right)+a\right)\Gamma(m-1,bx)-\cosh\left((m-2)\log\left(-b\right)-a\right)\Gamma(m-1,-bx)+\Gamma(m-1,-bx)\sinh\left((m-2)\log\left(-b\right)-a\right)-\Gamma(m-1,bx)\sinh\left((m-2)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) - cosh((m - 2)*log(-b) - a
)*gamma(m - 1, -b*x) + gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gamma
(m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cosh(b*x+a),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cosh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-2} \cosh(a + b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*cosh(a + b*x),x)
```

```
[Out] int(x^(m - 2)*cosh(a + b*x), x)
```

3.87 $\int x^{-3+m}\cosh(a+bx)\,dx$

Optimal. Leaf size=59

$$-\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx)-\frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)$$

```
[Out] -1/2*b^2*exp(a)*x^m*GAMMA(-2+m,-b*x)/((-b*x)^m)-1/2*b^2*x^m*GAMMA(-2+m,b*x)
/exp(a)/((b*x)^m)
```

Rubi [A]
time = 0.05, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3388, 2212}

$$-\frac{1}{2}e^ab^2x^m(-bx)^{-m}\text{Gamma}(m-2,-bx)-\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\text{Gamma}(m-2,bx)$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cosh[a + b*x],x]
```

```
[Out] -1/2*(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(-(b*x))^m - (b^2*x^m*Gamma[-2 + m
, b*x])/(2*E^a*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\cosh(a+bx)\,dx &= \frac{1}{2}\int e^{-i(ia+ibx)}x^{-3+m}\,dx+\frac{1}{2}\int e^{i(ia+ibx)}x^{-3+m}\,dx\\ &= -\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx)-\frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 55, normalized size = 0.93

$$\frac{1}{2}b^2e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(-2+m,-bx)-(bx)^{-m}\Gamma(-2+m,bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cosh[a + b*x],x]
```

```
[Out] (b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) - Gamma[-2 + m, b*x
]/(b*x)^m))/(2*E^a)
```

Maple [C] Result contains higher order function than in optimal. Order 5 vs. order 4.
time = 0.33, size = 71, normalized size = 1.20

method	result	size
meijerg	$\frac{x^{-2+m}\,\mathrm{hypergeom}\left(\left[-1+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\cosh(a)}{-2+m}+\frac{b\,x^{-1+m}\,\mathrm{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{b^2x^2}{4}\right)\sinh(a)}{-1+m}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*b^2*x^2)*cosh(a)+b/(
-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*b^2*x^2)*sinh(a)
```

Maxima [A]
time = 0.09, size = 55, normalized size = 0.93

$$-\frac{1}{2}\left(bx\right)^{-m+2}x^{m-2}e^{(-a)}\Gamma(m-2,bx)-\frac{1}{2}\left(-bx\right)^{-m+2}x^{m-2}e^{a}\Gamma(m-2,-bx)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="maxima")
```

```
[Out] -1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m + 2
)*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Fricas [A]
time = 0.11, size = 86, normalized size = 1.46

$$-\frac{\cosh\left((m-3)\log\left(b\right)+a\right)\Gamma(m-2,bx)-\cosh\left((m-3)\log\left(-b\right)-a\right)\Gamma(m-2,-bx)+\Gamma(m-2,-bx)\sinh\left((m-3)\log\left(-b\right)-a\right)-\Gamma(m-2,bx)\sinh\left((m-3)\log\left(b\right)+a\right)}{2\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) - cosh((m - 3)*log(-b) - a
)*gamma(m - 2, -b*x) + gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gamma
(m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cosh(b*x+a),x)
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="giac")
[Out] integrate(x^(m - 3)*cosh(b*x + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int x^{m-3}\cosh(a+b\,x)\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*cosh(a + b*x),x)
[Out] int(x^(m - 3)*cosh(a + b*x), x)
```

3.88 $\int x^{3+m} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=86

$$\frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}$$

```
[Out] 1/2*x^(4+m)/(4+m)-2^(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-2^
(-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.11, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\text{Gamma}(m+4,-2bx)}{b^4} - \frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\text{Gamma}(m+4,2bx)}{b^4} + \frac{x^{m+4}}{2(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4*
(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{3+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{3+m}}{2}+\frac{1}{2}x^{3+m}\cosh(2a+2bx)\right)dx\\ &= \frac{x^{4+m}}{2(4+m)}+\frac{1}{2}\int x^{3+m}\cosh(2a+2bx)\,dx\\ &= \frac{x^{4+m}}{2(4+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{3+m}\,dx\\ &= \frac{x^{4+m}}{2(4+m)}-\frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+}{b^4}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 79, normalized size = 0.92

$$\frac{1}{64}x^m\left(\frac{32x^4}{4+m}-\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(4+m,2bx)}{b^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))^m
) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int x^{m+3}\left(\cosh^2(bx+a)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m+3)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(m+3)*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.07, size = 71, normalized size = 0.83

$$-\frac{1}{4}(2\,bx)^{-m-4}\,x^{m+4}e^{(-2\,a)}\Gamma(m+4,2\,bx)-\frac{1}{4}(-2\,bx)^{-m-4}\,x^{m+4}e^{(2\,a)}\Gamma(m+4,-2\,bx)+\frac{x^{m+4}}{2\,(m+4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 4)*x^(m + 4)*e^(-2*a)*gamma(m + 4, 2*b*x) - 1/4*(-2*b*x)
^(-m - 4)*x^(m + 4)*e^(2*a)*gamma(m + 4, -2*b*x) + 1/2*x^(m + 4)/(m + 4)
```

Fricas [A]
time = 0.10, size = 136, normalized size = 1.58

$$\frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)-(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma(m+4,2\,bx)+(m+4)\cosh\left((m+3)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+4,-2\,bx)+(m+4)\Gamma(m+4,2\,bx)\sinh\left((m+3)\log\left(2\,b\right)+2\,a\right)-(m+4)\Gamma(m+4,-2\,bx)\sinh\left((m+3)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+3)\log\left(x\right)\right)}{8\,(bm+4\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 3)*log(x)) - (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*gamm
a(m + 4, 2*b*x) + (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*b*x
) + (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) - (m + 4)*gamm
a(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*log(x))
)/(b*m + 4*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+3}\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(3+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 3)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 3)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+3}\,{\cosh\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 3)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 3)*cosh(a + b*x)^2, x)
```

3.89 $\int x^{2+m} \cosh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}$$

```
[Out] 1/2*x^(3+m)/(3+m)+2^(-5-m)*exp(2*a)*x^m*GAMMA(3+m,-2*b*x)/b^3/((-b*x)^m)-2^
(-5-m)*x^m*GAMMA(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\text{Gamma}(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\text{Gamma}(m+3,2bx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3*
(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{2+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{2+m}}{2}+\frac{1}{2}x^{2+m}\cosh(2a+2bx)\right)\,dx \\
&= \frac{x^{3+m}}{2(3+m)}+\frac{1}{2}\int x^{2+m}\cosh(2a+2bx)\,dx \\
&= \frac{x^{3+m}}{2(3+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{2+m}\,dx \\
&= \frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+}{b^3}
\end{aligned}
$$

Mathematica [**A**]
time = 0.07, size = 78, normalized size = 0.92

$$
\frac{1}{32}x^m\left(\frac{16x^3}{3+m}+\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(3+m,2bx)}{b^3}\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))^m
) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Maple [**F**]
time = 0.51, size = 0, normalized size = 0.00

$$
\int x^{2+m}\left(\cosh^2(bx+a)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(2+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(2+m)*cosh(b*x+a)^2,x)
```

Maxima [**A**]
time = 0.08, size = 71, normalized size = 0.84

$$
-\frac{1}{4}(2\,bx)^{-m-3}x^{m+3}e^{(-2\,a)}\Gamma(m+3,2\,bx)-\frac{1}{4}(-2\,bx)^{-m-3}x^{m+3}e^{(2\,a)}\Gamma(m+3,-2\,bx)+\frac{x^{m+3}}{2\,(m+3)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 3)*x^(m + 3)*e^(-2*a)*gamma(m + 3, 2*b*x) - 1/4*(-2*b*x)
^(-m - 3)*x^(m + 3)*e^(2*a)*gamma(m + 3, -2*b*x) + 1/2*x^(m + 3)/(m + 3)
```

Fricas [A]
time = 0.08, size = 136, normalized size = 1.60

$$\frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right)-(m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma(m+3,2\,bx)+(m+3)\cosh\left((m+2)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+3,-2\,bx)+(m+3)\Gamma(m+3,2\,bx)\sinh\left((m+2)\log\left(2\,b\right)+2\,a\right)-(m+3)\Gamma(m+3,-2\,bx)\sinh\left((m+2)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+2)\log\left(x\right)\right)}{8\,(bm+3\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 2)*log(x)) - (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*gamm
a(m + 3, 2*b*x) + (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*b*x
) + (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) - (m + 3)*gamm
a(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*log(x))
)/(b*m + 3*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+2}\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(2+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 2)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 2)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+2}\,{\cosh\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 2)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 2)*cosh(a + b*x)^2, x)
```

3.90 $\int x^{1+m} \cosh^2(a+bx)\, dx$

Optimal. Leaf size=86

$$\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}$$

```
[Out] 1/2*x^(2+m)/(2+m)-2^(-4-m)*exp(2*a)*x^m*GAMMA(2+m,-2*b*x)/b^2/((-b*x)^m)-2^
(-4-m)*x^m*GAMMA(2+m,2*b*x)/b^2/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3393, 3388, 2212}

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\text{Gamma}(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\text{Gamma}(m+2,2bx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2*
(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{1+m}\cosh^2(a+bx)\,dx = \int\left(\frac{x^{1+m}}{2}+\frac{1}{2}x^{1+m}\cosh(2a+2bx)\right)dx$$
$$= \frac{x^{2+m}}{2(2+m)}+\frac{1}{2}\int x^{1+m}\cosh(2a+2bx)\,dx$$
$$= \frac{x^{2+m}}{2(2+m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{1+m}\,dx$$
$$= \frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+}{b^2}$$

Mathematica [A]
time = 0.08, size = 79, normalized size = 0.92

$$\frac{1}{16}x^m\left(\frac{8x^2}{2+m}-\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(2+m,2bx)}{b^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(1 + m)*Cosh[a + b*x]^2,x]
[Out] (x^m*((8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^m)
 - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Maple [F]
time = 0.55, size = 0, normalized size = 0.00

$$\int x^{1+m}\left(\cosh^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(1+m)*cosh(b*x+a)^2,x)
[Out] int(x^(1+m)*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.08, size = 71, normalized size = 0.83

$$-\frac{1}{4}\left(2\,bx\right)^{-m-2}x^{m+2}e^{(-2\,a)}\Gamma(m+2,2\,bx)-\frac{1}{4}\left(-2\,bx\right)^{-m-2}x^{m+2}e^{(2\,a)}\Gamma(m+2,-2\,bx)+\frac{x^{m+2}}{2\,(m+2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*(2*b*x)^(-m - 2)*x^(m + 2)*e^(-2*a)*gamma(m + 2, 2*b*x) - 1/4*(-2*b*x)
^(-m - 2)*x^(m + 2)*e^(2*a)*gamma(m + 2, -2*b*x) + 1/2*x^(m + 2)/(m + 2)
```

Fricas [A]
time = 0.16, size = 136, normalized size = 1.58

$$\frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)-(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma(m+2,2\,bx)+(m+2)\cosh\left((m+1)\log\left(-2\,b\right)-2\,a\right)\Gamma(m+2,-2\,bx)+(m+2)\Gamma(m+2,2\,bx)\sinh\left((m+1)\log\left(2\,b\right)+2\,a\right)-(m+2)\Gamma(m+2,-2\,bx)\sinh\left((m+1)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m+1)\log\left(x\right)\right)}{8\,(bm+2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m + 1)*log(x)) - (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*gamm
a(m + 2, 2*b*x) + (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*b*x
) + (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) - (m + 2)*gamm
a(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*log(x))
)/(b*m + 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m+1}\cosh^{2}\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(1+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m + 1)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m + 1)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m+1}\,\mathrm{cosh}\left(a+b\,x\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m + 1)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m + 1)*cosh(a + b*x)^2, x)
```

3.91 $\int x^m \cosh^2(a+bx)\,dx$

Optimal. Leaf size=85

$$\frac{x^{1+m}}{2(1+m)}+\frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b}-\frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

```
[Out] 1/2*x^(1+m)/(1+m)+2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(-
3-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3393, 3388, 2212}

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\text{Gamma}(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\text{Gamma}(m+1,2bx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^m*Cosh[a + b*x]^2,x]
```

```
[Out] x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(-
(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^m \cosh^2(a+bx)\,dx = \int \left(\frac{x^m}{2} + \frac{1}{2}x^m \cosh(2a+2bx)\right)\,dx$$
$$= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2}\int x^m \cosh(2a+2bx)\,dx$$
$$= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4}\int e^{-i(2ia+2ibx)}x^m\,dx + \frac{1}{4}\int e^{i(2ia+2ibx)}x^m\,dx$$
$$= \frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+r}{b}$$

Mathematica [A]
time = 0.06, size = 76, normalized size = 0.89

$$\frac{1}{8}x^m\left(\frac{4x}{1+m} + \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(1+m,2bx)}{b}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^m*Cosh[a + b*x]^2,x]
```

```
[Out] (x^m*((4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) - G
amma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Maple [F]
time = 0.54, size = 0, normalized size = 0.00

$$\int x^m\left(\cosh^2\left(bx+a\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(b*x+a)^2,x)
```

```
[Out] int(x^m*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.07, size = 71, normalized size = 0.84

$$-\frac{1}{4}\left(2\,bx\right)^{-m-1}x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) - \frac{1}{4}\left(-2\,bx\right)^{-m-1}x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) + \frac{x^{m+1}}{2\,(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x)
^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) + 1/2*x^(m + 1)/(m + 1)
```

Fricas [A]
time = 0.10, size = 122, normalized size = 1.44

$$\frac{4\,bx\cosh\left(m\log\left(x\right)\right) - \left(m+1\right)\cosh\left(m\log\left(2\,b\right) + 2\,a\right)\Gamma\left(m+1, 2\,bx\right) + \left(m+1\right)\cosh\left(m\log\left(-2\,b\right) - 2\,a\right)\Gamma\left(m+1, -2\,bx\right) + \left(m+1\right)\Gamma\left(m+1, 2\,bx\right)\sinh\left(m\log\left(2\,b\right) + 2\,a\right) - \left(m+1\right)\Gamma\left(m+1, -2\,bx\right)\sinh\left(m\log\left(-2\,b\right) - 2\,a\right) + 4\,bx\sinh\left(m\log\left(x\right)\right)}{8\,\left(bm+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh(m*log(x)) - (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*b
*x) + (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + (m + 1)*gamma(
m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - (m + 1)*gamma(m + 1, -2*b*x)*sinh(m*
log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^m \cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**m*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**m*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^m*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^m*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^m\,{\mathrm{cosh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^m*cosh(a + b*x)^2,x)
```

```
[Out] int(x^m*cosh(a + b*x)^2, x)
```

3.92 $\int x^{-1+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=72

$$\frac{x^m}{2m}-2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx)-2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)$$

```
[Out] 1/2*x^m/m-2^(-2-m)*exp(2*a)*x^m*GAMMA(m,-2*b*x)/((-b*x)^m)-2^(-2-m)*x^m*GAM
MA(m,2*b*x)/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.08, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.214$, Rules used = {3393, 3388, 2212}

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\text{Gamma}(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\text{Gamma}(m,2bx)+\frac{x^m}{2m}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-1 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2 -
 m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$
\begin{aligned}
\int x^{-1+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{-1+m}}{2}+\frac{1}{2}x^{-1+m}\cosh(2a+2bx)\right)\,dx\\
&= \frac{x^m}{2m}+\frac{1}{2}\int x^{-1+m}\cosh(2a+2bx)\,dx\\
&= \frac{x^m}{2m}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-1+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-1+m}\,dx\\
&= \frac{x^m}{2m}-2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx)-2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 64, normalized size = 0.89

$$
\frac{1}{4}x^m\left(\frac{2}{m}-2^{-m}e^{2a}(-bx)^{-m}\Gamma(m,-2bx)-2^{-m}e^{-2a}(bx)^{-m}\Gamma(m,2bx)\right)
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-1 + m)*Cosh[a + b*x]^2,x]
[Out] (x^m*(2/m - (E^(2*a)*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) - Gamma[m, 2*b*x]/(
2^m*E^(2*a)*(b*x)^m)))/4
```

Maple [F]
time = 0.65, size = 0, normalized size = 0.00

$$
\int x^{-1+m}\left(\cosh^2\left(bx+a\right)\right)\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-1+m)*cosh(b*x+a)^2,x)
[Out] int(x^(-1+m)*cosh(b*x+a)^2,x)
```

Maxima [A]
time = 0.08, size = 55, normalized size = 0.76

$$
-\frac{x^m e^{(-2a)}\Gamma(m,2bx)}{4(2bx)^m}-\frac{x^m e^{(2a)}\Gamma(m,-2bx)}{4(-2bx)^m}+\frac{x^m}{2m}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
[Out] -1/4*x^m*e^(-2*a)*gamma(m, 2*b*x)/(2*b*x)^m - 1/4*x^m*e^(2*a)*gamma(m, -2*b
*x)/(-2*b*x)^m + 1/2*x^m/m
```

Fricas [A]
time = 0.09, size = 117, normalized size = 1.62

$$\frac{4\,bx\cosh\left((m-1)\log\left(x\right)\right)-m\cosh\left((m-1)\log\left(2\,b\right)+2\,a\right)\Gamma\left(m,2\,bx\right)+m\cosh\left((m-1)\log\left(-2\,b\right)-2\,a\right)\Gamma\left(m,-2\,bx\right)+m\Gamma\left(m,2\,bx\right)\sinh\left((m-1)\log\left(2\,b\right)+2\,a\right)-m\Gamma\left(m,-2\,bx\right)\sinh\left((m-1)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-1)\log\left(x\right)\right)}{8\,bm}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 1)*log(x)) - m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m, 2
*b*x) + m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) + m*gamma(m, 2*b*x
)*sinh((m - 1)*log(2*b) + 2*a) - m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2*b)
- 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-1}\cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-1+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 1)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 1)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-1}\,{\mathrm{cosh}\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 1)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 1)*cosh(a + b*x)^2, x)
```

3.93 $\int x^{-2+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=83

$$-\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)$$

```
[Out] -1/2*x^(-1+m)/(1-m)+2^(-1-m)*b*exp(2*a)*x^m*GAMMA(-1+m,-2*b*x)/((-b*x)^m)-2
^(-1-m)*b*x^m*GAMMA(-1+m,2*b*x)/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3393, 3388, 2212}

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\text{Gamma}(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\text{Gamma}(m-1,2bx)-\frac{x^{m-1}}{2(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-2 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-1 + m)/(1 - m) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])/
(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \int \left(\frac{x^{-2+m}}{2}+\frac{1}{2}x^{-2+m}\cosh(2a+2bx)\right)\,dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{2}\int x^{-2+m}\cosh(2a+2bx)\,dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-2+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-2+m}\,dx$$
$$= -\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}$$

Mathematica [A]
time = 0.07, size = 73, normalized size = 0.88

$$\frac{1}{2}x^m\left(\frac{1}{(-1+m)x}+2^{-m}be^{2a}(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-m}be^{-2a}(bx)^{-m}\Gamma(-1+m,2bx)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-2 + m)*Cosh[a + b*x]^2,x]
[Out] (x^m*(1/((-1 + m)*x) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m) -
 (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Maple [F]
time = 0.66, size = 0, normalized size = 0.00

$$\int x^{-2+m}\left(\cosh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-2+m)*cosh(b*x+a)^2,x)
[Out] int(x^(-2+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(m-2>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.15, size = 136, normalized size = 1.64

$$\frac{4\,bx\cosh\left((m-2)\log\left(x\right)\right)-(m-1)\cosh\left((m-2)\log\left(2\,b\right)+2\,a\right)\Gamma(m-1,2\,bx)+(m-1)\cosh\left((m-2)\log\left(-2\,b\right)-2\,a\right)\Gamma(m-1,-2\,bx)+(m-1)\Gamma(m-1,2\,bx)\sinh\left((m-2)\log\left(2\,b\right)+2\,a\right)-(m-1)\Gamma(m-1,-2\,bx)\sinh\left((m-2)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-2)\log\left(x\right)\right)}{8\,(bm-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 2)*log(x)) - (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*gamm
a(m - 1, 2*b*x) + (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*b*x
) + (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) - (m - 1)*gamm
a(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*log(x))
)/(b*m - b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-2}\cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-2+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 2)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 2)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-2}\,{\cosh\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 2)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 2)*cosh(a + b*x)^2, x)
```

3.94 $\int x^{-3+m}\cosh^2(a+bx)\,dx$

Optimal. Leaf size=84

$$-\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

```
[Out] -1/2*x^(-2+m)/(2-m)-b^2*exp(2*a)*x^m*GAMMA(-2+m,-2*b*x)/(2^m)/((-b*x)^m)-b^
2*x^m*GAMMA(-2+m,2*b*x)/(2^m)/exp(2*a)/((b*x)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3393, 3388, 2212}

$$-e^{2a}b^22^{-m}x^m(-bx)^{-m}\text{Gamma}(m-2,-2bx)-e^{-2a}b^22^{-m}x^m(bx)^{-m}\text{Gamma}(m-2,2bx)-\frac{x^{m-2}}{2(2-m)}$$

Antiderivative was successfully verified.

```
[In] Int[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rubi steps

$$\begin{aligned}\int x^{-3+m}\cosh^2(a+bx)\,dx &= \int\left(\frac{x^{-3+m}}{2}+\frac{1}{2}x^{-3+m}\cosh(2a+2bx)\right)dx\\ &= -\frac{x^{-2+m}}{2(2-m)}+\frac{1}{2}\int x^{-3+m}\cosh(2a+2bx)\,dx\\ &= -\frac{x^{-2+m}}{2(2-m)}+\frac{1}{4}\int e^{-i(2ia+2ibx)}x^{-3+m}\,dx+\frac{1}{4}\int e^{i(2ia+2ibx)}x^{-3+m}\,dx\\ &= -\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(\end{aligned}$$

Mathematica [A]
time = 0.07, size = 84, normalized size = 1.00

$$-\frac{x^{-2+m}}{2(2-m)}-2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx)-2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

```
[Out] -1/2*x^(-2 + m)/(2 - m) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(b
*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Maple [F]
time = 0.64, size = 0, normalized size = 0.00

$$\int x^{-3+m}\left(\cosh^2(bx+a)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(-3+m)*cosh(b*x+a)^2,x)
```

```
[Out] int(x^(-3+m)*cosh(b*x+a)^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(m-3>0)', see `assume?` for more det
ails)Is
```

Fricas [A]
time = 0.14, size = 136, normalized size = 1.62

$$\frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right)-(m-2)\cosh\left((m-3)\log\left(2\,b\right)+2\,a\right)\Gamma(m-2,2\,bx)+(m-2)\cosh\left((m-3)\log\left(-2\,b\right)-2\,a\right)\Gamma(m-2,-2\,bx)+(m-2)\Gamma(m-2,2\,bx)\sinh\left((m-3)\log\left(2\,b\right)+2\,a\right)-(m-2)\Gamma(m-2,-2\,bx)\sinh\left((m-3)\log\left(-2\,b\right)-2\,a\right)+4\,bx\sinh\left((m-3)\log\left(x\right)\right)}{8\,(bm-2\,b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b*x*cosh((m - 3)*log(x)) - (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*gamm
a(m - 2, 2*b*x) + (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*b*x
) + (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) - (m - 2)*gamm
a(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*log(x))
)/(b*m - 2*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{m-3}\cosh^2\left(a+bx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**(-3+m)*cosh(b*x+a)**2,x)
```

```
[Out] Integral(x**(m - 3)*cosh(a + b*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

```
[Out] integrate(x^(m - 3)*cosh(b*x + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^{m-3}\,{\cosh\left(a+b\,x\right)}^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^(m - 3)*cosh(a + b*x)^2,x)
```

```
[Out] int(x^(m - 3)*cosh(a + b*x)^2, x)
```

3.95 $\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\operatorname{sech}(x)} \right) dx$

Optimal. Leaf size=24

$$-\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}$$

```
[Out] -4/9/sech(x)^(3/2)+2/3*x*sinh(x)/sech(x)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}} - \frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
```

```
[Out] -4/(9*Sech[x]^(3/2)) + (2*x*Sinh[x])/(3*Sqrt[Sech[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}
\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3} x \sqrt{\operatorname{sech}(x)} \right) dx &= -\left(\frac{1}{3} \int x \sqrt{\operatorname{sech}(x)}\, dx \right) + \int \frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}\, dx \\
&= -\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}} + \frac{1}{3} \int x \sqrt{\operatorname{sech}(x)}\, dx - \frac{1}{3} \Big(\sqrt{\cosh(x)}\, \surd \\
&= -\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)} + \frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 17, normalized size = 0.71

$$\frac{2(-2+3x\tanh(x))}{9\mathrm{sech}^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
[Out] (2*(-2 + 3*x*Tanh[x]))/(9*Sech[x]^(3/2))
```

Maple [F]
time = 1.12, size = 0, normalized size = 0.00

$$\int \frac{x}{\mathrm{sech}\left(x\right)^{\frac{3}{2}}}-\frac{x\sqrt{\mathrm{sech}\left(x\right)}}{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
[Out] int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{3x}{\mathrm{sech}^{\frac{3}{2}}\left(x\right)}\right)dx+\int x\sqrt{\mathrm{sech}\left(x\right)}\,dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(3/2)-1/3*x*sech(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x/sech(x)**(3/2), x) + Integral(x*sqrt(sech(x)), x))/3
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{x\,\sqrt{\frac{1}{\cosh\left(x\right)}}}{3}-\frac{x}{{\left(\frac{1}{\cosh\left(x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(x))^(3/2) - (x*(1/cosh(x))^(1/2))/3,x)
```

```
[Out] -int((x*(1/cosh(x))^(1/2))/3 - x/(1/cosh(x))^(3/2), x)
```

3.96 $\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx$

Optimal. Leaf size=24

$$-\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}$$

```
[Out] -4/25/sech(x)^(5/2)+2/5*x*sinh(x)/sech(x)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {4272, 4274}

$$\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
```

```
[Out] -4/(25*Sech[x]^(5/2)) + (2*x*Sinh[x])/(5*Sech[x]^(3/2))
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx &= -\left(\frac{3}{5}\int \frac{x}{\sqrt{\operatorname{sech}(x)}}\,dx\right)+\int \frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}\,dx\\ &= -\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{3}{5}\int \frac{x}{\sqrt{\operatorname{sech}(x)}}\,dx-\frac{1}{5}\Big(3\sqrt{\cosh(x)}\ \sqrt{}\\ &= -\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 17, normalized size = 0.71

$$\frac{2(-2+5x\tanh(x))}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
[Out] (2*(-2 + 5*x*Tanh[x]))/(25*Sech[x]^(5/2))
```

Maple [F]
time = 1.08, size = 0, normalized size = 0.00

$$\int \frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
[Out] int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{5x}{\operatorname{sech}^{\frac{5}{2}}(x)}\right)dx+\int\frac{3x}{\sqrt{\operatorname{sech}(x)}}\,dx}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(5/2)-3/5*x/sech(x)**(1/2),x)
```

```
[Out] -(Integral(-5*x/sech(x)**(5/2), x) + Integral(3*x/sqrt(sech(x)), x))/5
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$-\int \frac{3\,x}{5\,\sqrt{\frac{1}{\cosh\left(x\right)}}}-\frac{x}{{\left(\frac{1}{\cosh\left(x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(x))^(5/2) - (3*x)/(5*(1/cosh(x))^(1/2)),x)
```

```
[Out] -int((3*x)/(5*(1/cosh(x))^(1/2)) - x/(1/cosh(x))^(5/2), x)
```

3.97 $\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) dx$

Optimal. Leaf size=47

$$-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

```
[Out] -4/49/sech(x)^(7/2)-20/63/sech(x)^(3/2)+2/7*x*sinh(x)/sech(x)^(5/2)+10/21*x
*sinh(x)/sech(x)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {4272, 4274}

$$-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
```

```
[Out] -4/(49*Sech[x]^(7/2)) - 20/(63*Sech[x]^(3/2)) + (2*x*Sinh[x])/(7*Sech[x]^(5
/2)) + (10*x*Sinh[x])/(21*Sqrt[Sech[x]])
```

Rule 4272

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[d*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist[(n + 1)/(b^2*n), Int[(c
+ d*x)*(b*Csc[e + f*x])^(n + 2), x], x] + Simp[(c + d*x)*Cos[e + f*x]*((b*C
sc[e + f*x])^(n + 1)/(b*f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && LtQ[n, -
1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$\int\left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{sech}(x)}\right)dx=-\left(\frac{5}{21}\int x\sqrt{\operatorname{sech}(x)}\,dx\right)+\int\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}\,dx$$
$$=-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{5}{7}\int\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx-\frac{1}{21}\Big(5\sqrt{\cosh(x)}$$
$$=-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}+\frac{5}{21}\int$$
$$=-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

Mathematica [**A**]
time = 0.09, size = 45, normalized size = 0.96

$$\sqrt{\operatorname{sech}(x)}\left(-\frac{167}{882}-\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)+\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
[Out] Sqrt[Sech[x]]*(-167/882 - (88*Cosh[2*x])/441 - Cosh[4*x]/98 + (13*x*Sinh[2*
x])/42 + (x*Sinh[4*x])/28)
```

Maple [**F**]
time = 1.11, size = 0, normalized size = 0.00

$$\int\frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}}-\frac{5x\sqrt{\operatorname{sech}(x)}}{21}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
[Out] int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="maxima")
[Out] integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\left(-\frac{21x}{\operatorname{sech}^{\frac{7}{2}}(x)}\right)dx+\int 5x\sqrt{\operatorname{sech}(x)}\,dx}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)**(7/2)-5/21*x*sech(x)**(1/2),x)
```

```
[Out] -(Integral(-21*x/sech(x)**(7/2), x) + Integral(5*x*sqrt(sech(x)), x))/21
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int\frac{5\,x\,\sqrt{\frac{1}{\cosh(x)}}}{21}-\frac{x}{\left(\frac{1}{\cosh(x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(1/cosh(x))^(7/2) - (5*x*(1/cosh(x))^(1/2))/21,x)
```

```
[Out] -int((5*x*(1/cosh(x))^(1/2))/21 - x/(1/cosh(x))^(7/2), x)
```

3.98 $\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx$

Optimal. Leaf size=66

$$-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{16}{27}i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)\sqrt{\operatorname{sech}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}$$

```
[Out] -8/9*x/sech(x)^(3/2)+16/27*sinh(x)/sech(x)^(1/2)+2/3*x^2*sinh(x)/sech(x)^(1
/2)-16/27*I*(cosh(1/2*x)^2)^(1/2)/cosh(1/2*x)*EllipticF(I*sinh(1/2*x),2^(1/
2))*cosh(x)^(1/2)*sech(x)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {4273, 4274, 3854, 3856, 2720}

$$\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}-\frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\,F\left(\frac{ix}{2}\middle|2\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

```
[Out] (-8*x)/(9*Sech[x]^(3/2)) - ((16*I)/27)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]*
Sqrt[Sech[x]] + (16*Sinh[x])/(27*Sqrt[Sech[x]]) + (2*x^2*Sinh[x])/(3*Sqrt[S
ech[x]])
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4273

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^n/(f^2*n^2)), x] + (Dist
[(n + 1)/(b^2*n), Int[(c + d*x)^m*(b*Csc[e + f*x])^(n + 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^n, x], x]
+ Simp[(c + d*x)^m*Cos[e + f*x]*((b*Csc[e + f*x])^(n + 1)/(b*f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 1]
```

Rule 4274

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_.), x_Symb
ol] :> Dist[(b*Sin[e + f*x])^n*(b*Csc[e + f*x])^n, Int[(c + d*x)^m/(b*Sin[e
 + f*x])^n, x], x] /; FreeQ[{b, c, d, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rubi steps

$$
\begin{aligned}
\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx &= -\left(\frac{1}{3}\int x^2\sqrt{\operatorname{sech}(x)}\,dx\right)+\int \frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}\,dx\\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{3}\int x^2\sqrt{\operatorname{sech}(x)}\,dx+\frac{8}{9}\int \frac{1}{\operatorname{sech}^{\frac{3}{2}}(x)}\\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{8}{27}\int \sqrt{\operatorname{sech}(x)}\,dx\\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}+\frac{1}{27}\Big(8\sqrt{\cosh(x)}\sqrt{\operatorname{sec}}\\
&= -\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{16}{27}i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)\sqrt{\operatorname{sech}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}
\end{aligned}
$$

Mathematica [A]
time = 0.06, size = 55, normalized size = 0.83

$$\frac{1}{27}\sqrt{\operatorname{sech}(x)}\left(-12x-12x\cosh(2x)-16i\sqrt{\cosh(x)}\,F\left(\frac{ix}{2}\middle|2\right)+8\sinh(2x)+9x^2\sinh(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

```
[Out] (Sqrt[Sech[x]]*(-12*x - 12*x*Cosh[2*x] - (16*I)*Sqrt[Cosh[x]]*EllipticF[(I/
2)*x, 2] + 8*Sinh[2*x] + 9*x^2*Sinh[2*x]))/27
```

Maple [F]
time = 1.13, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}-\frac{x^{2} \sqrt{\operatorname{sech}\left(x\right)}}{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

```
[Out] int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \left(- \frac{3 x^{2}}{\operatorname{sech}^{\frac{3}{2}}{\left(x \right)}}\right)\, dx + \int x^{2} \sqrt{\operatorname{sech}{\left(x \right)}}\, dx}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/sech(x)**(3/2)-1/3*x**2*sech(x)**(1/2),x)
```

```
[Out] -(Integral(-3*x**2/sech(x)**(3/2), x) + Integral(x**2*sqrt(sech(x)), x))/3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2 \sqrt{\frac{1}{\cosh(x)}}}{3} - \frac{x^2}{\left(\frac{1}{\cosh(x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(1/cosh(x))^(3/2) - (x^2*(1/cosh(x))^(1/2))/3,x)
```

```
[Out] -int((x^2*(1/cosh(x))^(1/2))/3 - x^2/(1/cosh(x))^(3/2), x)
```

3.99 $\int (c+dx)^3(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cosh(e+fx)}{f^4}-\frac{3ad(c+dx)^2\cosh(e+fx)}{f^2}+\frac{6ad^2(c+dx)\sinh(e+fx)}{f^3}+\frac{a(c+dx)^3\sinh(e+fx)}{f}$$

```
[Out] 1/4*a*(d*x+c)^4/d-6*a*d^3*cosh(f*x+e)/f^4-3*a*d*(d*x+c)^2*cosh(f*x+e)/f^2+6
*a*d^2*(d*x+c)*sinh(f*x+e)/f^3+a*(d*x+c)^3*sinh(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$\frac{6ad^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3ad(c+dx)^2\cosh(e+fx)}{f^2}+\frac{a(c+dx)^3\sinh(e+fx)}{f}+\frac{a(c+dx)^4}{4d}-\frac{6ad^3\cosh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cosh[e + f*x])/f^4 - (3*a*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*a*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (a*(c + d*x)^3*S
inh[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = \int \left(a(c+dx)^3 + a(c+dx)^3\cosh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + a\int (c+dx)^3\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^4}{4d} + \frac{a(c+dx)^3\sinh(e+fx)}{f} - \frac{(3ad)\int (c+dx)^2\sinh(e+fx}{f}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{a(c+dx)^3\sinh(e+fx)}{f} +$$
$$= \frac{a(c+dx)^4}{4d} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6ad^2(c+dx)\sinh(e+fx)}{f^3}$$
$$= \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cosh(e+fx)}{f^4} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6ad^2}{}$$

Mathematica [A]
time = 0.31, size = 122, normalized size = 1.37

$$a\left(\frac{1}{4}x(4c^3+6c^2dx+4cd^2x^2+d^3x^3) - \frac{3d(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^4} + \frac{(c+dx)(c^2f^2+2cdf^2x+d^2(6+f^2x^2))\sinh(e+fx)}{f^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

```
[Out] a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*d*(c^2*f^2 + 2*c*
d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(87) = 174$.
time = 0.83, size = 482, normalized size = 5.42

method	result
risch	$\frac{a\,d^3x^4}{4} + ac\,d^2x^3 + \frac{3a\,c^2d\,x^2}{2} + c^3ax + \frac{a\,c^4}{4d} + \frac{a(d^3f^3x^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x-3c}{2f^4}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3} + \frac{d^3a\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3} - \frac{d^3ea(fx+e)^3}{f^3} - \frac{3d^3ea\left((fx+e\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3} + \frac{d^3a\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3} - \frac{d^3ea(fx+e)^3}{f^3} - \frac{3d^3ea\left((fx+e\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*d^3/f^3*a*(f*x+e)^4+d^3/f^3*a*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2*c
osh(f*x+e)+6*(f*x+e)*sinh(f*x+e)-6*cosh(f*x+e))-d^3/f^3*e*a*(f*x+e)^3-3*d^3
/f^3*e*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))+d^2/f^
2*c*a*(f*x+e)^3+3*d^2/f^2*c*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+
2*sinh(f*x+e))+3/2*d^3/f^3*e^2*a*(f*x+e)^2+3*d^3/f^3*e^2*a*((f*x+e)*sinh(f*
x+e)-cosh(f*x+e))-3*d^2/f^2*e*c*a*(f*x+e)^2-6*d^2/f^2*e*c*a*((f*x+e)*sinh(f
*x+e)-cosh(f*x+e))+3/2*d/f*c^2*a*(f*x+e)^2+3*d/f*c^2*a*((f*x+e)*sinh(f*x+e)
-cosh(f*x+e))-d^3/f^3*e^3*a*(f*x+e)-d^3/f^3*e^3*a*sinh(f*x+e)+3*d^2/f^2*e^2
*c*a*(f*x+e)+3*d^2/f^2*e^2*c*a*sinh(f*x+e)-3*d/f*e*c^2*a*(f*x+e)-3*d/f*e*c^
2*a*sinh(f*x+e)+c^3*a*(f*x+e)+c^3*a*sinh(f*x+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(91) = 182$.
time = 0.28, size = 250, normalized size = 2.81

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}\,ac^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}\,acd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{1}{2}\,ad^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) + \frac{ac^3\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*a*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*a*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + a*c^3*sinh(f
*x + e)/f
```

Fricas [**A**]
time = 0.45, size = 174, normalized size = 1.96

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 12\,(ad^3f^2x^2 + 2\,acd^2f^2x + ac^2df^2 + 2\,ad^3)\cosh(fx + \cosh(1) + \sinh(1)) + 4\,(ad^3f^3x^3 + 3\,acd^2f^3x^2 + ac^3f^3 + 6\,acd^2f + 3\,(ac^2df^3 + 2\,ad^3f)x)\sinh(fx + \cosh(1) + \sinh(1))}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
- 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 + 2*a*d^3)*cosh(f*x + c
osh(1) + sinh(1)) + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 + 6*a*
c*d^2*f + 3*(a*c^2*d*f^3 + 2*a*d^3*f)*x)*sinh(f*x + cosh(1) + sinh(1)))/f^4
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.
time = 0.26, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{ac^3\sinh(e+fx)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sinh(e+fx)}{f} - \frac{3ac^2d\cosh(e+fx)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sinh(e+fx)}{f} - \frac{6acd^2x\cosh(e+fx)}{f^2} + \frac{6acd^2\sinh(e+fx)}{f^3} + \frac{ad^3x^4}{4} + \frac{ad^3x^3\sinh(e+fx)}{f} - \frac{3ad^3x^2\cosh(e+fx)}{f^2} + \frac{6ad^3x\sinh(e+fx)}{f^3} - \frac{6ad^3\cosh(e+fx)}{f^4} & \text{for}\: f \neq 0 \\ (a\cosh(e) + a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + a*c**3*sinh(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**2
*d*x*sinh(e + f*x)/f - 3*a*c**2*d*cosh(e + f*x)/f**2 + a*c*d**2*x**3 + 3*a*
c*d**2*x**2*sinh(e + f*x)/f - 6*a*c*d**2*x*cosh(e + f*x)/f**2 + 6*a*c*d**2*
sinh(e + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sinh(e + f*x)/f - 3*a*d**3
*x**2*cosh(e + f*x)/f**2 + 6*a*d**3*x*sinh(e + f*x)/f**3 - 6*a*d**3*cosh(e
+ f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. 2(87) = 174.
time = 0.42, size = 258, normalized size = 2.90

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x - 3\,ad^3f^2x^2 + ac^3f^3 - 6\,acd^2f^2x - 3\,ac^2df^2 + 6\,ad^3fx + 6\,acd^2f - 6\,ad^3)e^{(fx+e)}}{2\,f^4} - \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + 3\,ad^3f^2x^2 + ac^3f^3 + 6\,acd^2f^2x + 3\,ac^2df^2 + 6\,ad^3fx + 6\,acd^2f + 6\,ad^3)e^{(-fx-e)}}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(a*d^3*f^3*x^
3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x - 3*a*d^3*f^2*x^2 + a*c^3*f^3 - 6*a
*c*d^2*f^2*x - 3*a*c^2*d*f^2 + 6*a*d^3*f*x + 6*a*c*d^2*f - 6*a*d^3)*e^(f*x
+ e)/f^4 - 1/2*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + 3*a*d
^3*f^2*x^2 + a*c^3*f^3 + 6*a*c*d^2*f^2*x + 3*a*c^2*d*f^2 + 6*a*d^3*f*x + 6*
a*c*d^2*f + 6*a*d^3)*e^(-f*x - e)/f^4
```

Mupad [B]
time = 1.02, size = 187, normalized size = 2.10

$$\frac{\sinh(e+f\,x)\,\left(a\,c^3\,f^2+6\,a\,c\,d^2\right)}{f^3} - \frac{3\cosh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{3\,x\sinh(e+f\,x)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{3\,a\,d^3\,x^2\cosh(e+f\,x)}{f^2} + \frac{a\,d^3\,x^3\sinh(e+f\,x)}{f} - \frac{6\,a\,c\,d^2\,x\cosh(e+f\,x)}{f^2} + \frac{3\,a\,c\,d^2\,x^2\sinh(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (sinh(e + f*x)*(a*c^3*f^2 + 6*a*c*d^2))/f^3 - (3*cosh(e + f*x)*(2*a*d^3 + a
*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*sinh(e + f*x)*(2*a*d^3 +
a*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (3*a*d^3*x^2*cosh(e +
 f*x))/f^2 + (a*d^3*x^3*sinh(e + f*x))/f - (6*a*c*d^2*x*cosh(e + f*x))/f^2
+ (3*a*c*d^2*x^2*sinh(e + f*x))/f
```

3.100 $\int (c+dx)^2(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{2ad^2\sinh(e+fx)}{f^3}+\frac{a(c+dx)^2\sinh(e+fx)}{f}$$

```
[Out] 1/3*a*(d*x+c)^3/d-2*a*d*(d*x+c)*cosh(f*x+e)/f^2+2*a*d^2*sinh(f*x+e)/f^3+a*(
d*x+c)^2*sinh(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3398, 3377, 2717}

$$-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{a(c+dx)^2\sinh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*a*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*a*d^2*Sinh
[e + f*x])/f^3 + (a*(c + d*x)^2*Sinh[e + f*x])/f
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \int \left(a(c+dx)^2 + a(c+dx)^2\cosh(e+fx)\right)\,dx$$
$$= \frac{a(c+dx)^3}{3d} + a\int (c+dx)^2\cosh(e+fx)\,dx$$
$$= \frac{a(c+dx)^3}{3d} + \frac{a(c+dx)^2\sinh(e+fx)}{f} - \frac{(2ad)\int (c+dx)\sinh(e+fx)}{f}$$
$$= \frac{a(c+dx)^3}{3d} - \frac{2ad(c+dx)\cosh(e+fx)}{f^2} + \frac{a(c+dx)^2\sinh(e+fx)}{f} + \cdot$$
$$= \frac{a(c+dx)^3}{3d} - \frac{2ad(c+dx)\cosh(e+fx)}{f^2} + \frac{2ad^2\sinh(e+fx)}{f^3} + \frac{a(c+}{}$$

Mathematica [A]
time = 0.21, size = 80, normalized size = 1.19

$$a\left(c^2x + cdx^2 + \frac{d^2x^3}{3} - \frac{2d(c+dx)\cosh(e+fx)}{f^2} + \frac{(c^2f^2 + 2cdf^2x + d^2(2+f^2x^2))\sinh(e+fx)}{f^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

```
[Out] a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - (2*d*(c + d*x)*Cosh[e + f*x])/f^2 + ((c^
2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(65) = 130$.
time = 0.84, size = 240, normalized size = 3.58

method	result
risch	$\frac{a\,d^2x^3}{3} + adc\,x^2 + a\,c^2x + \frac{a\,c^3}{3d} + \frac{a\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2\right)e^{fx+e}}{2f^3} - \frac{a\left(d^2x^2f^2+2cd\,f^2x+\right.}{}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2} + \frac{d^2a\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2} - \frac{d^2ea(fx+e)^2}{f^2} - \frac{2d^2ea((fx+e)\sinh(fx+e)-\cosh(fx+}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2} + \frac{d^2a\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2} - \frac{d^2ea(fx+e)^2}{f^2} - \frac{2d^2ea((fx+e)\sinh(fx+e)-\cosh(fx+}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a*(f*x+e)^3+d^2/f^2*a*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cos
h(f*x+e)+2*sinh(f*x+e))-d^2/f^2*e*a*(f*x+e)^2-2*d^2/f^2*e*a*((f*x+e)*sinh(f
*x+e)-cosh(f*x+e))+d/f*c*a*(f*x+e)^2+2*d/f*c*a*((f*x+e)*sinh(f*x+e)-cosh(f*
```

```
x+e))+d^2/f^2*e^2*a*(f*x+e)+d^2/f^2*e^2*a*sinh(f*x+e)-2*d/f*e*c*a*(f*x+e)-2
*d/f*e*c*a*sinh(f*x+e)+a*c^2*(f*x+e)+a*c^2*sinh(f*x+e))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.
time = 0.28, size = 149, normalized size = 2.22

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + acd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}\,ad^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{ac^2\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*c^2*sinh(f*x + e)
/f
```

Fricas [**A**]
time = 0.55, size = 108, normalized size = 1.61

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x - 6\,(ad^2fx + acdf)\cosh(fx + \cosh(1) + \sinh(1)) + 3\,(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 + 2\,ad^2)\sinh(fx + \cosh(1) + \sinh(1))}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(a*d^2*f*x + a*c*d
*f)*cosh(f*x + cosh(1) + sinh(1)) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^
2*f^2 + 2*a*d^2)*sinh(f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.16, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + \frac{ac^2\sinh(e+fx)}{f} + acdx^2 + \frac{2acdx\sinh(e+fx)}{f} - \frac{2acd\cosh(e+fx)}{f^2} + \frac{ad^2x^3}{3} + \frac{ad^2x^2\sinh(e+fx)}{f} - \frac{2ad^2x\cosh(e+fx)}{f^2} + \frac{2ad^2\sinh(e+fx)}{f^3} & \text{for } f \neq 0 \\ (a\cosh(e) + a)\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**2*x + a*c**2*sinh(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sinh(
e + f*x)/f - 2*a*c*d*cosh(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sinh(
e + f*x)/f - 2*a*d**2*x*cosh(e + f*x)/f**2 + 2*a*d**2*sinh(e + f*x)/f**3, N
e(f, 0)), ((a*cosh(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.
time = 0.41, size = 146, normalized size = 2.18

$$\frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2fx - 2\,acdf + 2\,ad^2)e^{(fx+e)}}{2\,f^3} - \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 + 2\,ad^2fx + 2\,acdf + 2\,ad^2)e^{(-fx-e)}}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x +
a*c^2*f^2 - 2*a*d^2*f*x - 2*a*c*d*f + 2*a*d^2)*e^(f*x + e)/f^3 - 1/2*(a*d^2
*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^2*f*x + 2*a*c*d*f + 2*a*d^2)*e
^(-f*x - e)/f^3
```

Mupad [B]
time = 0.14, size = 112, normalized size = 1.67

$$\frac{2\,a\,d^2\,\sinh(e+f\,x)-\frac{a\,f\,\left(6\,x\,\cosh(e+f\,x)\,d^2+6\,c\,\cosh(e+f\,x)\,d\right)}{3}+\frac{a\,f^2\,\left(3\,c^2\,\sinh(e+f\,x)+3\,d^2\,x^2\,\sinh(e+f\,x)+6\,c\,d\,x\,\sinh(e+f\,x)\right)}{3}}{f^3}+\frac{a\,\left(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (2*a*d^2*sinh(e + f*x) - (a*f*(6*d^2*x*cosh(e + f*x) + 6*c*d*cosh(e + f*x))
)/3 + (a*f^2*(3*c^2*sinh(e + f*x) + 3*d^2*x^2*sinh(e + f*x) + 6*c*d*x*sinh(
e + f*x)))/3)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

3.101 $\int (c+dx)(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}+\frac{a(c+dx)\sinh(e+fx)}{f}$$

```
[Out] 1/2*a*(d*x+c)^2/d-a*d*cosh(f*x+e)/f^2+a*(d*x+c)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)\sinh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (a*d*Cosh[e + f*x])/f^2 + (a*(c + d*x)*Sinh[e + f*x
])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)(a+a\cosh(e+fx))\,dx &= \int (a(c+dx)+a(c+dx)\cosh(e+fx))\,dx \\
&= \frac{a(c+dx)^2}{2d} + a\int (c+dx)\cosh(e+fx)\,dx \\
&= \frac{a(c+dx)^2}{2d} + \frac{a(c+dx)\sinh(e+fx)}{f} - \frac{(ad)\int \sinh(e+fx)\,dx}{f} \\
&= \frac{a(c+dx)^2}{2d} - \frac{ad\cosh(e+fx)}{f^2} + \frac{a(c+dx)\sinh(e+fx)}{f}
\end{aligned}
$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 1.16

$$
\frac{a(-2(e+fx)(de-2cf-dfx)-4d\cosh(e+fx)+4f(c+dx)\sinh(e+fx))}{4f^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) - 4*d*Cosh[e + f*x] + 4*f*(c + d*x)*
Sinh[e + f*x]))/(4*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.
time = 0.90, size = 91, normalized size = 2.02

method	result	size
risch	$\frac{ad\,x^2}{2}+acx+\frac{a(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}-\frac{a(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$	60
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{da((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{dea\sinh(fx+e)}{f}+ac(fx+e)+ac\sinh(fx+e)}{f}$	91
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{da((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{dea\sinh(fx+e)}{f}+ac(fx+e)+ac\sinh(fx+e)}{f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a*(f*x+e)^2+d/f*a*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-d/f*e*a*(f
*x+e)-d/f*e*a*sinh(f*x+e)+a*c*(f*x+e)+a*c*sinh(f*x+e))
```

Maxima [A]
time = 0.31, size = 70, normalized size = 1.56

$$
\frac{1}{2}\,adx^2+acx+\frac{1}{2}\,ad\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{ac\sinh\left(fx+e\right)}{f}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-
f*x - e)/f^2) + a*c*sinh(f*x + e)/f
```

Fricas [A]
time = 0.45, size = 57, normalized size = 1.27

$$\frac{adf^2x^2 + 2\,acf^2x - 2\,ad\cosh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right) + 2\left(adfx + acf\right)\sinh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*a*d*cosh(f*x + cosh(1) + sinh(1)) + 2*(a
*d*f*x + a*c*f)*sinh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.09, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{ac\sinh\left(e+fx\right)}{f} + \frac{adx^2}{2} + \frac{adx\sinh\left(e+fx\right)}{f} - \frac{ad\cosh\left(e+fx\right)}{f^2} & \text{for}\ f \neq 0 \\ \left(a\cosh\left(e\right) + a\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*c*sinh(e + f*x)/f + a*d*x**2/2 + a*d*x*sinh(e + f*x)/f
 - a*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a*cosh(e) + a)*(c*x + d*x**2/2), Tr
ue))
```

Giac [A]
time = 0.41, size = 64, normalized size = 1.42

$$\frac{1}{2}\,adx^2 + acx + \frac{(adfx + acf - ad)e^{(fx+e)}}{2\,f^2} - \frac{(adfx + acf + ad)e^{(-fx-e)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(a*d*f*x + a*c*f - a*d)*e^(f*x + e)/f^2 - 1/2*(a*
d*f*x + a*c*f + a*d)*e^(-f*x - e)/f^2
```

Mupad [B]
time = 0.08, size = 53, normalized size = 1.18

$$\frac{\frac{a\,f\,(2\,c\sinh(e+f\,x)+2\,d\,x\sinh(e+f\,x))}{2} - a\,d\cosh(e+f\,x)}{f^2} + \frac{a\,(d\,x^2+2\,c\,x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x),x)
```

```
[Out] ((a*f*(2*c*sinh(e + f*x) + 2*d*x*sinh(e + f*x)))/2 - a*d*cosh(e + f*x))/f^2
 + (a*(2*c*x + d*x^2))/2
```

3.102 $\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{a\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] a*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d+a*ln(d*x+c)/d-a*Shi(c*f/d+f*x)*sinh(-e+c*
f/d)/d
```

Rubi [A]
time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3398, 3384, 3379, 3382}

$$\frac{a\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (a*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(a*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{a\cosh(e+fx)}{c+dx}\right)dx \\
&= \frac{a\log(c+dx)}{d}+a\int \frac{\cosh(e+fx)}{c+dx}\,dx \\
&= \frac{a\log(c+dx)}{d}+\left(a\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(a\sinh\left(e-\frac{cf}{d}\right)\right. \\
&= \frac{a\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.84

$$\frac{a\left(\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)+\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x),x]
[Out] (a*(Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Log[c + d*x] + Sinh[e - (
c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Maple [A]
time = 1.14, size = 94, normalized size = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}-\frac{a\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{a\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*ln(d*x+c)/d-1/2*a/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*a/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.29, size = 72, normalized size = 1.12

$$-\frac{1}{2}\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(-c*f/d + e)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A]
time = 0.40, size = 122, normalized size = 1.91

$$\frac{\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\,a\log\left(dx+c\right)+\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((a*Ei((d*f*x + c*f)/d) + a*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1
) - d*sinh(1))/d) + 2*a*log(d*x + c) + (a*Ei((d*f*x + c*f)/d) - a*Ei(-(d*f*
x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{\cosh\left(e+fx\right)}{c+dx}\,dx+\int\frac{1}{c+dx}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] a*(Integral(cosh(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A]
time = 0.42, size = 67, normalized size = 1.05

$$\frac{a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(a*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + a*Ei(-(d*f*x + c*f)/d)*e^(-e + c
*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+a\cosh(e+f\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + a*cosh(e + f*x))/(c + d*x), x)
```

3.103 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{af\,\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)-a*cosh(f*x+e)/d/(d*x+c)+a*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2-
a*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.14, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3398, 3378, 3384, 3379, 3382}

$$\frac{af\,\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cosh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (a*Cosh[e + f*x])/(d*(c + d*x)) + (a*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (a*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{a\cosh(e+fx)}{(c+dx)^2}\right)dx\\
&= -\frac{a}{d(c+dx)}+a\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx\\
&= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{(af)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d}\\
&= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{\left(af\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(af\sinh\right.}{}\\
&= -\frac{a}{d(c+dx)}-\frac{a\cosh(e+fx)}{d(c+dx)}+\frac{af\operatorname{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.24, size = 68, normalized size = 0.78

$$\frac{a\left(-\frac{d(1+\cosh(e+fx))}{c+dx}+f\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+f\cosh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] (a*(-((d*(1 + Cosh[e + f*x]))/(c + d*x)) + f*CoshIntegral[f*(c/d + x)]*Sinh
[e - (c*f)/d] + f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d^2
```

Maple [A]
time = 1.06, size = 149, normalized size = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}-\frac{fa\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}+\frac{fa\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d^{2}}-\frac{fa\,\mathrm{e}^{fx+e}}{2d^{2}\left(\frac{cf}{d}+fx\right)}-\frac{fa\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\right.}{2d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/(d*x+c)-1/2*f*a*exp(-f*x-e)/d/(d*f*x+c*f)+1/2*f*a/d^2*exp((c*f-d*e)/d)
*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*a/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*a/d^2*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 89, normalized size = 1.02

$$-\frac{1}{2}\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_{2}\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_{2}\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^{2}x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^(-
c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*
d)
```

Fricas [A]
time = 0.45, size = 178, normalized size = 2.05

$$-\frac{2\,ad\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad-\left((adfx+acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(adfx+acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((adfx+acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(adfx+acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\left(d^{3}x+cd^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*d*cosh(f*x + cosh(1) + sinh(1)) + 2*a*d - ((a*d*f*x + a*c*f)*Ei((
d*f*x + c*f)/d) - (a*d*f*x + a*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*co
sh(1) - d*sinh(1))/d) - ((a*d*f*x + a*c*f)*Ei((d*f*x + c*f)/d) + (a*d*f*x +
 a*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/(d^3*
x + c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(90) = 180.
time = 0.43, size = 631, normalized size = 7.25

$$\frac{1}{2}a\left(\frac{\left(\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^{2}\operatorname{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-def^{2}\operatorname{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+cf^{3}\operatorname{Ei}\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-df^{2}e^{\left(\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^{2}}{\left(\left(dx+c\right)d^{4}\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^{5}e+cd^{4}f\right)f}-\frac{\left(\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^{2}\operatorname{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-def^{2}\operatorname{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+cf^{3}\operatorname{Ei}\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+df^{2}e^{\left(-\frac{\left(dx+c\right)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^{2}}{\left(\left(dx+c\right)d^{4}\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^{5}e+cd^{4}f\right)f}\right)-\frac{a}{\left(dx+c\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*a*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*f^
2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e
 - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
+ c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
+ c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d^5
*e + c*d^4*f)*f) - ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-(
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*
f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\cosh(e+f\,x)}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + a*cosh(e + f*x))/(c + d*x)^2, x)
```

3.104 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}+\frac{af^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}\right.}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2+1/2*a*f^2*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^3-1/2*a*cosh(f
*x+e)/d/(d*x+c)^2-1/2*a*f^2*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2*a*f*sinh(
f*x+e)/d^2/(d*x+c)
```

Rubi [A]
time = 0.16, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3379, 3382}

$$\frac{af^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (a*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (a*f^2*Cosh[
e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (a*f*Sinh[e + f*x])/(2*
d^2*(c + d*x)) + (a*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d
^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx = \int \left(\frac{a}{(c+dx)^3}+\frac{a\cosh(e+fx)}{(c+dx)^3}\right)dx$$
$$= -\frac{a}{2d(c+dx)^2}+a\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}+\frac{(af)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx}{2d}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{(af^2)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{2d^2}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(af^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cos}{}}{2d^2}$$
$$= -\frac{a}{2d(c+dx)^2}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}+\frac{af^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{af\sinh(e}{2d^2(c+}$$

Mathematica [A]
time = 0.33, size = 90, normalized size = 0.73

$$\frac{a\left(f^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(d+d\cosh(e+fx)+f(c+dx)\sinh(e+fx))}{(c+dx)^2}+f^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (a*(f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(d + d*Cosh[e + f*
x] + f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + f^2*Sinh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)]))/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.
time = 1.05, size = 296, normalized size = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2}+\frac{f^3 a\,\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 a\,\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 a\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 a\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/(d*x+c)^2+1/4*f^3*a*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x+1/4*f^3*a*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2*a
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*a/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*a*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*a*f
^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*a*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 100, normalized size = 0.81

$$-\frac{1}{2}\,a\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}\right)-\frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*a*(e^(c*f/d - e)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e^
(-c*f/d + e)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^
3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(119) = 238$.
time = 0.37, size = 293, normalized size = 2.38

$$-\frac{2\,ad^2\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad^2-\left(\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\left(ad^2fx+acdf\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-\left(\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-\left(ad^2f^2x^2+2\,acdf^2x+ac^2f^2\right)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{4\left(d^5x^2+2\,cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*a*d^2*cosh(f*x + cosh(1) + sinh(1)) + 2*a*d^2 - ((a*d^2*f^2*x^2 + 2
*a*c*d*f^2*x + a*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*d^2*f^2*x^2 + 2*a*c*d*f^
2*x + a*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/
```

```
d) + 2*(a*d^2*f*x + a*c*d*f)*sinh(f*x + cosh(1) + sinh(1)) - ((a*d^2*f^2*x^
2 + 2*a*c*d*f^2*x + a*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*d^2*f^2*x^2 + 2*a*c
*d*f^2*x + a*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh
(1))/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(115) = 230.
time = 0.42, size = 316, normalized size = 2.57

$$\frac{ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 2\,acdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + ac^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + ac^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} - ad^2fxe^{(fx+e)} + ad^2fxe^{(-fx-e)} - acdfe^{(fx+e)} + acdfe^{(-fx-e)} - ad^2e^{(fx+e)} - ad^2e^{(-fx-e)} - 2\,ad^2}{4\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(a*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + a*d^2*f^2*x^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 2*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*
f/d) + 2*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a*c^2*f^2*Ei((d*
f*x + c*f)/d)*e^(e - c*f/d) + a*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d)
 - a*d^2*f*x*e^(f*x + e) + a*d^2*f*x*e^(-f*x - e) - a*c*d*f*e^(f*x + e) + a
*c*d*f*e^(-f*x - e) - a*d^2*e^(f*x + e) - a*d^2*e^(-f*x - e) - 2*a*d^2)/(d^
5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+a\cosh(e+f\,x)}{{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))/(c + d*x)^3,x)
```

```
[Out] int((a + a*cosh(e + f*x))/(c + d*x)^3, x)
```

3.105 $\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=237

$$\frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cosh(e+fx)}{f^4}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}-$$

```
[Out] 3/4*a^2*c*d^2*x/f^2+3/8*a^2*d^3*x^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*cosh
(f*x+e)/f^4-6*a^2*d*(d*x+c)^2*cosh(f*x+e)/f^2-3/8*a^2*d^3*cosh(f*x+e)^2/f^4
-3/4*a^2*d*(d*x+c)^2*cosh(f*x+e)^2/f^2+12*a^2*d^2*(d*x+c)*sinh(f*x+e)/f^3+2
*a^2*(d*x+c)^3*sinh(f*x+e)/f+3/4*a^2*d^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f^
3+1/2*a^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.19, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2718, 3392, 32, 3391}

$$\frac{12a^2d^2(c+dx)\sinh(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3a^2cd^2x}{4f^2}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^3\sinh(e+fx)}{f}+\frac{a^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2(c+dx)^4}{8d}-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}-\frac{12a^2d^3\cosh(e+fx)}{f^4}+\frac{3a^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (3*a^2*c*d^2*x)/(4*f^2) + (3*a^2*d^3*x^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*
d) - (12*a^2*d^3*Cosh[e + f*x])/f^4 - (6*a^2*d*(c + d*x)^2*Cosh[e + f*x])/f
^2 - (3*a^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*a^2*d*(c + d*x)^2*Cosh[e + f*
x]^2)/(4*f^2) + (12*a^2*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (2*a^2*(c + d*x)
^3*Sinh[e + f*x])/f + (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*
f^3) + (a^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2a^2(c+dx)^3\cosh(e+fx)+a^2(c+dx)^3\cosh^2(e\right.\\ &= \frac{a^2(c+dx)^4}{4d}+a^2\int (c+dx)^3\cosh^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^3\\ &= \frac{a^2(c+dx)^4}{4d}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)^3\sinh(e+}{f}\\ &= \frac{3a^2(c+dx)^4}{8d}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}\\ &= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}-\\ &= \frac{3a^2cd^2x}{4f^2}+\frac{3a^2d^3x^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cosh(e+fx)}{f^4}-\frac{6a^2d(}{}\end{aligned}$$

Mathematica [A]

time = 0.99, size = 217, normalized size = 0.92

$$\frac{a^2(-96d(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-3d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh(2(e+fx))+2f(3f^3x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+16(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\sinh(e+fx)+(c+dx)\left(2c^2f^2+4cdf^2x+d^2(3+2f^2x^2)\right)\sinh(2(e+fx))))}{16f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(-96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*d
*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*f*(3
*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + (c + d*x)*(2*c^2*f^2 + 4
*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)])))/(16*f^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. $2(223) = 446$.
time = 0.92, size = 1071, normalized size = 4.52

method	result
risch	$\frac{3a^2d^3x^4}{8} + \frac{3a^2c\,d^2x^3}{2} + \frac{9a^2c^2d\,x^2}{4} + \frac{3c^3a^2x}{2} + \frac{3a^2c^4}{8d} + \frac{a^2\left(4d^3f^3x^3+12c\,d^2f^3x^2+12c^2d\,f^3x-6d^3f^2x^2+4c^3f^3-\right.}{32f^4}$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-6*d^2/f^2*e*c*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-
1/4*cosh(f*x+e)^2)+3*d^2/f^2*e^2*c*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x
+1/2*e)-3*d/f*e*c^2*a^2*(f*x+e)+3*d^2/f^2*e^2*c*a^2*(f*x+e)+d^2/f^2*c*a^2*(
f*x+e)^3+6*d^2/f^2*c*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sin
h(f*x+e))-3*d^3/f^3*e*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e
)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)+3*
d^2/f^2*c*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f*x
+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)+6*d^3/f^3*e^2*
a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+6*d/f*c^2*a^2*((f*x+e)*sinh(f*x+e)-co
sh(f*x+e))+3*d/f*c^2*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2
-1/4*cosh(f*x+e)^2)-d^3/f^3*e^3*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/
2*e)+2*c^3*a^2*sinh(f*x+e)+c^3*a^2*(f*x+e)+c^3*a^2*(1/2*sinh(f*x+e)*cosh(f*
x+e)+1/2*f*x+1/2*e)-12*d^2/f^2*e*c*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+d^
3/f^3*a^2*(1/2*(f*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^
2*cosh(f*x+e)^2+3/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(
f*x+e)^2)+2*d^3/f^3*a^2*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2*cosh(f*x+e)+6*(f
*x+e)*sinh(f*x+e)-6*cosh(f*x+e))+1/4*d^3/f^3*a^2*(f*x+e)^4+3*d^3/f^3*e^2*a^
2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-6*d
^3/f^3*e*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))-d^
3/f^3*e*a^2*(f*x+e)^3-d^3/f^3*e^3*a^2*(f*x+e)-2*d^3/f^3*e^3*a^2*sinh(f*x+e)
+3/2*d^3/f^3*e^2*a^2*(f*x+e)^2+3/2*d/f*c^2*a^2*(f*x+e)^2-3*d^2/f^2*e*c*a^2*
(f*x+e)^2+6*d^2/f^2*e^2*c*a^2*sinh(f*x+e)-6*d/f*e*c^2*a^2*sinh(f*x+e)-3*d/f
*e*c^2*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(233) = 466$.
time = 0.29, size = 554, normalized size = 2.34

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c^
2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/32*(
4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
a^2*d^3 + 1/8*a^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a^2*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a^2*c^3*sinh(f*x + e)/f
```

Fricas [A]
time = 0.48, size = 410, normalized size = 1.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(6*a^2*d^3*f^4*x^4 + 24*a^2*c*d^2*f^4*x^3 + 36*a^2*c^2*d*f^4*x^2 + 24*
a^2*c^3*f^4*x - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^2
+ a^2*d^3)*cosh(f*x + cosh(1) + sinh(1))^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c
*d^2*f^2*x + 2*a^2*c^2*d*f^2 + a^2*d^3)*sinh(f*x + cosh(1) + sinh(1))^2 - 9
6*(a^2*d^3*f^2*x^2 + 2*a^2*c*d^2*f^2*x + a^2*c^2*d*f^2 + 2*a^2*d^3)*cosh(f*
x + cosh(1) + sinh(1)) + 4*(8*a^2*d^3*f^3*x^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^
2*c^3*f^3 + 48*a^2*c*d^2*f + 24*(a^2*c^2*d*f^3 + 2*a^2*d^3*f)*x + (2*a^2*d^
3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 2*a^2*c^3*f^3 + 3*a^2*c*d^2*f + 3*(2*a^2*
c^2*d*f^3 + a^2*d^3*f)*x)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1)
 + sinh(1)))/f^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(243) = 486$.
time = 0.47, size = 779, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c**3*x*sinh(e + f*x)**2/2 + a**2*c**3*x*cosh(e + f*x)**2/2
 + a**2*c**3*x + a**2*c**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**3*
sinh(e + f*x)/f - 3*a**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*a**2*c**2*d*x**
2*cosh(e + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sinh(e + f*x)
*cosh(e + f*x)/(2*f) + 6*a**2*c**2*d*x*sinh(e + f*x)/f - 3*a**2*c**2*d*cosh
(e + f*x)**2/(4*f**2) - 6*a**2*c**2*d*cosh(e + f*x)/f**2 - a**2*c*d**2*x**3
*sinh(e + f*x)**2/2 + a**2*c*d**2*x**3*cosh(e + f*x)**2/2 + a**2*c*d**2*x**
3 + 3*a**2*c*d**2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 6*a**2*c*d**2*x*
*2*sinh(e + f*x)/f - 3*a**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*a**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) - 12*a**2*c*d**2*x*cosh(e + f*x)/f**2 + 3*a
**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 12*a**2*c*d**2*sinh(e + f
*x)/f**3 - a**2*d**3*x**4*sinh(e + f*x)**2/8 + a**2*d**3*x**4*cosh(e + f*x)
**2/8 + a**2*d**3*x**4/4 + a**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f)
 + 2*a**2*d**3*x**3*sinh(e + f*x)/f - 3*a**2*d**3*x**2*sinh(e + f*x)**2/(8*
f**2) - 3*a**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) - 6*a**2*d**3*x**2*cosh(
e + f*x)/f**2 + 3*a**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 12*a**
2*d**3*x*sinh(e + f*x)/f**3 - 3*a**2*d**3*cosh(e + f*x)**2/(8*f**4) - 12*a*
*2*d**3*cosh(e + f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**3*x + 3*c**2
*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 577 vs. 2(223) = 446.
time = 0.43, size = 577, normalized size = 2.43

$$\frac{3}{8}a^2d^3x^4 + \frac{3}{2}a^2cd^2x^3 + \frac{9}{4}a^2c^2dx^2 + \frac{3}{2}a^2c^3x + \frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x - 6a^2d^3f^2x^2 + 4a^2c^3f^3 - 12a^2cd^2f^2x - 6a^2c^2df^2 + 6a^2d^3fx + 6a^2cd^2f - 3a^2d^3)e^{(2fx+2e)}}{32f^4} + \frac{(a^2d^3f^3x^3 + 3a^2cd^2f^3x^2 + 3a^2c^2df^3x - 3a^2d^3f^2x^2 + a^2c^3f^3 - 6a^2cd^2f^2x - 3a^2c^2df^2 + 6a^2d^3fx + 6a^2cd^2f - 6a^2d^3)e^{(fx+e)}}{f^4} - \frac{(a^2d^3f^3x^3 + 3a^2cd^2f^3x^2 + 3a^2c^2df^3x + 3a^2d^3f^2x^2 + a^2c^3f^3 + 6a^2cd^2f^2x + 3a^2c^2df^2 + 6a^2d^3fx + 6a^2cd^2f + 6a^2d^3)e^{(-fx-e)}}{f^4} - \frac{(4a^2d^3f^3x^3 + 12a^2cd^2f^3x^2 + 12a^2c^2df^3x + 6a^2d^3f^2x^2 + 4a^2c^3f^3 + 12a^2cd^2f^2x + 6a^2c^2df^2 + 6a^2d^3fx + 6a^2cd^2f + 3a^2d^3)e^{(-2fx-2e)}}{32f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x + 1
/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^2*
d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a^2*
d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a^2*d^3*f^3*x^3
 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x - 3*a^2*d^3*f^2*x^2 + a^2*c^3*f^
3 - 6*a^2*c*d^2*f^2*x - 3*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f - 6
*a^2*d^3)*e^(f*x + e)/f^4 - (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*
c^2*d*f^3*x + 3*a^2*d^3*f^2*x^2 + a^2*c^3*f^3 + 6*a^2*c*d^2*f^2*x + 3*a^2*c
^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 6*a^2*d^3)*e^(-f*x - e)/f^4 - 1/
32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x + 6*a^2*d
^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^2 + 6*a^2*d
^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B]
time = 2.25, size = 452, normalized size = 1.91

$$\frac{16\,a^2\,c^3\,f^3\,\mathrm{sinh}\left(e+f\,x\right)-\frac{3\,a^2\,d^3\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{2}-96\,a^2\,d^3\,\mathrm{cosh}\left(e+f\,x\right)+12\,a^2\,c^3\,f^4\,x+2\,a^2\,c^3\,f^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+3\,a^2\,d^3\,f^4\,x^4+96\,a^2\,c\,d^2\,f\,\mathrm{sinh}\left(e+f\,x\right)+96\,a^2\,d^3\,f\,x\,\mathrm{sinh}\left(e+f\,x\right)-3\,a^2\,d^3\,f^2\,x^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+2\,a^2\,d^3\,f^3\,x^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-48\,a^2\,c^2\,d\,f^2\,\mathrm{cosh}\left(e+f\,x\right)+3\,a^2\,c\,d^2\,f\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+3\,a^2\,d^3\,f\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-3\,a^2\,c^2\,d\,f^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+18\,a^2\,c^2\,d\,f^4\,x^2+12\,a^2\,c\,d^2\,f^4\,x^3-48\,a^2\,d^3\,f^2\,x^2\,\mathrm{cosh}\left(e+f\,x\right)+16\,a^2\,d^3\,f^3\,x^3\,\mathrm{sinh}\left(e+f\,x\right)-6\,a^2\,c\,d^2\,f^2\,x\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)+6\,a^2\,c^2\,d\,f^3\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)+48\,a^2\,c\,d^2\,f^3\,x^2\,\mathrm{sinh}\left(e+f\,x\right)+6\,a^2\,c\,d^2\,f^3\,x^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)-96\,a^2\,c\,d^2\,f^2\,x\,\mathrm{cosh}\left(e+f\,x\right)+48\,a^2\,c^2\,d\,f^3\,x\,\mathrm{sinh}\left(e+f\,x\right)}{8\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] (16*a^2*c^3*f^3*sinh(e + f*x) - (3*a^2*d^3*cosh(2*e + 2*f*x))/2 - 96*a^2*d^
3*cosh(e + f*x) + 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sinh(2*e + 2*f*x) + 3*a^
2*d^3*f^4*x^4 + 96*a^2*c*d^2*f*sinh(e + f*x) + 96*a^2*d^3*f*x*sinh(e + f*x)
 - 3*a^2*d^3*f^2*x^2*cosh(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sinh(2*e + 2*f*x
) - 48*a^2*c^2*d*f^2*cosh(e + f*x) + 3*a^2*c*d^2*f*sinh(2*e + 2*f*x) + 3*a^
2*d^3*f*x*sinh(2*e + 2*f*x) - 3*a^2*c^2*d*f^2*cosh(2*e + 2*f*x) + 18*a^2*c^
2*d*f^4*x^2 + 12*a^2*c*d^2*f^4*x^3 - 48*a^2*d^3*f^2*x^2*cosh(e + f*x) + 16*
a^2*d^3*f^3*x^3*sinh(e + f*x) - 6*a^2*c*d^2*f^2*x*cosh(2*e + 2*f*x) + 6*a^2
*c^2*d*f^3*x*sinh(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^2*sinh(e + f*x) + 6*a^2
*c*d^2*f^3*x^2*sinh(2*e + 2*f*x) - 96*a^2*c*d^2*f^2*x*cosh(e + f*x) + 48*a^
2*c^2*d*f^3*x*sinh(e + f*x))/(8*f^4)
```

3.106 $\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=168

$$\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{4a^2d^2\sinh(e+fx)}{f^3}+\frac{2a^2(c+\cdots}{}$$

```
[Out] 1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d-4*a^2*d*(d*x+c)*cosh(f*x+e)/f^2-1/2*a
^2*d*(d*x+c)*cosh(f*x+e)^2/f^2+4*a^2*d^2*sinh(f*x+e)/f^3+2*a^2*(d*x+c)^2*si
nh(f*x+e)/f+1/4*a^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*cosh(
f*x+e)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.14, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2717, 3392, 32, 2715, 8}

$$-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\sinh(e+fx)}{f^3}+\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{a^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) - (4*a^2*d*(c + d*x)*Cosh[e +
 f*x])/f^2 - (a^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2*f^2) + (4*a^2*d^2*Sinh[e
+ f*x])/f^3 + (2*a^2*(c + d*x)^2*Sinh[e + f*x])/f + (a^2*d^2*Cosh[e + f*x]*
Sinh[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/(2*f
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)^2+2a^2(c+dx)^2\cosh(e+fx)+a^2(c+dx)^2\cosh^2(e\right.\\
&= \frac{a^2(c+dx)^3}{3d}+a^2\int (c+dx)^2\cosh^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)^2\\
&= \frac{a^2(c+dx)^3}{3d}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{2a^2(c+dx)^2\sinh(e+f}{f}\\
&= \frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\cosh^2(e+}{2f^2}\\
&= \frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\mathrm{co}}{2f}\end{aligned}$$

Mathematica [A]

time = 0.34, size = 192, normalized size = 1.14

$$\frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3-32df(c+dx)\cosh(e+fx)-2df(c+dx)\cosh(2(e+fx))+32d^2\sinh(e+fx)+16c^2f^2\sinh(e+fx)+32cdf^2x\sinh(e+fx)+16d^2f^2x^2\sinh(e+fx)+d^2\sinh(2(e+fx))+2c^2f^2\sinh(2(e+fx))+4cdf^2x\sinh(2(e+fx))+2d^2f^2x^2\sinh(2(e+fx)))}{8f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 32*d*f*(c + d*x)*Cosh
[e + f*x] - 2*d*f*(c + d*x)*Cosh[2*(e + f*x)] + 32*d^2*Sinh[e + f*x] + 16*c
^2*f^2*Sinh[e + f*x] + 32*c*d*f^2*x*Sinh[e + f*x] + 16*d^2*f^2*x^2*Sinh[e +
 f*x] + d^2*Sinh[2*(e + f*x)] + 2*c^2*f^2*Sinh[2*(e + f*x)] + 4*c*d*f^2*x*S
inh[2*(e + f*x)] + 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(158) = 316$.
time = 0.91, size = 541, normalized size = 3.22

method	result
risch	$\frac{a^2 d^2 x^3}{2} + \frac{3a^2 cd\, x^2}{2} + \frac{3a^2 c^2 x}{2} + \frac{a^2 c^3}{2d} + \frac{a^2 \left(2d^2 x^2 f^2+4cd\, f^2 x+2c^2 f^2-2d^2 f x-2cdf+d^2\right) \mathrm{e}^{2fx+2e}}{16 f^3} + \frac{a^2 \left(d^2 x^2 f^2+\right.}{}$
derivativedivides	$\frac{\frac{d^2 a^2 (fx+e)^3}{3f^2} + \frac{2d^2 a^2 \left((fx+e)^2 \sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2} + \frac{d^2 a^2 \left(\frac{(fx+e)^2 \cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}\right.}{}}{}$
default	$\frac{\frac{d^2 a^2 (fx+e)^3}{3f^2} + \frac{2d^2 a^2 \left((fx+e)^2 \sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2} + \frac{d^2 a^2 \left(\frac{(fx+e)^2 \cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a^2*(f*x+e)^3+2*d^2/f^2*a^2*((f*x+e)^2*sinh(f*x+e)-2*(f*x+
e)*cosh(f*x+e)+2*sinh(f*x+e))+d^2/f^2*a^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f
*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1
/4*f*x+1/4*e)-d^2/f^2*e*a^2*(f*x+e)^2-4*d^2/f^2*e*a^2*((f*x+e)*sinh(f*x+e)-
cosh(f*x+e))-2*d^2/f^2*e*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+
e)^2-1/4*cosh(f*x+e)^2)+d/f*c*a^2*(f*x+e)^2+4*d/f*c*a^2*((f*x+e)*sinh(f*x+e
)-cosh(f*x+e))+2*d/f*c*a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)
^2-1/4*cosh(f*x+e)^2)+d^2/f^2*e^2*a^2*(f*x+e)+2*d^2/f^2*e^2*a^2*sinh(f*x+e)
+d^2/f^2*e^2*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)-2*d/f*e*c*a^2*
(f*x+e)-4*d/f*e*c*a^2*sinh(f*x+e)-2*d/f*e*c*a^2*(1/2*sinh(f*x+e)*cosh(f*x+e
)+1/2*f*x+1/2*e)+a^2*c^2*(f*x+e)+2*a^2*c^2*sinh(f*x+e)+a^2*c^2*(1/2*sinh(f*
x+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 344 vs. $2(166) = 332$.
time = 0.29, size = 344, normalized size = 2.05

$$\frac{1}{3} a^2 d^2 x^3 + a^2 c d x^2 + \frac{1}{8} \left(4 x^2 + \frac{\left(2 f x e^{(2 e)} - e^{(2 e)}\right) e^{(2 f x)}}{f^2} - \frac{(2 f x + 1) e^{(-2 f x - 2 e)}}{f^2}\right) a^2 c d + \frac{1}{48} \left(8 x^3 + \frac{3 \left(2 f^2 x^2 e^{(2 e)} - 2 f x e^{(2 e)} + e^{(2 e)}\right) e^{(2 f x)}}{f^3} - \frac{3 \left(2 f^2 x^2 + 2 f x + 1\right) e^{(-2 f x - 2 e)}}{f^3}\right) a^2 d^2 + \frac{1}{8} a^2 c^2 \left(4 x + \frac{e^{(2 f x + 2 e)}}{f} - \frac{e^{(-2 f x - 2 e)}}{f}\right) + a^2 c^2 x + 2 a^2 c d \left(\frac{\left(f x e^{e} - e^{e}\right) e^{(f x)}}{f^2} - \frac{(f x + 1) e^{(-f x - e)}}{f^2}\right) + a^2 d^2 \left(\frac{\left(f^2 x^2 e^{e} - 2 f x e^{e} + 2 e^{e}\right) e^{(f x)}}{f^3} - \frac{\left(f^2 x^2 + 2 f x + 2\right) e^{(-f x - e)}}{f^3}\right) + \frac{2 a^2 c^2 \sinh\left(f x + e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 + 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x + e^(2*f*x + 2*e
)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a^2*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a^2*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a^2*c^2*sinh(
f*x + e)/f
```

Fricas [A]
time = 0.55, size = 242, normalized size = 1.44

$$\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2+6\,a^2c^2f^3x-\left(a^2d^2fx+a^2cdf\right)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-\left(a^2d^2fx+a^2cdf\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-16\left(a^2d^2fx+a^2cdf\right)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+\left(8\,a^2d^2f^2x^2+16\,a^2cdf^2x+8\,a^2c^2f^2+16\,a^2d^2+\left(2\,a^2d^2f^2x^2+4\,a^2cdf^2x+2\,a^2c^2f^2+a^2d^2\right)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{4\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 6*a^2*c^2*f^3*x - (a^2*d^2*f*x
 + a^2*c*d*f)*cosh(f*x + cosh(1) + sinh(1))^2 - (a^2*d^2*f*x + a^2*c*d*f)*s
inh(f*x + cosh(1) + sinh(1))^2 - 16*(a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + co
sh(1) + sinh(1)) + (8*a^2*d^2*f^2*x^2 + 16*a^2*c*d*f^2*x + 8*a^2*c^2*f^2 +
16*a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 + a^2*d^2
)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.
time = 0.30, size = 456, normalized size = 2.71

$$\begin{cases} -\frac{a^2c^2x\sinh^2(e+fx)}{2}+\frac{a^2c^2x\cosh^2(e+fx)}{2}+a^2c^2x+\frac{a^2c^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{2a^2c^2\sinh(e+fx)}{f}-\frac{a^2cdx^2\sinh^2(e+fx)}{2}+\frac{a^2cdx^2\cosh^2(e+fx)}{2}+a^2cdx^2+\frac{a^2cdx\sinh(e+fx)\cosh(e+fx)}{f}+\frac{4a^2cdx\sinh(e+fx)}{f}-\frac{a^2cd\cosh^2(e+fx)}{2f^2}-\frac{4a^2cd\cosh(e+fx)}{f^2}-\frac{a^2d^2x^3\sinh^2(e+fx)}{6}+\frac{a^2d^2x^3\cosh^2(e+fx)}{6}+\frac{a^2d^2x^3}{3}+\frac{a^2d^2x^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{2a^2d^2x^2\sinh(e+fx)}{f}-\frac{a^2d^2x\sinh^2(e+fx)}{4f^2}-\frac{a^2d^2x\cosh^2(e+fx)}{4f^2}-\frac{4a^2d^2x\cosh(e+fx)}{f^2}+\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{4a^2d^2\sinh(e+fx)}{f^3} & \text{for}\ f\neq 0 \\ \left(a\cosh(e)+a\right)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c**2*x*sinh(e + f*x)**2/2 + a**2*c**2*x*cosh(e + f*x)**2/2
 + a**2*c**2*x + a**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**2*
sinh(e + f*x)/f - a**2*c*d*x**2*sinh(e + f*x)**2/2 + a**2*c*d*x**2*cosh(e +
 f*x)**2/2 + a**2*c*d*x**2 + a**2*c*d*x*sinh(e + f*x)*cosh(e + f*x)/f + 4*a
**2*c*d*x*sinh(e + f*x)/f - a**2*c*d*cosh(e + f*x)**2/(2*f**2) - 4*a**2*c*d
*cosh(e + f*x)/f**2 - a**2*d**2*x**3*sinh(e + f*x)**2/6 + a**2*d**2*x**3*co
sh(e + f*x)**2/6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sinh(e + f*x)*cosh(e +
 f*x)/(2*f) + 2*a**2*d**2*x**2*sinh(e + f*x)/f - a**2*d**2*x*sinh(e + f*x)*
*2/(4*f**2) - a**2*d**2*x*cosh(e + f*x)**2/(4*f**2) - 4*a**2*d**2*x*cosh(e
+ f*x)/f**2 + a**2*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 4*a**2*d**2*
sinh(e + f*x)/f**3, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**2*x + c*d*x**2 + d**
2*x**3/3), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(158) = 316.
time = 0.43, size = 329, normalized size = 1.96

$$\frac{1}{2}a^2d^2x^3 + \frac{3}{2}a^2cdx^2 + \frac{3}{2}a^2c^2x + \frac{(2a^2d^2f^2x^2 + 4a^2cdf^2x + 2a^2c^2f^2 - 2a^2d^2fx - 2a^2cdf + a^2d^2)e^{(2fx+2e)}}{16f^3} + \frac{(a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2 - 2a^2d^2fx - 2a^2cdf + 2a^2d^2)e^{(fx+e)}}{f^3} - \frac{(a^2d^2f^2x^2 + 2a^2cdf^2x + a^2c^2f^2 + 2a^2d^2fx + 2a^2cdf + 2a^2d^2)e^{(-fx-e)}}{f^3} - \frac{(2a^2d^2f^2x^2 + 4a^2cdf^2x + 2a^2c^2f^2 + 2a^2d^2fx + 2a^2cdf + a^2d^2)e^{(-2fx-2e)}}{16f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/16*(2*a^2*d^2*f^2*x^2
 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^2)
*e^(2*f*x + 2*e)/f^3 + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 - 2
*a^2*d^2*f*x - 2*a^2*c*d*f + 2*a^2*d^2)*e^(f*x + e)/f^3 - (a^2*d^2*f^2*x^2
+ 2*a^2*c*d*f^2*x + a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + 2*a^2*d^2)*
e^(-f*x - e)/f^3 - 1/16*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^
2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [**B**]
time = 1.29, size = 257, normalized size = 1.53

$$\frac{16a^2d^2\sinh(e+fx) + \frac{a^2d^2\sinh(2e+2fx)}{2} + 8a^2c^2f^2\sinh(e+fx) + 6a^2c^2f^3x + a^2c^2f^2\sinh(2e+2fx) + 2a^2d^2f^3x^3 - a^2cdf\cosh(2e+2fx) - 16a^2d^2fx\cosh(e+fx) + a^2d^2f^2x^2\sinh(2e+2fx) + 6a^2cdf^3x^2 - a^2d^2fx\cosh(2e+2fx) - 16a^2cdf\cosh(e+fx) + 8a^2d^2f^2x^2\sinh(e+fx) + 16a^2cdf^2x\sinh(e+fx) + 2a^2cdf^2x\sinh(2e+2fx)}{4f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] (16*a^2*d^2*sinh(e + f*x) + (a^2*d^2*sinh(2*e + 2*f*x))/2 + 8*a^2*c^2*f^2*s
inh(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sinh(2*e + 2*f*x) + 2*a^2*d^2*
f^3*x^3 - a^2*c*d*f*cosh(2*e + 2*f*x) - 16*a^2*d^2*f*x*cosh(e + f*x) + a^2*
d^2*f^2*x^2*sinh(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 - a^2*d^2*f*x*cosh(2*e +
2*f*x) - 16*a^2*c*d*f*cosh(e + f*x) + 8*a^2*d^2*f^2*x^2*sinh(e + f*x) + 16*
a^2*c*d*f^2*x*sinh(e + f*x) + 2*a^2*c*d*f^2*x*sinh(2*e + 2*f*x))/(4*f^3)
```

3.107 $\int (c+dx)(a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=118

$$\frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2a^2d\cosh(e+fx)}{f^2}-\frac{a^2d\cosh^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+d}{}$$

```
[Out] 1/2*a^2*c*x+1/4*a^2*d*x^2+1/2*a^2*(d*x+c)^2/d-2*a^2*d*cosh(f*x+e)/f^2-1/4*a
^2*d*cosh(f*x+e)^2/f^2+2*a^2*(d*x+c)*sinh(f*x+e)/f+1/2*a^2*(d*x+c)*cosh(f*x
+e)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.07, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2718, 3391}

$$\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^2}{2d}+\frac{1}{2}a^2cx-\frac{a^2d\cosh^2(e+fx)}{4f^2}-\frac{2a^2d\cosh(e+fx)}{f^2}+\frac{1}{4}a^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*c*x)/2 + (a^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a^2*d*Cosh[e + f
*x])/f^2 - (a^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sinh[e + f*x]
)/f + (a^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+a\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2a^2(c+dx)\cosh(e+fx)+a^2(c+dx)\cosh^2(e+fx)\right.\\ &= \frac{a^2(c+dx)^2}{2d}+a^2\int (c+dx)\cosh^2(e+fx)\,dx+\left(2a^2\right)\int (c+dx)\cosh\\ &= \frac{a^2(c+dx)^2}{2d}-\frac{a^2d\cosh^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+\\ &= \frac{1}{2}a^2cx+\frac{1}{4}a^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2a^2d\cosh(e+fx)}{f^2}-\frac{a^2d\cosh^2(e+}{4f^2}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 81, normalized size = 0.69

$$\frac{a^2(-6(e+fx)(-2cf+d(e-fx))-16d\cosh(e+fx)-d\cosh(2(e+fx))+16f(c+dx)\sinh(e+fx)+2f(c+dx)\sinh(2(e+fx)))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*d*Cosh[e + f*x] - d*Cosh[2*(
e + f*x)] + 16*f*(c + d*x)*Sinh[e + f*x] + 2*f*(c + d*x)*Sinh[2*(e + f*x)])
)/(8*f^2)
```

Maple [A]
time = 0.95, size = 211, normalized size = 1.79

method	result
risch	$\frac{3d\,a^2x^2}{4}+\frac{3a^2cx}{2}+\frac{a^2(2dxf+2cf-d)e^{2fx+2e}}{16f^2}+\frac{a^2(dxf+cf-d)e^{fx+e}}{f^2}-\frac{a^2(dxf+cf+d)e^{-fx-e}}{f^2}-\frac{a^2(2dxf+2cf}{16}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2d\,a^2((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2d\,a^2((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a^2*(f*x+e)^2+2*d/f*a^2*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+d/f*
a^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-d
/f*e*a^2*(f*x+e)-2*d/f*e*a^2*sinh(f*x+e)-d/f*e*a^2*(1/2*sinh(f*x+e)*cosh(f*
x+e)+1/2*f*x+1/2*e)+a^2*c*(f*x+e)+2*a^2*c*sinh(f*x+e)+a^2*c*(1/2*sinh(f*x+e
)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.27, size = 176, normalized size = 1.49

$$\frac{1}{2}\,a^2dx^2+\frac{1}{16}\left(4\,x^2+\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}-\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2d+\frac{1}{8}\,a^2c\left(4\,x+\frac{e^{(2\,fx+2\,e)}}{f}-\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx+a^2d\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{2\,a^2c\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x + e^(2*f*x + 2*e)/f -
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x
+ 1)*e^(-f*x - e)/f^2) + 2*a^2*c*sinh(f*x + e)/f
```

Fricas [A]
time = 0.50, size = 128, normalized size = 1.08

$$\frac{6\,a^2df^2x^2+12\,a^2cf^2x-a^2d\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-a^2d\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-16\,a^2d\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+4\left(4\,a^2dfx+4\,a^2cf+\left(a^2dfx+a^2cf\right)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{8\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(6*a^2*d*f^2*x^2 + 12*a^2*c*f^2*x - a^2*d*cosh(f*x + cosh(1) + sinh(1))
^2 - a^2*d*sinh(f*x + cosh(1) + sinh(1))^2 - 16*a^2*d*cosh(f*x + cosh(1) +
sinh(1)) + 4*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d*f*x + a^2*c*f)*cosh(f*x + co
sh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.17, size = 219, normalized size = 1.86

$$\begin{cases} -\frac{a^2cx\sinh^2{(e+fx)}}{2}+\frac{a^2cx\cosh^2{(e+fx)}}{2}+a^2cx+\frac{a^2c\sinh{(e+fx)}\cosh{(e+fx)}}{2f}+\frac{2a^2c\sinh{(e+fx)}}{f}-\frac{a^2dx^2\sinh^2{(e+fx)}}{4}+\frac{a^2dx^2\cosh^2{(e+fx)}}{4}+\frac{a^2dx^2}{2}+\frac{a^2dx\sinh{(e+fx)}\cosh{(e+fx)}}{2f}+\frac{2a^2dx\sinh{(e+fx)}}{f}-\frac{a^2d\cosh^2{(e+fx)}}{4f^2}-\frac{2a^2d\cosh{(e+fx)}}{f^2} & \text{for}\: f\neq 0 \\ \left(a\cosh{(e)}+a\right)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-a**2*c*x*sinh(e + f*x)**2/2 + a**2*c*x*cosh(e + f*x)**2/2 + a**
2*c*x + a**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c*sinh(e + f*x)/f
 - a**2*d*x**2*sinh(e + f*x)**2/4 + a**2*d*x**2*cosh(e + f*x)**2/4 + a**2*d
*x**2/2 + a**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*d*x*sinh(e +
f*x)/f - a**2*d*cosh(e + f*x)**2/(4*f**2) - 2*a**2*d*cosh(e + f*x)/f**2, Ne
(f, 0)), ((a*cosh(e) + a)**2*(c*x + d*x**2/2), True))
```

Giac [A]
time = 0.43, size = 151, normalized size = 1.28

$$\frac{3}{4}\,a^{2}dx^{2}+\frac{3}{2}\,a^{2}cx+\frac{(2\,a^{2}dfx+2\,a^{2}cf-a^{2}d)e^{(2\,fx+2\,e)}}{16\,f^{2}}+\frac{(a^{2}dfx+a^{2}cf-a^{2}d)e^{(fx+e)}}{f^{2}}-\frac{(a^{2}dfx+a^{2}cf+a^{2}d)e^{(-fx-e)}}{f^{2}}-\frac{(2\,a^{2}dfx+2\,a^{2}cf+a^{2}d)e^{(-2\,fx-2\,e)}}{16\,f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*f
*x + 2*e)/f^2 + (a^2*d*f*x + a^2*c*f - a^2*d)*e^(f*x + e)/f^2 - (a^2*d*f*x
+ a^2*c*f + a^2*d)*e^(-f*x - e)/f^2 - 1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^2*d
)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B]
time = 0.13, size = 123, normalized size = 1.04

$$\frac{3\,a^2\,d\,x^2}{4}+\frac{3\,a^2\,c\,x}{2}-\frac{a^2\,d\,{\mathrm{cosh}\left(e+f\,x\right)}^2}{4\,f^2}-\frac{2\,a^2\,d\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{2\,a^2\,c\,\mathrm{sinh}\left(e+f\,x\right)}{f}+\frac{a^2\,c\,\mathrm{cosh}\left(e+f\,x\right)\,\mathrm{sinh}\left(e+f\,x\right)}{2\,f}+\frac{2\,a^2\,d\,x\,\mathrm{sinh}\left(e+f\,x\right)}{f}+\frac{a^2\,d\,x\,\mathrm{cosh}\left(e+f\,x\right)\,\mathrm{sinh}\left(e+f\,x\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2*(c + d*x),x)
```

```
[Out] (3*a^2*d*x^2)/4 + (3*a^2*c*x)/2 - (a^2*d*cosh(e + f*x)^2)/(4*f^2) - (2*a^2*
d*cosh(e + f*x))/f^2 + (2*a^2*c*sinh(e + f*x))/f + (a^2*c*cosh(e + f*x)*sin
h(e + f*x))/(2*f) + (2*a^2*d*x*sinh(e + f*x))/f + (a^2*d*x*cosh(e + f*x)*si
nh(e + f*x))/(2*f)
```

3.108 $\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=145

$$\frac{2a^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}+\frac{2a^2\sinh\left(e-\frac{cf}{d}\right)\operatorname{Sh}}{d}$$

```
[Out] 1/2*a^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+2*a^2*Chi(c*f/d+f*x)*cosh(-
e+c*f/d)/d+3/2*a^2*ln(d*x+c)/d-1/2*a^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d
)/d-2*a^2*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

Rubi [A]
time = 0.25, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3393, 3384, 3379, 3382}

$$\frac{2a^2\operatorname{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a^2\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{2a^2\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{3a^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (2*a^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (3*a^2*Log[c + d*x])/(2*
d) + (2*a^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d + (a^2*Sinh[2*
e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{c+dx}\,dx\\ &= \left(4a^2\right)\int \left(\frac{3}{8(c+dx)}+\frac{\cosh(e+fx)}{2(c+dx)}+\frac{\cosh(2e+2fx)}{8(c+dx)}\right)dx\\ &= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}a^2\int \frac{\cosh(2e+2fx)}{c+dx}\,dx+\left(2a^2\right)\int \frac{\cosh(e+fx)}{c+dx}\,dx\\ &= \frac{3a^2\log(c+dx)}{2d}+\frac{1}{2}\left(a^2\cosh\left(2e-\frac{2cf}{d}\right)\right)\int \frac{\cosh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx+\Big(2a^2\,\mathrm{co}\\ &= \frac{2a^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{3a^2\,\mathrm{lo}}{}\end{aligned}$$

Mathematica [A]
time = 0.15, size = 113, normalized size = 0.78

$$\frac{a^2\left(4\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)+4\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (a^2*(4*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Cosh[2*e - (2*c*f)/d]
*CoshIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] + 4*Sinh[e - (c*f)/d]*Sin
hIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
/d]))/(2*d)
```

Maple [A]
time = 4.39, size = 191, normalized size = 1.32

method	result
risch	$-\frac{a^{2} \mathrm{e}^{\frac{cf-de}{d}} \operatorname{expIntegral}\left(1, fx+e+\frac{cf-de}{d}\right)}{d}-\frac{a^{2} \mathrm{e}^{-\frac{cf-de}{d}} \operatorname{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{3 a^{2} \ln(dx+c)}{2 d}-\frac{a^{2} \mathrm{e}^{-\frac{2(cf-de)}{d}} \exp$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -a^2/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a^2/d*exp(-(c*f-d*e)/d)*Ei(
1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d-1/4*a^2/d*exp(-2*(c*f-d*e)/d)*Ei(
1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/4*a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(
c*f-d*e)/d)
```

Maxima [A]
time = 0.33, size = 153, normalized size = 1.06

$$-\frac{1}{4} a^{2}\left(\frac{e^{\left(\frac{2 cf}{d}-2 e\right)} E_{1}\left(\frac{2 (dx+c) f}{d}\right)}{d}+\frac{e^{\left(-\frac{2 cf}{d}+2 e\right)} E_{1}\left(-\frac{2 (dx+c) f}{d}\right)}{d}-\frac{2 \log(dx+c)}{d}\right)-a^{2}\left(\frac{e^{\left(\frac{cf}{d}-e\right)} E_{1}\left(\frac{(dx+c) f}{d}\right)}{d}+\frac{e^{\left(-\frac{cf}{d}+e\right)} E_{1}\left(-\frac{(dx+c) f}{d}\right)}{d}\right)+\frac{a^{2} \log(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(e^(2*c*f/d - 2*e)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(-2*c*
f/d + 2*e)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a^2*
(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(-c*f/d + e)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A]
time = 0.44, size = 251, normalized size = 1.73

$$\frac{6 a^{2} \log(dx+c)+4\left(a^{2} \operatorname{Ei}\left(\frac{dfx+cf}{d}\right)+a^{2} \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)\right) \cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(a^{2} \operatorname{Ei}\left(\frac{2(dfx+cf)}{d}\right)+a^{2} \operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right) \cosh\left(-\frac{2(cf-d\cosh(1)-d\sinh(1))}{d}\right)+4\left(a^{2} \operatorname{Ei}\left(\frac{dfx+cf}{d}\right)-a^{2} \operatorname{Ei}\left(-\frac{dfx+cf}{d}\right)\right) \sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(a^{2} \operatorname{Ei}\left(\frac{2(dfx+cf)}{d}\right)-a^{2} \operatorname{Ei}\left(-\frac{2(dfx+cf)}{d}\right)\right) \sinh\left(-\frac{2(cf-d\cosh(1)-d\sinh(1))}{d}\right)}{4 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(6*a^2*log(d*x + c) + 4*(a^2*Ei((d*f*x + c*f)/d) + a^2*Ei(-(d*f*x + c*f
)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) + (a^2*Ei(2*(d*f*x + c*f)/d) +
 a^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d) + 4*(
a^2*Ei((d*f*x + c*f)/d) - a^2*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1)
- d*sinh(1))/d) + (a^2*Ei(2*(d*f*x + c*f)/d) - a^2*Ei(-2*(d*f*x + c*f)/d))*
sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{2 \cosh(e+fx)}{c+dx}\, dx+\int \frac{\cosh^{2}(e+fx)}{c+dx}\, dx+\int \frac{1}{c+dx}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c),x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c + d*x), x) + Integral(cosh(e + f*x)**2/(c
 + d*x), x) + Integral(1/(c + d*x), x))
```

Giac [A]
time = 0.42, size = 135, normalized size = 0.93

$$\frac{a^2 \mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right) e^{\left(2\,e-\frac{2\,cf}{d}\right)} + 4\,a^2 \mathrm{Ei}\left(\frac{dfx+cf}{d}\right) e^{\left(e-\frac{cf}{d}\right)} + 4\,a^2 \mathrm{Ei}\left(-\frac{dfx+cf}{d}\right) e^{\left(-e+\frac{cf}{d}\right)} + a^2 \mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right) e^{\left(-2\,e+\frac{2\,cf}{d}\right)} + 6\,a^2 \log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a^2*Ei((d*f*x + c*f)/d
)*e^(e - c*f/d) + 4*a^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a^2*Ei(-2*(d*
f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 6*a^2*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+a\cosh\left(e+f\,x\right)\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + a*cosh(e + f*x))^2/(c + d*x), x)
```

3.109 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=157

$$-\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{a^2 f\,\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2 f\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2a^2 f\cosh\left(e\right.}{}$$

```
[Out] -4*a^2*cosh(1/2*f*x+1/2*e)^4/d/(d*x+c)+2*a^2*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x
)/d^2+a^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^2-a^2*f*Chi(2*c*f/d+2*f
*x)*sinh(-2*e+2*c*f/d)/d^2-2*a^2*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.24, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3399, 3394, 3384, 3379, 3382}

$$\frac{a^2 f\,\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2 f\,\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2a^2 f\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}+\frac{a^2 f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-4*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)) + (a^2*f*CoshIntegral[(2*c*f)/
d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d^2 + (2*a^2*f*CoshIntegral[(c*f)/d + f*x
]*Sinh[e - (c*f)/d])/d^2 + (2*a^2*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d^2 + (a^2*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])
/d^2
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx &= \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{(c+dx)^2}\,dx \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(8ia^2f\right)\int\left(-\frac{i\sinh(e+fx)}{4(c+dx)}-\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(a^2f\right)\int\frac{\sinh(2e+2fx)}{c+dx}\,dx}{d}+\frac{\left(2a^2f\right)\int\frac{\sinh(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{\left(a^2f\cosh\left(2e-\frac{2cf}{d}\right)\right)\int\frac{\sinh\left(\frac{2cf}{d}+2fx\right)}{c+dx}\,dx}{d}+\frac{\left(2a^2f\cosh\right.}{} \\
&= -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{a^2f\,\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2a^2f\,\mathrm{Chi}\left(\frac{cf}{d}+\right.}{}
\end{aligned}$$

Mathematica [A]
time = 0.47, size = 207, normalized size = 1.32

$$\frac{a^2\left(-3d-4d\cosh(e+fx)-d\cosh(2(e+fx))+2f(c+dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+4f(c+dx)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4cf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4dfx\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+2cf\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)+2dfx\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{2d^2(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (a^2*(-3*d - 4*d*Cosh[e + f*x] - d*Cosh[2*(e + f*x)] + 2*f*(c + d*x)*CoshIn
tegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CoshIntegra
l[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*c*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(
c/d + x)] + 4*d*f*x*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 2*c*f*Cos
```

```
h[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cosh[2*e - (2*
c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Maple [A]
time = 3.50, size = 308, normalized size = 1.96

method	result
risch	$-\frac{f\,a^{2} {\mathrm e}^{-f x -e}}{d \left(d x f +c f \right)}+\frac{f\,a^{2} {\mathrm e}^{\frac{c f -d e}{d}} \operatorname{expIntegral}\left(1, f x +e +\frac{c f -d e}{d}\right)}{d^{2}}-\frac{f\,a^{2} {\mathrm e}^{f x +e}}{d^{2} \left(\frac{c f}{d}+f x \right)}-\frac{f\,a^{2} {\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{expIntegral}\left(1, -f x -e -\frac{c f -d e}{d}\right)}{d^{2}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -f*a^2*exp(-f*x-e)/d/(d*f*x+c*f)+f*a^2/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f
-d*e)/d)-1/d^2*f*a^2*exp(f*x+e)/(c*f/d+f*x)-1/d^2*f*a^2*exp(-(c*f-d*e)/d)*E
i(1,-f*x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)-1/4*f*a^2*exp(-2*f*x-2*e)/d/(d*f*
x+c*f)+1/2*f*a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4/d
^2*f*a^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2/d^2*f*a^2*exp(-2*(c*f-d*e)/d)*Ei(1,
-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A]
time = 0.33, size = 186, normalized size = 1.18

$$-\frac{1}{4}\,a^{2} \left(\frac{e^{\left(\frac{2\,c f}{d} - 2\,e\right)} E_{2}\left(\frac{2\,{\left(d x + c\right)} f}{d}\right)}{{\left(d x + c\right)} d} + \frac{e^{\left(-\frac{2\,c f}{d} + 2\,e\right)} E_{2}\left(-\frac{2\,{\left(d x + c\right)} f}{d}\right)}{{\left(d x + c\right)} d} + \frac{2}{d^{2} x + c d}\right) - a^{2} \left(\frac{e^{\left(\frac{c f}{d} - e\right)} E_{2}\left(\frac{{\left(d x + c\right)} f}{d}\right)}{{\left(d x + c\right)} d} + \frac{e^{\left(-\frac{c f}{d} + e\right)} E_{2}\left(-\frac{{\left(d x + c\right)} f}{d}\right)}{{\left(d x + c\right)} d}\right) - \frac{a^{2}}{d^{2} x + c d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(e^(2*c*f/d - 2*e)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d
) + e^(-2*c*f/d + 2*e)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) +
2/(d^2*x + c*d)) - a^2*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*
x + c)*d) + e^(-c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(161) = 322$.
time = 0.40, size = 544, normalized size = 3.46

$$-\frac{a^{2} d \cosh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} + 4\,a^{2} d \cosh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right) + 3\,a^{2} d + {\left(a^{2} d + {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) \cosh\left(-\frac{2\,{\left(c f - d \cosh\left(1\right) - d \sinh\left(1\right)\right)}}{d}\right)\right)} \sinh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} - 2\,{\left({\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{d f x + c f}{d}\right) - {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(-\frac{d f x + c f}{d}\right)\right)} \cosh\left(-\frac{c f - d \cosh\left(1\right) - d \sinh\left(1\right)}{d}\right) - {\left({\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) \cosh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} - {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(-\frac{2\,{\left(d f x + c f\right)}}{d}\right)\right)} \cosh\left(-\frac{2\,{\left(c f - d \cosh\left(1\right) - d \sinh\left(1\right)\right)}}{d}\right) - 2\,{\left({\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{d f x + c f}{d}\right) + {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(-\frac{d f x + c f}{d}\right)\right)} \sinh\left(-\frac{c f - d \cosh\left(1\right) - d \sinh\left(1\right)}{d}\right) - {\left({\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) \cosh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} - {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(\frac{2\,{\left(d f x + c f\right)}}{d}\right) \sinh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} + {\left(a^{2} d f x + a^{2} c f\right)} {\rm Ei}\left(-\frac{2\,{\left(d f x + c f\right)}}{d}\right)\right)} \sinh\left(-\frac{2\,{\left(c f - d \cosh\left(1\right) - d \sinh\left(1\right)\right)}}{d}\right)}{2\,{\left({\left(d^{3} x + c d^{2}\right)} \cosh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2} - {\left(d^{3} x + c d^{2}\right)} \sinh\left(f x + \cosh\left(1\right) + \sinh\left(1\right)\right)^{2}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + 4*a^2*d*cosh(f*x + cosh(1) +
sinh(1)) + 3*a^2*d + (a^2*d + (a^2*d*f*x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d)*c
```

```
osh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))*sinh(f*x + cosh(1) + sinh(1))^2 -
2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-(d
*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((a^2*d*f*x + a^2*
c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (a^2*d*f*x + a
^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d) -
2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*Ei(-(d
*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((a^2*d*f*x + a^2*
c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (a^2*d*f*x + a
^2*c*f)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(1))^2 + (a^2*d*f*x
+ a^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d)
)/((d^3*x + c*d^2)*cosh(f*x + cosh(1) + sinh(1))^2 - (d^3*x + c*d^2)*sinh(f
*x + cosh(1) + sinh(1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{2\cosh(e+fx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{\cosh^2(e+fx)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(
cosh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*c*
d*x + d**2*x**2), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. 2(156) = 312.
time = 0.47, size = 1134, normalized size = 7.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) -
 2*a^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(2*(d*e - c*f)/d) + 2*a^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a^2*(d
*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a^2*d*e*f^2*Ei(((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4
*a^2*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d
)*e^((d*e - c*f)/d) - 4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f
^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
```

```
d*e - c*f)/d) + 4*a^2*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c)
 + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*a^2*c*f^3*Ei(-((d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 2*(d*x
+ c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*(d*e/(d*x
 + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 2*a^2*d*e
*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e
^(-2*(d*e - c*f)/d) - 2*a^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - a^2*d*f^2*e^(2*(d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a^2*d*f^2*e^((d*x + c)*(d*e/(d*
x + c) - c*f/(d*x + c) + f)/d) - 4*a^2*d*f^2*e^(-(d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f)/d) - a^2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f)/d) - 6*a^2*d*f^2)*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d^5*e + c*d^4*f)*f)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+a\cosh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + a*cosh(e + f*x))^2/(c + d*x)^2, x)
```

3.110 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=207

$$-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2}+\frac{a^2f^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^3}+\frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}-\frac{4a^2f\cosh^3\left(\right.}{d}$$

```
[Out] a^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3+a^2*f^2*Chi(c*f/d+f*x)*co
sh(-e+c*f/d)/d^3-2*a^2*cosh(1/2*f*x+1/2*e)^4/d/(d*x+c)^2-a^2*f^2*Shi(2*c*f/
d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-a^2*f^2*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-4
*a^2*f*cosh(1/2*f*x+1/2*e)^3*sinh(1/2*f*x+1/2*e)/d^2/(d*x+c)
```

Rubi [A]
time = 0.35, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3399, 3395, 3393, 3384, 3379, 3382}

$$\frac{a^2f^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{a^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}+\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{4a^2f\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)}-\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (-2*a^2*Cosh[e/2 + (f*x)/2]^4)/(d*(c + d*x)^2) + (a^2*f^2*Cosh[e - (c*f)/d]
*CoshIntegral[(c*f)/d + f*x])/d^3 + (a^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshInte
gral[(2*c*f)/d + 2*f*x])/d^3 - (4*a^2*f*Cosh[e/2 + (f*x)/2]^3*Sinh[e/2 + (f
*x)/2])/(d^2*(c + d*x)) + (a^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d +
 f*x])/d^3 + (a^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x]
)/d^3
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
```

```
NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx = \left(4a^2\right)\int \frac{\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)}{(c+dx)^3}\,dx$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(6a^2f^2\right)\int \frac{\cosh^2(}{c\text{-}}}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} - \frac{\left(6a^2f^2\right)\int\left(\frac{1}{2(c+}\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} + \frac{\left(a^2f^2\right)\int \frac{\cosh(2e\text{-}}{c+d}}{d^2}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} - \frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)} + \frac{\left(a^2f^2\cosh\left(2e\right.\right.}{}$$

$$= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2} + \frac{a^2f^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^3} + \frac{a^2f^2\cosh\left(2e-\right.}{}$$

Mathematica [A]
time = 0.75, size = 353, normalized size = 1.71

$$\frac{a^2\left(-3d^2-4d^2\cosh(e+fx)-d^2\cosh(2(e+fx))+4f^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+4f^2(c+dx)^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)-4cdf\sinh(e+fx)-4d^2fx\sinh(e+fx)-2cdf\sinh(2(e+fx))-2d^2fx\sinh(2(e+fx))+4c^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+8cdf^2x\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4d^2f^2x^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4c^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+8cdf^2x\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+4d^2f^2x^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)\right)}{4d^3(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] (a^2*(-3*d^2 - 4*d^2*Cosh[e + f*x] - d^2*Cosh[2*(e + f*x)] + 4*f^2*(c + d*x
)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + 4*f^2*(c + d*x)^2*Cosh[2*
e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 4*c*d*f*Sinh[e + f*x] - 4*
d^2*f*x*Sinh[e + f*x] - 2*c*d*f*Sinh[2*(e + f*x)] - 2*d^2*f*x*Sinh[2*(e + f
*x)] + 4*c^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 8*c*d*f^2*x*
Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Sinh[e - (c*f)/
d]*SinhIntegral[f*(c/d + x)] + 4*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral
[(2*f*(c + d*x))/d] + 8*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(
c + d*x))/d] + 4*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d
*x))/d]))/(4*d^3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(199) = 398$.
time = 3.44, size = 618, normalized size = 2.99

method	result
risch	$\frac{f^3a^2\mathrm{e}^{-fx-e}x}{2d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3a^2\mathrm{e}^{-fx-e}c}{2d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2a^2\mathrm{e}^{-fx-e}}{2d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2a^2\mathrm{e}^{\frac{cf-de}{d}}\text{expIntegral}\left(1,fx+e+\right.}{2d^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f^3*a^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*f^3*a^2*e
xp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*f^2*a^2*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*f^2*a^2/d^3*exp((c*f-d*e)/d)*Ei(1,f
*x+e+(c*f-d*e)/d)-1/2*a^2*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*a^2*f^2/d^3*
exp(f*x+e)/(c*f/d+f*x)-1/2*a^2*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d
*e)/d)-3/4*a^2/d/(d*x+c)^2+1/4*f^3*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d
*f^2*x+c^2*f^2)*x+1/4*f^3*a^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+
c^2*f^2)*c-1/8*f^2*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-
1/2*f^2*a^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/8*a^2*f^
2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*a^2*f^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*
x)-1/2*a^2*f^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A]
time = 0.34, size = 206, normalized size = 1.00

$$-\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}+\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-a^2\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(2*c*f/d - 2*e)*exp_integral_
e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(-2*c*f/d + 2*e)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a^2*(e^(c*f/d - e)*exp_integral_e(3,
 (d*x + c)*f/d)/((d*x + c)^2*d) + e^(-c*f/d + e)*exp_integral_e(3, -(d*x +
c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(206) = 412$.
time = 0.37, size = 840, normalized size = 4.06

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(a^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + 4*a^2*d^2*cosh(f*x + cosh(1
) + sinh(1)) + 3*a^2*d^2 + (a^2*d^2 + 2*(a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x
+ a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/
d))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x
 + a^2*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x +
a^2*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) -
 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)
*cosh(f*x + cosh(1) + sinh(1))^2 + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2
*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d)
+ 4*(a^2*d^2*f*x + a^2*c*d*f + (a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + cosh(1)
 + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)) - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*
d*f^2*x + a^2*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f
^2*x + a^2*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1
))/d) - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x +
c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*
x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(1))^2 - (a
^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sin
h(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/((d^5*x^2 + 2*c*d^4*x + c^2*d^3)*cos
h(f*x + cosh(1) + sinh(1))^2 - (d^5*x^2 + 2*c*d^4*x + c^2*d^3)*sinh(f*x + c
osh(1) + sinh(1))^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int \frac{2\cosh{\left(e+fx \right)}}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\, dx+\int \frac{\cosh^{2}{\left(e+fx \right)}}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\, dx+\int \frac{1}{c^{3}+3c^{2}dx+3cd^{2}x^{2}+d^{3}x^{3}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] a**2*(Integral(2*cosh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(cosh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 682 vs. $2(199) = 398$.
time = 0.43, size = 682, normalized size = 3.29

$$\frac{4a^2d^2f^2x^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}+4a^2d^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4a^2d^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+4a^2d^2f^2x^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)}+8a^2cdf^2x\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}+8a^2cdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+8a^2cdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+8a^2cdf^2x\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)}+4a^2c^2f^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)}+4a^2c^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4a^2c^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+4a^2c^2f^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)}-2a^2d^2fxe^{(2fx+2e)}-4a^2d^2fxe^{(fx+e)}+4a^2d^2fxe^{(-fx-e)}+2a^2d^2fxe^{(-2fx-2e)}-2a^2cdfe^{(2fx+2e)}-4a^2cdfe^{(fx+e)}+4a^2cdfe^{(-fx-e)}+2a^2cdfe^{(-2fx-2e)}-a^2d^2e^{(2fx+2e)}-4a^2d^2e^{(fx+e)}-4a^2d^2e^{(-fx-e)}-a^2d^2e^{(-2fx-2e)}-6a^2d^2}{8\left(d^5x^2+2cd^4x+c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a^2*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*a^2*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a^2*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a^2*d^2*f*x*e^(f*x + e) + 4*a^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*a^2*c*d*f*e^(2*f*x + 2*e) - 4*a^2*c*d*f*e^(f*x + e) + 4*a^2*c*d*f*e^(-f*x - e) + 2*a^2*c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) - 4*a^2*d^2*e^(f*x + e) - 4*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e) - 6*a^2*d^2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+a\cosh(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2/(c + d*x)^3,x)
```

```
[Out] int((a + a*cosh(e + f*x))^2/(c + d*x)^3, x)
```

3.111 $\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{(c+dx)^3}{af}-\frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2}-\frac{12d^2(c+dx)\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}+\frac{12d^3\mathrm{PolyLog}\left(3,-e^{e+fx}\right)}{af^4}+\frac{(c+}{}$$

```
[Out] (d*x+c)^3/a/f-6*d*(d*x+c)^2*ln(1+exp(f*x+e))/a/f^2-12*d^2*(d*x+c)*polylog(2
,-exp(f*x+e))/a/f^3+12*d^3*polylog(3,-exp(f*x+e))/a/f^4+(d*x+c)^3*tanh(1/2*
f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.19, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3399, 4269, 3799, 2221, 2611, 2320, 6724}

$$-\frac{12d^2(c+dx)\mathrm{Li}_2\left(-e^{e+fx}\right)}{af^3}-\frac{6d(c+dx)^2\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(c+dx)^3}{af}+\frac{12d^3\mathrm{Li}_3\left(-e^{e+fx}\right)}{af^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (c + d*x)^3/(a*f) - (6*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a*f^2) - (12*d^
2*(c + d*x)*PolyLog[2, -E^(e + f*x)])/(a*f^3) + (12*d^3*PolyLog[3, -E^(e +
f*x)])/(a*f^4) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \frac{\int (c+dx)^3\csc^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{2a}$$

$$= \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(3d)\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$

$$= \frac{(c+dx)^3}{af} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{(6d)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)^2}{1+e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{(12d^2)\int}{}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{(c+dx}{}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{(c+dx}{}$$

$$= \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{af^3} + \frac{12d^3\text{Li}}{}$$

Mathematica [A]
time = 1.58, size = 158, normalized size = 1.35

$$\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(2d\cosh\left(\frac{1}{2}(e+fx)\right)\left(\frac{e^ef^3x(3c^2+3cdx+d^2x^2)}{1+e^e}-3f^2(c+dx)^2\log\left(1+e^{e+fx}\right)-6df(c+dx)\text{PolyLog}(2,-e^{e+fx})+6d^2\text{PolyLog}(3,-e^{e+fx})\right)+f^3(c+dx)^3\text{sech}\left(\frac{e}{2}\right)\sinh\left(\frac{fx}{2}\right)\right)}{af^4(1+\cosh(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (2*Cosh[(e + f*x)/2]*(2*d*Cosh[(e + f*x)/2]*((E^e*f^3*x*(3*c^2 + 3*c*d*x +
d^2*x^2))/(1 + E^e) - 3*f^2*(c + d*x)^2*Log[1 + E^(e + f*x)] - 6*d*f*(c + d
*x)*PolyLog[2, -E^(e + f*x)] + 6*d^2*PolyLog[3, -E^(e + f*x)]) + f^3*(c + d
*x)^3*Sech[e/2]*Sinh[(f*x)/2]))/(a*f^4*(1 + Cosh[e + f*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(110) = 220$.
time = 1.68, size = 325, normalized size = 2.78

method	result
risch	$-\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(\mathrm{e}^{fx+e}+1)} - \frac{6d\,c^2\ln(\mathrm{e}^{fx+e}+1)}{a\,f^2} + \frac{6d\,c^2\ln(\mathrm{e}^{fx+e})}{a\,f^2} + \frac{6d^3e^2\ln(\mathrm{e}^{fx+e})}{a\,f^4} + \frac{2d^3x^3}{af} - \frac{6d^3e^2x}{a\,f^3} - \frac{4d^3e^3}{a\,f^4} -$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^3/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2/f*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/a/(exp(f*x+e)+1)-6/a/f^2*d*c^2*ln(exp(f*x+e)+1)+6/a/f^2*d*c^2*ln(exp(f*x+e))+6/a/f^4*d^3*e^2*ln(exp(f*x+e))+2/a/f*d^3*x^3-6/a/f^3*d^3*e^2*x-4/a/f^4*d^3*e^3-6/a/f^2*d^3*ln(exp(f*x+e)+1)*x^2-12/a/f^3*d^3*polylog(2,-exp(f*x+e))*x+12*d^3*polylog(3,-exp(f*x+e))/a/f^4-12/a/f^3*d^2*e*c*ln(exp(f*x+e))+6/a/f*d^2*c*x^2+12/a/f^2*d^2*c*e*x+6/a/f^3*d^2*c*e^2-12/a/f^2*d^2*c*ln(exp(f*x+e)+1)*x-12/a/f^3*d^2*c*polylog(2,-exp(f*x+e))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(113) = 226$.
time = 0.38, size = 239, normalized size = 2.04

$$6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right)+\frac{2\,c^3}{\left(ae^{(-fx-e)}+a\right)f}-\frac{2\,\left(d^3x^3+3\,cd^2x^2\right)}{afe^{(fx+e)}+af}-\frac{12\,\left(fx\log\left(e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)cd^2}{af^3}-\frac{6\,\left(f^2x^2\log\left(e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(fx+e)}\right)-2\,\mathrm{Li}_3\left(-e^{(fx+e)}\right)\right)d^3}{af^4}+\frac{2\,\left(d^3f^3x^3+3\,cd^2f^3x^2\right)}{af^4}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="maxima")

[Out] 6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-e))/(a*f^2)) + 2*c^3/((a*e^(-f*x - e) + a)*f) - 2*(d^3*x^3 + 3*c*d^2*x^2)/(a*f*e^(f*x + e) + a*f) - 12*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e)) - 2*polylog(3, -e^(f*x + e)))*d^3/(a*f^4) + 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a*f^4)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(113) = 226$.
time = 0.42, size = 672, normalized size = 5.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="fricas")

[Out] -2*(c^3*f^3 - 3*c^2*d*f^2*cosh(1) + 3*c*d^2*f*cosh(1)^2 - d^3*cosh(1)^3 - d^3*sinh(1)^3 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + 3*c^2*d*f^2*cosh(1) - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2)*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)) + (d^3*f*x + c*d^2*f)*sinh(f*x + cosh(1) + sinh(1)))*dilog(-cosh(f*x +

```
 cosh(1) + sinh(1)) - sinh(f*x + cosh(1) + sinh(1))) + 3*(d^3*f^2*x^2 + 2*c
*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*cosh(f*x
 + cosh(1) + sinh(1)) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sinh(f*x
+ cosh(1) + sinh(1)))*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x + cosh(1
) + sinh(1)) + 1) - 6*(d^3*cosh(f*x + cosh(1) + sinh(1)) + d^3*sinh(f*x + c
osh(1) + sinh(1)) + d^3)*polylog(3, -cosh(f*x + cosh(1) + sinh(1)) - sinh(f
*x + cosh(1) + sinh(1))) - 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2
)*sinh(1) - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + 3*c^2*d*f^2*co
sh(1) - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 - 3*(c*d^2*f -
d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2)*
sinh(1))*sinh(f*x + cosh(1) + sinh(1)))/(a*f^4*cosh(f*x + cosh(1) + sinh(1)
) + a*f^4*sinh(f*x + cosh(1) + sinh(1)) + a*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cosh(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cosh(f*x+e)),x)
```

```
[Out] (Integral(c**3/(cosh(e + f*x) + 1), x) + Integral(d**3*x**3/(cosh(e + f*x)
+ 1), x) + Integral(3*c*d**2*x**2/(cosh(e + f*x) + 1), x) + Integral(3*c**2
*d*x/(cosh(e + f*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cosh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(c+d\,x)^3}{a+a\cosh(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cosh(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + a*cosh(e + f*x)), x)
```

3.112 $\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2}-\frac{4d^2\text{PolyLog}\left(2,-e^{e+fx}\right)}{af^3}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] (d*x+c)^2/a/f-4*d*(d*x+c)*ln(1+exp(f*x+e))/a/f^2-4*d^2*polylog(2,-exp(f*x+e
))/a/f^3+(d*x+c)^2*tanh(1/2*f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3399, 4269, 3799, 2221, 2317, 2438}

$$-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(c+dx)^2}{af}-\frac{4d^2\text{Li}_2\left(-e^{e+fx}\right)}{af^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (c + d*x)^2/(a*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(a*f^2) - (4*d^2*P
olyLog[2, -E^(e + f*x)])/(a*f^3) + ((c + d*x)^2*Tanh[e/2 + (f*x)/2])/(a*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
```

```
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx &= \frac{\int (c+dx)^2\csc^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{2a}\\
&= \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{(2d)\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}\\
&= \frac{(c+dx)^2}{af}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{(4d)\int \frac{e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}(c+dx)}{1+e^{2\left(\frac{e}{2}+\frac{fx}{2}\right)}}\,dx}{af}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(4d^2)\int \text{lo}\ldots}{\ldots}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{(4d^2)\,\text{Sub}\ldots}{\ldots}\\
&= \frac{(c+dx)^2}{af}-\frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2}-\frac{4d^2\text{Li}_2\left(-e^{e+fx}\right)}{af^3}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}\ldots\right.}{af}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.44, size = 295, normalized size = 3.35

$$\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\text{sech}\left(\frac{e}{2}\right)\left(2cdf\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2\cosh\left(\frac{e}{2}\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+fx\sinh\left(\frac{e}{2}\right)\right)+d^2\cosh\left(\frac{1}{2}(e+fx)\right)\left(2\cosh\left(\frac{e}{2}\right)\left(-i\left(f\pi x-2\pi\log\left(1+e^{fx}\right)-2ifx\log\left(1-e^{-fx-2\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)}\right)+2\pi\log\left(\cosh\left(\frac{fx}{2}\right)\right)\right)-2\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)\left(fx+2\log\left(1-e^{-fx-2\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)}\right)-2\log\left(i\sinh\left(\frac{fx}{2}+\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)\right)\right)\right)+2\text{PolyLog}\left(2,e^{-fx-2\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)}\right)\right)+e^{-\tanh^{-1}\left(\coth\left(\frac{e}{2}\right)\right)}f^2x^2\sqrt{-\text{csch}^2\left(\frac{e}{2}\right)}\sinh\left(\frac{e}{2}\right)\right)+f^2(c+dx)^2\sinh\left(\frac{fx}{2}\right)\right)}{af^3(1+\cosh(e+fx))}$$

Warning: Unable to verify antiderivative.

[In] Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]

[Out] (2*Cosh[(e + f*x)/2]*Sech[e/2]*(2*c*d*f*Cosh[(e + f*x)/2]*(-2*Cosh[e/2]*Log [Cosh[(e + f*x)/2]] + f*x*Sinh[e/2]) + d^2*Cosh[(e + f*x)/2]*(2*Cosh[e/2]*((-I)*(f*Pi*x - 2*Pi*Log[1 + E^(f*x)] - (2*I)*f*x*Log[1 - E^(-(f*x) - 2*ArcT anh[Coth[e/2]])] + 2*Pi*Log[Cosh[(f*x)/2]]) - 2*ArcTanh[Coth[e/2]]*(f*x + 2 *Log[1 - E^(-(f*x) - 2*ArcTanh[Coth[e/2]])] - 2*Log[I*Sinh[(f*x)/2 + ArcTan h[Coth[e/2]]]]) + 2*PolyLog[2, E^(-(f*x) - 2*ArcTanh[Coth[e/2]])]) + (f^2*x ^2*Sqrt[-Csch[e/2]^2]*Sinh[e/2])/E^ArcTanh[Coth[e/2]]) + f^2*(c + d*x)^2*Si nh[(f*x)/2]))/(a*f^3*(1 + Cosh[e + f*x]))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(82) = 164$.
time = 1.51, size = 174, normalized size = 1.98

method	result
risch	$-\frac{2(d^2x^2+2cdx+c^2)}{fa(\mathrm{e}^{fx+e}+1)}-\frac{4dc\ln(\mathrm{e}^{fx+e}+1)}{a\,f^2}+\frac{4dc\ln(\mathrm{e}^{fx+e})}{a\,f^2}+\frac{2d^2x^2}{af}+\frac{4d^2ex}{a\,f^2}+\frac{2d^2e^2}{a\,f^3}-\frac{4d^2\ln(\mathrm{e}^{fx+e}+1)x}{a\,f^2}-\frac{4d^2\,\mathrm{polylog}(}{a\,f}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^2/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)

[Out] -2/f*(d^2*x^2+2*c*d*x+c^2)/a/(exp(f*x+e)+1)-4/a/f^2*d*c*ln(exp(f*x+e)+1)+4/ a/f^2*d*c*ln(exp(f*x+e))+2/a/f*d^2*x^2+4/a/f^2*d^2*e*x+2/a/f^3*d^2*e^2-4/a/ f^2*d^2*ln(exp(f*x+e)+1)*x-4*d^2*polylog(2,-exp(f*x+e))/a/f^3-4/a/f^3*d^2*e *ln(exp(f*x+e))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")

[Out] -2*d^2*(x^2/(a*f*e^(f*x + e) + a*f) - 2*integrate(x/(a*f*e^(f*x + e) + a*f) , x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-e))/(a*f^2)) + 2*c^2/((a*e^(-f*x - e) + a)*f)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(84) = 168$.
time = 0.50, size = 359, normalized size = 4.08

$$-\frac{2\left(c^2f^2-2cdf\cosh(1)+d^2\cosh(1)^2+d^2\sinh(1)^2-\left(d^2f^2x^2+2cdf^2x+2cdf\cosh(1)-d^2\cosh(1)^2-d^2\sinh(1)^2+2(cdf-d^2\cosh(1))\sinh(1)\right)\cosh(fx+\cosh(1)+\sinh(1))+2\left(d^2\cosh(fx+\cosh(1)+\sinh(1))+d^2\sinh(fx+\cosh(1)+\sinh(1))+d^2\right)\mathrm{Li}_2(-\cosh(fx+\cosh(1)+\sinh(1))-\sinh(fx+\cosh(1)+\sinh(1)))+2\left(d^2fx+cdf+(d^2fx+cdf)\cosh(fx+\cosh(1)+\sinh(1))+(d^2fx+cdf)\sinh(fx+\cosh(1)+\sinh(1))\right)\log(\cosh(fx+\cosh(1)+\sinh(1))+\sinh(fx+\cosh(1)+\sinh(1))+1)-2(cdf-d^2\cosh(1))\sinh(1)-\left(d^2f^2x^2+2cdf^2x+2cdf\cosh(1)-d^2\cosh(1)^2-d^2\sinh(1)^2+2(cdf-d^2\cosh(1))\sinh(1)\right)\sinh(fx+\cosh(1)+\sinh(1))\right)}{af^3\cosh(fx+\cosh(1)+\sinh(1))+af^3\sinh(fx+\cosh(1)+\sinh(1))+af^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -2*(c^2*f^2 - 2*c*d*f*cosh(1) + d^2*cosh(1)^2 + d^2*sinh(1)^2 - (d^2*f^2*x^
2 + 2*c*d*f^2*x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*
f - d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 2*(d^2*cosh(f*x +
 cosh(1) + sinh(1)) + d^2*sinh(f*x + cosh(1) + sinh(1)) + d^2)*dilog(-cosh(
f*x + cosh(1) + sinh(1)) - sinh(f*x + cosh(1) + sinh(1))) + 2*(d^2*f*x + c*
d*f + (d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1)) + (d^2*f*x + c*d*f)*s
inh(f*x + cosh(1) + sinh(1)))*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x
+ cosh(1) + sinh(1)) + 1) - 2*(c*d*f - d^2*cosh(1))*sinh(1) - (d^2*f^2*x^2
+ 2*c*d*f^2*x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*f
- d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1) + sinh(1)))/(a*f^3*cosh(f*x + co
sh(1) + sinh(1)) + a*f^3*sinh(f*x + cosh(1) + sinh(1)) + a*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cosh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cosh(e+fx)+1}\,dx + \int \frac{2cdx}{\cosh(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

```
[Out] (Integral(c**2/(cosh(e + f*x) + 1), x) + Integral(d**2*x**2/(cosh(e + f*x)
+ 1), x) + Integral(2*c*d*x/(cosh(e + f*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cosh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(c+d\,x\right)}^2}{a+a\,\cosh\left(e+f\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cosh(e + f*x)),x)
```

```
[Out] int((c + d*x)^2/(a + a*cosh(e + f*x)), x)
```

3.113 $\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=49

$$-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

```
[Out] -2*d*ln(cosh(1/2*f*x+1/2*e))/a/f^2+(d*x+c)*tanh(1/2*f*x+1/2*e)/a/f
```

Rubi [A]
time = 0.05, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3399, 4269, 3556}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(a*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(
a*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{\int (c+dx)\csc^2\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{2a}$$
$$= \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\,dx}{af}$$
$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

Mathematica [A]
time = 0.16, size = 70, normalized size = 1.43

$$\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2d\cosh\left(\frac{1}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(1+\cosh(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

```
[Out] (2*Cosh[(e + f*x)/2]*(-2*d*Cosh[(e + f*x)/2]*Log[Cosh[(e + f*x)/2]] + f*(c
+ d*x)*Sinh[(e + f*x)/2]))/(a*f^2*(1 + Cosh[e + f*x]))
```

Maple [A]
time = 1.23, size = 63, normalized size = 1.29

method	result	size
risch	$\frac{2dx}{af}+\frac{2de}{a\,f^2}-\frac{2(dx+c)}{af\left(e^{fx+e}+1\right)}-\frac{2d\ln\left(e^{fx+e}+1\right)}{a\,f^2}$	63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2*d/a/f*x+2*d/a/f^2*e-2*(d*x+c)/a/f/(exp(f*x+e)+1)-2*d/a/f^2*ln(exp(f*x+e)+
1)
```

Maxima [A]
time = 0.26, size = 76, normalized size = 1.55

$$2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right)+\frac{2\,c}{\left(ae^{(-fx-e)}+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-e))/
(a*f^2)) + 2*c/((a*e^(-f*x - e) + a)*f)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(43) = 86$.
time = 0.41, size = 116, normalized size = 2.37

$$\frac{2\,(dfx\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+dfx\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-cf-(d\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+d\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+d)\log\left(\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+1\right))}{af^2\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+af^2\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 2*(d*f*x*cosh(f*x + cosh(1) + sinh(1)) + d*f*x*sinh(f*x + cosh(1) + sinh(1)
) - c*f - (d*cosh(f*x + cosh(1) + sinh(1)) + d*sinh(f*x + cosh(1) + sinh(1)
) + d)*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x + cosh(1) + sinh(1)) +
1))/(a*f^2*cosh(f*x + cosh(1) + sinh(1)) + a*f^2*sinh(f*x + cosh(1) + sinh(
1)) + a*f^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.
time = 0.33, size = 76, normalized size = 1.55

$$\begin{cases} \frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}+\frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}-\frac{dx}{af}+\frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cosh\left(e\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] Piecewise(((c*tanh(e/2 + f*x/2)/(a*f) + d*x*tanh(e/2 + f*x/2)/(a*f) - d*x/(a
*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/2)
/(a*cosh(e) + a), True))
```

Giac [A]
time = 0.41, size = 66, normalized size = 1.35

$$\frac{2\left(dfxe^{(fx+e)}-de^{(fx+e)}\log\left(e^{(fx+e)}+1\right)-cf-d\log\left(e^{(fx+e)}+1\right)\right)}{af^2e^{(fx+e)}+af^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f - d*log(e^(
f*x + e) + 1))/(a*f^2*e^(f*x + e) + a*f^2)
```

Mupad [B]
time = 0.90, size = 53, normalized size = 1.08

$$\frac{2\,d\,x}{a\,f}-\frac{2\,\left(c+d\,x\right)}{a\,f\,\left({\mathrm{e}}^{e+f\,x}+1\right)}-\frac{2\,d\,\ln\left({\mathrm{e}}^{f\,x}\,{\mathrm{e}}^{e}+1\right)}{a\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cosh(e + f*x)),x)
```

```
[Out] (2*d*x)/(a*f) - (2*(c + d*x))/(a*f*(exp(e + f*x) + 1)) - (2*d*log(exp(f*x)*
exp(e) + 1))/(a*f^2)
```

3.114 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(c+dx)(a+a\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Mathematica [A]
time = 6.38, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+a\cosh\,(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -2*d*integrate(1/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e +
2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) - 2/(a*d*f*x + a*c*f + (a*d*f*x
*e^e + a*c*f*e^e)*e^(f*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (a*d*x + a*c)*cosh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\cosh{(e+fx)}+c+dx\cosh{(e+fx)}+dx}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral(1/(c*cosh(e + f*x) + c + d*x*cosh(e + f*x) + d*x), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + a\cosh(e + f\,x))\,\,(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cosh(e + f*x))*(c + d*x)),x)
```

```
[Out] int(1/((a + a*cosh(e + f*x))*(c + d*x)), x)
```

3.115 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))} dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))} dx = \int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))} dx$$

Mathematica [A]
time = 6.49, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))} dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+a\cosh(fx+e))} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -4*d*integrate(1/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f +
 (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a*c^3*f*e^e)*
e^(f*x)), x) - 2/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e +
2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2)
*cosh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\cosh(e+fx)+c^2+2cdx\cosh(e+fx)+2cdx+d^2x^2\cosh(e+fx)+d^2x^2}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral(1/(c**2*cosh(e + f*x) + c**2 + 2*c*d*x*cosh(e + f*x) + 2*c*d*x + d
**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+a\cosh\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cosh(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + a*cosh(e + f*x))*(c + d*x)^2), x)
```

3.116 $\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=255

$$\frac{(c+dx)^3}{3a^2f}-\frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2}+\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}-\frac{4d^2(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{a^2f^3}+\frac{4d^3\text{Poly}}{}$$

```
[Out] 1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*ln(1+exp(f*x+e))/a^2/f^2+4*d^3*ln(cosh(1/
2*f*x+1/2*e))/a^2/f^4-4*d^2*(d*x+c)*polylog(2,-exp(f*x+e))/a^2/f^3+4*d^3*po
lylog(3,-exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*sech(1/2*f*x+1/2*e)^2/a^2/f^2-
2*d^2*(d*x+c)*tanh(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^3*tanh(1/2*f*x+1/2*e)
/a^2/f+1/6*(d*x+c)^3*sech(1/2*f*x+1/2*e)^2*tanh(1/2*f*x+1/2*e)/a^2/f
```

Rubi [A]

time = 0.25, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 4269, 3556, 3799, 2221, 2611, 2320, 6724}

$$-\frac{4d^2(c+dx)\text{Li}_2\left(-e^{e+fx}\right)}{a^2f^3}-\frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3}-\frac{2d(c+dx)^2\log\left(e^{e+fx}+1\right)}{a^2f^2}+\frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{(c+dx)^3}{3a^2f}+\frac{4d^3\text{Li}_3\left(-e^{e+fx}\right)}{a^2f^4}+\frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^3/(3*a^2*f) - (2*d*(c + d*x)^2*Log[1 + E^(e + f*x)])/(a^2*f^2) +
(4*d^3*Log[Cosh[e/2 + (f*x)/2]])/(a^2*f^4) - (4*d^2*(c + d*x)*PolyLog[2, -E
^(e + f*x)])/(a^2*f^3) + (4*d^3*PolyLog[3, -E^(e + f*x)])/(a^2*f^4) + (d*(c
 + d*x)^2*Sech[e/2 + (f*x)/2]^2)/(2*a^2*f^2) - (2*d^2*(c + d*x)*Tanh[e/2 +
(f*x)/2])/(a^2*f^3) + ((c + d*x)^3*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d
*x)^3*Sech[e/2 + (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
```

```
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)^3 \csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{4a^2}$$

$$= \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} + \frac{(c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx$$

$$= \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\right.}{3a^2f}$$

$$= \frac{(c+dx)^3}{3a^2f} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} - \frac{2d^2(c+dx}{}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} + \frac{d(c+dx}{}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+}{}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+}{}$$

$$= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} - \frac{4d^2(c+}{}$$

Mathematica [A]
time = 2.67, size = 492, normalized size = 1.93

$$\frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(\frac{8d\cosh^3\left(\frac{1}{2}(e+fx)\right)\left(-6d^2e^efx+3c^2e^ef^3x+3cde^ef^3x^2+d^2e^ef^3x^3+6d^2\log\left(1+e^{e+fx}\right)+6d^2e^e\log\left(1+e^{e+fx}\right)-3c^2f^2\log\left(1+e^{e+fx}\right)-3c^2e^ef^2\log\left(1+e^{e+fx}\right)-6cdf^2x\log\left(1+e^{e+fx}\right)-6cde^ef^2x\log\left(1+e^{e+fx}\right)-3d^2f^2x^2\log\left(1+e^{e+fx}\right)-3d^2e^ef^2x^2\log\left(1+e^{e+fx}\right)-6d(1+e^e)f(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)+6d^2(1+e^e)\text{PolyLog}\left(3,-e^{e+fx}\right)\right)}{1+e^e}+f(c+dx)\text{sech}\left(\frac{e}{2}\right)\left(3df(c+dx)\cosh\left(\frac{fx}{2}\right)+3df(c+dx)\cosh\left(e+\frac{fx}{2}\right)-12d^2\sinh\left(\frac{fx}{2}\right)+3c^2f^2\sinh\left(\frac{fx}{2}\right)+6cdf^2x\sinh\left(\frac{fx}{2}\right)+3d^2f^2x^2\sinh\left(\frac{fx}{2}\right)+6d^2\sinh\left(e+\frac{fx}{2}\right)-6d^2\sinh\left(e+\frac{3fx}{2}\right)+c^2f^2\sinh\left(e+\frac{3fx}{2}\right)+2cdf^2x\sinh\left(e+\frac{3fx}{2}\right)+d^2f^2x^2\sinh\left(e+\frac{3fx}{2}\right)\right)\right)}{3a^2f^4(1+\cosh(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (Cosh[(e + f*x)/2]*((8*d*Cosh[(e + f*x)/2]^3*(-6*d^2*E^e*f*x + 3*c^2*E^e*f^
3*x + 3*c*d*E^e*f^3*x^2 + d^2*E^e*f^3*x^3 + 6*d^2*Log[1 + E^(e + f*x)] + 6*
d^2*E^e*Log[1 + E^(e + f*x)] - 3*c^2*f^2*Log[1 + E^(e + f*x)] - 3*c^2*E^e*f
^2*Log[1 + E^(e + f*x)] - 6*c*d*f^2*x*Log[1 + E^(e + f*x)] - 6*c*d*E^e*f^2*
x*Log[1 + E^(e + f*x)] - 3*d^2*f^2*x^2*Log[1 + E^(e + f*x)] - 3*d^2*E^e*f^2
*x^2*Log[1 + E^(e + f*x)] - 6*d*(1 + E^e)*f*(c + d*x)*PolyLog[2, -E^(e + f*
```

```
x)] + 6*d^2*(1 + E^e)*PolyLog[3, -E^(e + f*x)]))/(1 + E^e) + f*(c + d*x)*Se
ch[e/2]*(3*d*f*(c + d*x)*Cosh[(f*x)/2] + 3*d*f*(c + d*x)*Cosh[e + (f*x)/2]
- 12*d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] + 6*c*d*f^2*x*Sinh[(f*x)/2
] + 3*d^2*f^2*x^2*Sinh[(f*x)/2] + 6*d^2*Sinh[e + (f*x)/2] - 6*d^2*Sinh[e +
(3*f*x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f^2*x*Sinh[e + (3*f*x)/2]
+ d^2*f^2*x^2*Sinh[e + (3*f*x)/2])))/(3*a^2*f^4*(1 + Cosh[e + f*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(220) = 440$.
time = 2.04, size = 600, normalized size = 2.35

method	result
risch	$-\frac{2\left(3f^2d^3x^3\mathrm{e}^{fx+e}+9f^2c\,d^2x^2\mathrm{e}^{fx+e}+d^3f^2x^3-3d^3f\,x^2\mathrm{e}^{2fx+2e}+9f^2c^2dx\,\mathrm{e}^{fx+e}+3c\,d^2f^2x^2-6c\,d^2fx\,\mathrm{e}^{2fx+2e}-3f\,d^3x^2\mathrm{e}^{fx+e}+3f^2}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*f^2*d^3*x^3*exp(f*x+e)+9*f^2*c*d^2*x^2*exp(f*x+e)+d^3*f^2*x^3-3*d^3
*f*x^2*exp(2*f*x+2*e)+9*f^2*c^2*d*x*exp(f*x+e)+3*c*d^2*f^2*x^2-6*c*d^2*f*x*
exp(2*f*x+2*e)-3*f*d^3*x^2*exp(f*x+e)+3*f^2*c^3*exp(f*x+e)+3*c^2*d*f^2*x-3*
c^2*d*f*exp(2*f*x+2*e)-6*f*c*d^2*x*exp(f*x+e)-6*d^3*x*exp(2*f*x+2*e)+c^3*f^
2-3*f*c^2*d*exp(f*x+e)-6*c*d^2*exp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c*d^2*
exp(f*x+e)-6*d^3*x-6*c*d^2)/f^3/a^2/(exp(f*x+e)+1)^3+2/3/a^2/f*d^3*x^3+2/a^
2/f*d^2*c*x^2+2/a^2/f^3*d^2*c*e^2-4/a^2/f^3*d^2*c*polylog(2,-exp(f*x+e))+4/
a^2/f^2*d^2*c*e*x-2/a^2/f^2*d^3*ln(exp(f*x+e)+1)*x^2-4/a^2/f^3*d^3*polylog(
2,-exp(f*x+e))*x-2/a^2/f^2*d*c^2*ln(exp(f*x+e)+1)+2/a^2/f^2*d*c^2*ln(exp(f*
x+e))+2/a^2/f^4*d^3*e^2*ln(exp(f*x+e))-2/a^2/f^3*d^3*e^2*x-4/a^2/f^2*d^2*ln
(exp(f*x+e)+1)*c*x-4/a^2/f^3*d^2*c*e*ln(exp(f*x+e))-4/3/a^2/f^4*d^3*e^3+4*d
^3*polylog(3,-exp(f*x+e))/a^2/f^4+4/a^2/f^4*d^3*ln(exp(f*x+e)+1)-4/a^2/f^4*
d^3*ln(exp(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(228) = 456$.
time = 0.42, size = 633, normalized size = 2.48

$$2c^2d\left(\frac{fxe^{(3fx+3e)}+\left(3fxe^{(2e)}+e^{(2e)}\right)e^{(2fx)}+e^{(fx+e)}}{a^2f^2e^{(3fx+3e)}+3a^2f^2e^{(2fx+2e)}+3a^2f^2e^{(fx+e)}+a^2f^2}-\frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)+\frac{2}{3}c^3\left(\frac{3e^{(-fx-e)}}{\left(3a^2e^{(-fx-e)}+3a^2e^{(-2fx-2e)}+a^2e^{(-3fx-3e)}+a^2\right)f}+\frac{1}{\left(3a^2e^{(-fx-e)}+3a^2e^{(-2fx-2e)}+a^2e^{(-3fx-3e)}+a^2\right)f}\right)-\frac{2\left(d^3f^2x^3+3cd^2f^2x^2-6d^3x-6cd^2-3\left(d^3fx^2e^{(2e)}+2cd^2e^{(2e)}+2\left(cd^2f+d^3\right)xe^{(2e)}\right)e^{(2fx)}+3\left(d^3f^2x^3e^e-4cd^2e^e+\left(3cd^2f^2-d^3f\right)x^2e^e-2\left(cd^2f+2d^3\right)xe^e\right)e^{(fx)}\right)}{3\left(a^2f^3e^{(3fx+3e)}+3a^2f^3e^{(2fx+2e)}+3a^2f^3e^{(fx+e)}+a^2f^3\right)}-\frac{4\left(fx\log\left(e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)cd^2}{a^2f^3}-\frac{4d^3x}{a^2f^3}-\frac{2\left(f^2x^2\log\left(e^{(fx+e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(fx+e)}\right)-2\mathrm{Li}_3\left(-e^{(fx+e)}\right)\right)d^3}{a^2f^4}+\frac{4d^3\log\left(e^{(fx+e)}+1\right)}{a^2f^4}+\frac{2\left(d^3f^3x^3+3cd^2f^3x^2\right)}{3a^2f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*c^2*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f*
x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^
(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^3*(
3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*
```

```
x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*
e^(-3*f*x - 3*e) + a^2)*f)) - 2/3*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 - 6*d^3*x
- 6*c*d^2 - 3*(d^3*f*x^2*e^(2*e) + 2*c*d^2*e^(2*e) + 2*(c*d^2*f + d^3)*x*e^
(2*e))*e^(2*f*x) + 3*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*d^2*f^2 - d^3*f)
*x^2*e^e - 2*(c*d^2*f + 2*d^3)*x*e^e)*e^(f*x))/(a^2*f^3*e^(3*f*x + 3*e) + 3
*a^2*f^3*e^(2*f*x + 2*e) + 3*a^2*f^3*e^(f*x + e) + a^2*f^3) - 4*(f*x*log(e^
(f*x + e) + 1) + dilog(-e^(f*x + e)))*c*d^2/(a^2*f^3) - 4*d^3*x/(a^2*f^3) -
 2*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x + e)) - 2*polylog(3,
 -e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f*x + e) + 1)/(a^2*f^4) + 2/3*
(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2683 vs. $2(228) = 456$.
time = 0.45, size = 2683, normalized size = 10.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(c^3*f^3 + 3*c*d^2*f*cosh(1)^2 - d^3*cosh(1)^3 - d^3*sinh(1)^3 - 6*c*d
^2*f - (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3
 + d^3*sinh(1)^3 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^3 - 2*d
^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1)
+ d^3*cosh(1)^2 - 2*d^3)*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^3 - (d^3*f^
3*x^3 + 3*c*d^2*f^3*x^2 - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)
^3 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^3 - 2*d^3*f)*x + 3*(c
^2*d*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^
2 - 2*d^3)*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^3 - 3*(d^3*f^3*x^3 - 3*c*
d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 + c^2*d*f^2 + 2*c*d^2*f + (
3*c*d^2*f^3 + d^3*f^2)*x^2 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + (3*c^2*d
*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*
f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2 - 2*d^3)*sinh(1))*cosh(f*x + cosh(1
) + sinh(1))^2 + 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 - 3*(d^3*f^3*x^3 - 3*c
*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 + c^2*d*f^2 + 2*c*d^2*f +
(3*c*d^2*f^3 + d^3*f^2)*x^2 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + (3*c^2*
d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*cosh(1) + (d^3*f^3
*x^3 + 3*c*d^2*f^3*x^2 - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^
3 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^3 - 2*d^3*f)*x + 3*(c^
2*d*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2
 - 2*d^3)*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 3*(c^2*d*f^2 - 2*c*d^2*f
*cosh(1) + d^3*cosh(1)^2 - 2*d^3)*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2
- 3*(c^2*d*f^2 - 2*d^3)*cosh(1) - 3*(d^3*f^2*x^2 - c^3*f^3 - 3*c*d^2*f*cosh
(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 + c^2*d*f^2 + 4*c*d^2*f - 3*(c*d^2*f
- d^3*cosh(1))*sinh(1)^2 + 2*(c*d^2*f^2 - d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3)*
```

```
cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2 - 2*d^3)*sinh(1)
)*cosh(f*x + cosh(1) + sinh(1)) + 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f
)*cosh(f*x + cosh(1) + sinh(1))^3 + (d^3*f*x + c*d^2*f)*sinh(f*x + cosh(1)
+ sinh(1))^3 + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sinh(1))^2 + 3*(d
^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(
f*x + cosh(1) + sinh(1))^2 + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sin
h(1)) + 3*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sin
h(1))^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + c
osh(1) + sinh(1)))*dilog(-cosh(f*x + cosh(1) + sinh(1)) - sinh(f*x + cosh(1
) + sinh(1))) + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 +
 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + cosh(1) + sinh(1))^3 + (d^3*
f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*sinh(f*x + cosh(1) + sinh(1))^
3 - 2*d^3 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x +
cosh(1) + sinh(1))^2 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3 +
 (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*x + cosh(1) + sin
h(1)))*sinh(f*x + cosh(1) + sinh(1))^2 + 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c
^2*d*f^2 - 2*d^3)*cosh(f*x + cosh(1) + sinh(1)) + 3*(d^3*f^2*x^2 + 2*c*d^2*
f^2*x + c^2*d*f^2 - 2*d^3 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^
3)*cosh(f*x + cosh(1) + sinh(1))^2 + 2*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d
*f^2 - 2*d^3)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))
*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x + cosh(1) + sinh(1)) + 1) - 6
*(d^3*cosh(f*x + cosh(1) + sinh(1))^3 + d^3*sinh(f*x + cosh(1) + sinh(1))^3
 + 3*d^3*cosh(f*x + cosh(1) + sinh(1))^2 + 3*d^3*cosh(f*x + cosh(1) + sinh(
1)) + d^3 + 3*(d^3*cosh(f*x + cosh(1) + sinh(1)) + d^3)*sinh(f*x + cosh(1)
+ sinh(1))^2 + 3*(d^3*cosh(f*x + cosh(1) + sinh(1))^2 + 2*d^3*cosh(f*x + co
sh(1) + sinh(1)) + d^3)*sinh(f*x + cosh(1) + sinh(1)))*polylog(3, -cosh(f*x
 + cosh(1) + sinh(1)) - sinh(f*x + cosh(1) + sinh(1))) - 3*(c^2*d*f^2 - 2*c
*d^2*f*cosh(1) + d^3*cosh(1)^2 - 2*d^3)*sinh(1) - 3*(d^3*f^2*x^2 - c^3*f^3
- 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh(1)^3 + c^2*d*f^2 + 4*c*d^2
*f + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 +
 d^3*sinh(1)^3 - 3*(c*d^2*f - d^3*cosh(1))*sinh(1)^2 + 3*(c^2*d*f^3 - 2*d^3
*f)*x + 3*(c^2*d*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) +
d^3*cosh(1)^2 - 2*d^3)*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - 3*(c*d^2*
f - d^3*cosh(1))*sinh(1)^2 + 2*(c*d^2*f^2 - d^3*f)*x + 3*(c^2*d*f^2 - 2*d^3
)*cosh(1) + 2*(d^3*f^3*x^3 - 3*c*d^2*f*cosh(1)^2 + d^3*cosh(1)^3 + d^3*sinh
(1)^3 + c^2*d*f^2 + 2*c*d^2*f + (3*c*d^2*f^3 + d^3*f^2)*x^2 - 3*(c*d^2*f -
d^3*cosh(1))*sinh(1)^2 + (3*c^2*d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x + 3*(c^2*d
*f^2 - 2*d^3)*cosh(1) + 3*(c^2*d*f^2 - 2*c*d^2*f*cosh(1) + d^3*cosh(1)^2 -
2*d^3)*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 3*(c^2*d*f^2 - 2*c*d^2*f*co
sh(1) + d^3*cosh(1)^2 - 2*d^3)*sinh(1))*sinh(f*x + cosh(1) + sinh(1)))/(a^2
*f^4*cosh(f*x + cosh(1) + sinh(1))^3 + a^2*f^4*sinh(f*x + cosh(1) + sinh(1)
)^3 + 3*a^2*f^4*cosh(f*x + cosh(1) + sinh(1))^2...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] (Integral(c**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**3
*x**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c*d**2*x**2
/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cosh(e
 + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{(a+a\cosh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + a*cosh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^3/(a + a*cosh(e + f*x))^2, x)
```

3.117 $\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=200

$$\frac{(c+dx)^2}{3a^2f}-\frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2}-\frac{4d^2\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{3a^2f^3}+\frac{d(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}-\frac{2d^2\tanh\left(\frac{e}{2}+\ldots\right.}{3a^2f^3}$$

```
[Out] 1/3*(d*x+c)^2/a^2/f-4/3*d*(d*x+c)*ln(1+exp(f*x+e))/a^2/f^2-4/3*d^2*polylog(
2,-exp(f*x+e))/a^2/f^3+1/3*d*(d*x+c)*sech(1/2*f*x+1/2*e)^2/a^2/f^2-2/3*d^2*
tanh(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^2*tanh(1/2*f*x+1/2*e)/a^2/f+1/6*(d*
x+c)^2*sech(1/2*f*x+1/2*e)^2*tanh(1/2*f*x+1/2*e)/a^2/f
```

Rubi [A]
time = 0.18, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3399, 4271, 3852, 8, 4269, 3799, 2221, 2317, 2438}

$$-\frac{4d(c+dx)\log\left(e^{e+fx}+1\right)}{3a^2f^2}+\frac{d(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{(c+dx)^2}{3a^2f}-\frac{4d^2\mathrm{Li}_2\left(-e^{e+fx}\right)}{3a^2f^3}-\frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (c + d*x)^2/(3*a^2*f) - (4*d*(c + d*x)*Log[1 + E^(e + f*x)])/(3*a^2*f^2) -
(4*d^2*PolyLog[2, -E^(e + f*x)])/(3*a^2*f^3) + (d*(c + d*x)*Sech[e/2 + (f*x
)/2]^2)/(3*a^2*f^2) - (2*d^2*Tanh[e/2 + (f*x)/2])/(3*a^2*f^3) + ((c + d*x)^
2*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)^2*Sech[e/2 + (f*x)/2]^2*Tanh[
e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3799

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Dist[2*I, Int[(
c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
/; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)^2 \csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{4a^2}$$

$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+d}{}$$

$$= \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right.}{6a^2f}$$

$$= \frac{(c+dx)^2}{3a^2f} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh}{3a^2f}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tan}{3}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} + \frac{d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tan}{3}$$

$$= \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\text{Li}_2\left(-e^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\text{sech}^2}{3a^2f^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.27, size = 637, normalized size = 3.18

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (-16*c*d*Cosh[e/2 + (f*x)/2]^4*Sech[e/2]*(Cosh[e/2]*Log[Cosh[e/2]*Cosh[(f*x
)/2] + Sinh[e/2]*Sinh[(f*x)/2]] - (f*x*Sinh[e/2])/2))/(3*f^2*(a + a*Cosh[e
+ f*x])^2*(Cosh[e/2]^2 - Sinh[e/2]^2)) - (16*d^2*Cosh[e/2 + (f*x)/2]^4*Csch
[e/2]*((f^2*x^2)/(4*E^ArcTanh[Coth[e/2]]) - (I*Coth[e/2]*(-1/2*(f*x*(-Pi +
(2*I)*ArcTanh[Coth[e/2]])) - Pi*Log[1 + E^(f*x)] - 2*((I/2)*f*x + I*ArcTanh
[Coth[e/2]])*Log[1 - E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))] + Pi*Log
[Cosh[(f*x)/2]] + (2*I)*ArcTanh[Coth[e/2]]*Log[I*Sinh[(f*x)/2 + ArcTanh[Cot
h[e/2]]]] + I*PolyLog[2, E^((2*I)*((I/2)*f*x + I*ArcTanh[Coth[e/2]]))]))/Sq
rt[1 - Coth[e/2]^2])*Sech[e/2])/(3*f^3*(a + a*Cosh[e + f*x])^2*Sqrt[Csch[e/
2]^2*(-Cosh[e/2]^2 + Sinh[e/2]^2)]) + (Cosh[e/2 + (f*x)/2]*Sech[e/2]*(2*c*d
*f*Cosh[(f*x)/2] + 2*d^2*f*x*Cosh[(f*x)/2] + 2*c*d*f*Cosh[e + (f*x)/2] + 2*
d^2*f*x*Cosh[e + (f*x)/2] - 4*d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] +
 6*c*d*f^2*x*Sinh[(f*x)/2] + 3*d^2*f^2*x^2*Sinh[(f*x)/2] + 2*d^2*Sinh[e + (
f*x)/2] - 2*d^2*Sinh[e + (3*f*x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f
```

```
^2*x*Sinh[e + (3*f*x)/2] + d^2*f^2*x^2*Sinh[e + (3*f*x)/2]))/(3*f^3*(a + a*
Cosh[e + f*x])^2)
```

Maple [A]
time = 1.60, size = 313, normalized size = 1.56

method	result
risch	$-\frac{2\left(3 f^2 d^2 x^2 \mathrm{e}^{f x+e}+6 f^2 c d x\,\mathrm{e}^{f x+e}+d^2 x^2 f^2-2 d^2 f x\,\mathrm{e}^{2 f x+2 e}+3 f^2 c^2 \mathrm{e}^{f x+e}+2 c d\, f^2 x-2 c d f\,\mathrm{e}^{2 f x+2 e}-2 f\, d^2 x\,\mathrm{e}^{f x+e}+c^2 f^2-2 f c d\,\mathrm{e}^{f x+e}\right.}{3 f^3 a^2\left(\mathrm{e}^{f x+e}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] -2/3*(3*f^2*d^2*x^2*exp(f*x+e)+6*f^2*c*d*x*exp(f*x+e)+d^2*x^2*f^2-2*d^2*f*x
*exp(2*f*x+2*e)+3*f^2*c^2*exp(f*x+e)+2*c*d*f^2*x-2*c*d*f*exp(2*f*x+2*e)-2*f
*d^2*x*exp(f*x+e)+c^2*f^2-2*f*c*d*exp(f*x+e)-2*d^2*exp(2*f*x+2*e)-4*d^2*exp
(f*x+e)-2*d^2)/f^3/a^2/(exp(f*x+e)+1)^3-4/3/a^2*d/f^2*c*ln(exp(f*x+e)+1)+4/
3/a^2*d/f^2*c*ln(exp(f*x+e))+2/3/a^2*d^2/f*x^2+4/3/a^2*d^2/f^2*e*x+2/3/a^2*
d^2/f^3*e^2-4/3/a^2*d^2/f^2*ln(exp(f*x+e)+1)*x-4/3*d^2*polylog(2,-exp(f*x+e
))/a^2/f^3-4/3/a^2*d^2/f^3*e*ln(exp(f*x+e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*d^2*((f^2*x^2 - 2*(f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + (3*f^2*x^2*e^e -
 2*f*x*e^e - 4*e^e)*e^(f*x) - 2)/(a^2*f^3*e^(3*f*x + 3*e) + 3*a^2*f^3*e^(2*
f*x + 2*e) + 3*a^2*f^3*e^(f*x + e) + a^2*f^3) - 6*integrate(1/3*x/(a^2*f*e^
(f*x + e) + a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) +
e^(2*e))*e^(2*f*x) + e^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2
*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-
e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2
*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a
^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1347 vs. $2(170) = 340$.
time = 0.45, size = 1347, normalized size = 6.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -2/3*(c^2*f^2 - 2*c*d*f*cosh(1) + d^2*cosh(1)^2 - (d^2*f^2*x^2 + 2*c*d*f^2*
x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*f - d^2*cosh(1
))*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^3 + d^2*sinh(1)^2 - (d^2*f^2*x^2
+ 2*c*d*f^2*x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*f
- d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^3 - (3*d^2*f^2*x^2 +
6*c*d*f*cosh(1) - 3*d^2*cosh(1)^2 - 3*d^2*sinh(1)^2 + 2*c*d*f + 2*d^2 + 2*(
3*c*d*f^2 + d^2*f)*x + 6*(c*d*f - d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1)
+ sinh(1))^2 - (3*d^2*f^2*x^2 + 6*c*d*f*cosh(1) - 3*d^2*cosh(1)^2 - 3*d^2*s
inh(1)^2 + 2*c*d*f + 2*d^2 + 2*(3*c*d*f^2 + d^2*f)*x + 3*(d^2*f^2*x^2 + 2*c
*d*f^2*x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*f - d^2
*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 6*(c*d*f - d^2*cosh(1))*
sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*d^2 + (3*c^2*f^2 - 2*d^2*f*x -
 6*c*d*f*cosh(1) + 3*d^2*cosh(1)^2 + 3*d^2*sinh(1)^2 - 2*c*d*f - 4*d^2 - 6*
(c*d*f - d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 2*(d^2*cosh(
f*x + cosh(1) + sinh(1))^3 + d^2*sinh(f*x + cosh(1) + sinh(1))^3 + 3*d^2*co
sh(f*x + cosh(1) + sinh(1))^2 + 3*d^2*cosh(f*x + cosh(1) + sinh(1)) + 3*(d^
2*cosh(f*x + cosh(1) + sinh(1)) + d^2)*sinh(f*x + cosh(1) + sinh(1))^2 + d^
2 + 3*(d^2*cosh(f*x + cosh(1) + sinh(1))^2 + 2*d^2*cosh(f*x + cosh(1) + sin
h(1)) + d^2)*sinh(f*x + cosh(1) + sinh(1)))*dilog(-cosh(f*x + cosh(1) + sin
h(1)) - sinh(f*x + cosh(1) + sinh(1))) + 2*(d^2*f*x + (d^2*f*x + c*d*f)*cos
h(f*x + cosh(1) + sinh(1))^3 + (d^2*f*x + c*d*f)*sinh(f*x + cosh(1) + sinh(
1))^3 + c*d*f + 3*(d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1))^2 + 3*(d^
2*f*x + c*d*f + (d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x +
 cosh(1) + sinh(1))^2 + 3*(d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1)) +
 3*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1))^2 + 2
*(d^2*f*x + c*d*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh
(1)))*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x + cosh(1) + sinh(1)) + 1
) - 2*(c*d*f - d^2*cosh(1))*sinh(1) + (3*c^2*f^2 - 2*d^2*f*x - 6*c*d*f*cosh
(1) + 3*d^2*cosh(1)^2 + 3*d^2*sinh(1)^2 - 2*c*d*f - 3*(d^2*f^2*x^2 + 2*c*d*
f^2*x + 2*c*d*f*cosh(1) - d^2*cosh(1)^2 - d^2*sinh(1)^2 + 2*(c*d*f - d^2*co
sh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - 4*d^2 - 2*(3*d^2*f^2*x^2
+ 6*c*d*f*cosh(1) - 3*d^2*cosh(1)^2 - 3*d^2*sinh(1)^2 + 2*c*d*f + 2*d^2 + 2
*(3*c*d*f^2 + d^2*f)*x + 6*(c*d*f - d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1
) + sinh(1)) - 6*(c*d*f - d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1) + sinh(1
)))/(a^2*f^3*cosh(f*x + cosh(1) + sinh(1))^3 + a^2*f^3*sinh(f*x + cosh(1) +
 sinh(1))^3 + 3*a^2*f^3*cosh(f*x + cosh(1) + sinh(1))^2 + 3*a^2*f^3*cosh(f*
x + cosh(1) + sinh(1)) + a^2*f^3 + 3*(a^2*f^3*cosh(f*x + cosh(1) + sinh(1))
 + a^2*f^3)*sinh(f*x + cosh(1) + sinh(1))^2 + 3*(a^2*f^3*cosh(f*x + cosh(1)
 + sinh(1))^2 + 2*a^2*f^3*cosh(f*x + cosh(1) + sinh(1)) + a^2*f^3)*sinh(f*x
 + cosh(1) + sinh(1)))
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{c^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{2cdx}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] (Integral(c**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**2
*x**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(2*c*d*x/(cosh
(e + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^2/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{(a+a\cosh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + a*cosh(e + f*x))^2,x)
```

```
[Out] int((c + d*x)^2/(a + a*cosh(e + f*x))^2, x)
```

3.118 $\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=123

$$-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}+\frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

```
[Out] -2/3*d*ln(cosh(1/2*f*x+1/2*e))/a^2/f^2+1/6*d*sech(1/2*f*x+1/2*e)^2/a^2/f^2+
1/3*(d*x+c)*tanh(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)*sech(1/2*f*x+1/2*e)^2*tan
h(1/2*f*x+1/2*e)/a^2/f
```

Rubi [A]
time = 0.07, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3399, 4270, 4269, 3556}

$$\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}-\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (-2*d*Log[Cosh[e/2 + (f*x)/2]])/(3*a^2*f^2) + (d*Sech[e/2 + (f*x)/2]^2)/(6*
a^2*f^2) + ((c + d*x)*Tanh[e/2 + (f*x)/2])/(3*a^2*f) + ((c + d*x)*Sech[e/2
+ (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(6*a^2*f)
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3556

```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 4269

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*
Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int (c+dx)\csc^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx}{4a^2}$$
$$= \frac{d\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{\int (c+dx)\mathrm{sech}^2\left(\frac{e}{2}\right.}{6a^2}$$
$$= \frac{d\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\right.}{6a^2f}$$
$$= -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2} + \frac{d\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} + \frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+}{}$$

Mathematica [A]
time = 0.29, size = 114, normalized size = 0.93

$$\frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2d\cosh\left(\frac{3}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+\cosh\left(\frac{1}{2}(e+fx)\right)\left(2d-6d\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)\right)+f(c+dx)\left(3\sinh\left(\frac{1}{2}(e+fx)\right)+\sinh\left(\frac{3}{2}(e+fx)\right)\right)\right)}{3a^2f^2(1+\cosh(e+fx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (Cosh[(e + f*x)/2]*(-2*d*Cosh[(3*(e + f*x))/2]*Log[Cosh[(e + f*x)/2]] + Cos
h[(e + f*x)/2]*(2*d - 6*d*Log[Cosh[(e + f*x)/2]]) + f*(c + d*x)*(3*Sinh[(e
+ f*x)/2] + Sinh[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cosh[e + f*x])^2)
```

Maple [A]
time = 1.57, size = 108, normalized size = 0.88

method	result	size
risch	$\frac{2dx}{3a^2f}+\frac{2de}{3a^2f^2}-\frac{2\left(3dfx\,\mathrm{e}^{fx+e}+3cf\,\mathrm{e}^{fx+e}+dxf-d\,\mathrm{e}^{2fx+2e}+cf-d\,\mathrm{e}^{fx+e}\right)}{3f^2a^2\left(\mathrm{e}^{fx+e}+1\right)^3}-\frac{2d\ln\left(\mathrm{e}^{fx+e}+1\right)}{3a^2f^2}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*d/a^2/f*x+2/3*d/a^2/f^2*e-2/3*(3*d*f*x*exp(f*x+e)+3*c*f*exp(f*x+e)+d*x*
f-d*exp(2*f*x+2*e)+c*f-d*exp(f*x+e))/f^2/a^2/(exp(f*x+e)+1)^3-2/3*d/a^2/f^2
*ln(exp(f*x+e)+1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(100) = 200$.
time = 0.29, size = 255, normalized size = 2.07

$$\frac{2}{3}\,d\left(\frac{fxe^{(3\,fx+3\,e)} + \left(3\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)} + e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)} + 3\,a^2f^2e^{(2\,fx+2\,e)} + 3\,a^2f^2e^{(fx+e)} + a^2f^2} - \frac{\log\left(\left(e^{(fx+e)} + 1\right)e^{(-e)}\right)}{a^2f^2}\right) + \frac{2}{3}\,c\left(\frac{3\,e^{(-fx-e)}}{(3\,a^2e^{(-fx-e)} + 3\,a^2e^{(-2\,fx-2\,e)} + a^2e^{(-3\,fx-3\,e)} + a^2)f} + \frac{1}{(3\,a^2e^{(-fx-e)} + 3\,a^2e^{(-2\,fx-2\,e)} + a^2e^{(-3\,fx-3\,e)} + a^2)f}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2/3*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f*x
+ e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f
*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c*(3*e^
(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x -
3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-
3*f*x - 3*e) + a^2)*f))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(100) = 200$.
time = 0.38, size = 472, normalized size = 3.84

$$\frac{2\left(dfx\cosh\left(fx + \cosh(1) + \sinh(1)\right)^3 + dfx\sinh\left(fx + \cosh(1) + \sinh(1)\right)^3 + (3\,dfx + d)\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 + \left(3\,dfx\cosh\left(fx + \cosh(1) + \sinh(1)\right) + 3\,dfx + d\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right)^2 - cf - (3\,cf - d)\cosh\left(fx + \cosh(1) + \sinh(1)\right) - \left(d\cosh\left(fx + \cosh(1) + \sinh(1)\right)^3 + d\sinh\left(fx + \cosh(1) + \sinh(1)\right)^3 + 3\,d\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 3\left(d\cosh\left(fx + \cosh(1) + \sinh(1)\right) + d\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 3\,d\cosh\left(fx + \cosh(1) + \sinh(1)\right) + 3\left(d\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 2\,d\cosh\left(fx + \cosh(1) + \sinh(1)\right) + d\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right) + d\right)\log\left(\cosh\left(fx + \cosh(1) + \sinh(1)\right) + \sinh\left(fx + \cosh(1) + \sinh(1)\right) + 1\right) + \left(3\,dfx\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 - 3\,cf + 2\,(3\,dfx + d)\cosh\left(fx + \cosh(1) + \sinh(1)\right) + d\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right)\right)}{3\left(a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right)^3 + a^2f^2\sinh\left(fx + \cosh(1) + \sinh(1)\right)^3 + 3\,a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 3\,a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right) + a^2f^2 + 3\left(a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right) + a^2f^2\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 3\left(a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right)^2 + 2\,a^2f^2\cosh\left(fx + \cosh(1) + \sinh(1)\right) + a^2f^2\right)\sinh\left(fx + \cosh(1) + \sinh(1)\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 2/3*(d*f*x*cosh(f*x + cosh(1) + sinh(1))^3 + d*f*x*sinh(f*x + cosh(1) + sin
h(1))^3 + (3*d*f*x + d)*cosh(f*x + cosh(1) + sinh(1))^2 + (3*d*f*x*cosh(f*x
 + cosh(1) + sinh(1)) + 3*d*f*x + d)*sinh(f*x + cosh(1) + sinh(1))^2 - c*f
- (3*c*f - d)*cosh(f*x + cosh(1) + sinh(1)) - (d*cosh(f*x + cosh(1) + sinh(
1))^3 + d*sinh(f*x + cosh(1) + sinh(1))^3 + 3*d*cosh(f*x + cosh(1) + sinh(1
))^2 + 3*(d*cosh(f*x + cosh(1) + sinh(1)) + d)*sinh(f*x + cosh(1) + sinh(1)
)^2 + 3*d*cosh(f*x + cosh(1) + sinh(1)) + 3*(d*cosh(f*x + cosh(1) + sinh(1)
)^2 + 2*d*cosh(f*x + cosh(1) + sinh(1)) + d)*sinh(f*x + cosh(1) + sinh(1))
+ d)*log(cosh(f*x + cosh(1) + sinh(1)) + sinh(f*x + cosh(1) + sinh(1)) + 1)
 + (3*d*f*x*cosh(f*x + cosh(1) + sinh(1))^2 - 3*c*f + 2*(3*d*f*x + d)*cosh(
f*x + cosh(1) + sinh(1)) + d)*sinh(f*x + cosh(1) + sinh(1)))/(a^2*f^2*cosh(
f*x + cosh(1) + sinh(1))^3 + a^2*f^2*sinh(f*x + cosh(1) + sinh(1))^3 + 3*a^
2*f^2*cosh(f*x + cosh(1) + sinh(1))^2 + 3*a^2*f^2*cosh(f*x + cosh(1) + sinh
(1)) + a^2*f^2 + 3*(a^2*f^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*f^2)*sinh(f
*x + cosh(1) + sinh(1))^2 + 3*(a^2*f^2*cosh(f*x + cosh(1) + sinh(1))^2 + 2*
a^2*f^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*f^2)*sinh(f*x + cosh(1) + sinh(
1)))
```

Sympy [A]

time = 0.53, size = 156, normalized size = 1.27

$$\begin{cases} -\frac{c\tanh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}-\frac{dx\tanh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}+\frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f}-\frac{dx}{3a^2f}+\frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^2f^2}-\frac{d\tanh^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2} & \text{for } f\neq 0 \\ \frac{cx+\frac{dx^2}{2}}{(a\cosh(e)+a)^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((-c*tanh(e/2 + f*x/2)**3/(6*a**2*f) + c*tanh(e/2 + f*x/2)/(2*a**2
*f) - d*x*tanh(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tanh(e/2 + f*x/2)/(2*a**2*f
) - d*x/(3*a**2*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(3*a**2*f**2) - d*tanh(
e/2 + f*x/2)**2/(6*a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cosh(e) + a)
**2, True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(95) = 190.
time = 0.40, size = 192, normalized size = 1.56

$$\frac{2\left(dfxe^{(3fx+3e)}+3dfxe^{(2fx+2e)}-3cfe^{(fx+e)}-de^{(3fx+3e)}\log\left(e^{(fx+e)}+1\right)-3de^{(2fx+2e)}\log\left(e^{(fx+e)}+1\right)-3de^{(fx+e)}\log\left(e^{(fx+e)}+1\right)-cf+de^{(2fx+2e)}+de^{(fx+e)}-d\log\left(e^{(fx+e)}+1\right)\right)}{3\left(a^2f^2e^{(3fx+3e)}+3a^2f^2e^{(2fx+2e)}+3a^2f^2e^{(fx+e)}+a^2f^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 2/3*(d*f*x*e^(3*f*x + 3*e) + 3*d*f*x*e^(2*f*x + 2*e) - 3*c*f*e^(f*x + e) -
d*e^(3*f*x + 3*e)*log(e^(f*x + e) + 1) - 3*d*e^(2*f*x + 2*e)*log(e^(f*x + e
) + 1) - 3*d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f + d*e^(2*f*x + 2*e) + d
*e^(f*x + e) - d*log(e^(f*x + e) + 1))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2
*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2)
```

Mupad [B]
time = 0.89, size = 138, normalized size = 1.12

$$\frac{2\,d}{3\,a^2\,f^2\,\left({\mathrm{e}}^{e+f\,x}+1\right)}-\frac{2\,\left(d+c\,f+d\,f\,x\right)}{3\,a^2\,f^2\,\left(2\,{\mathrm{e}}^{e+f\,x}+{\mathrm{e}}^{2\,e+2\,f\,x}+1\right)}+\frac{2\,d\,x}{3\,a^2\,f}-\frac{2\,d\,\ln\left({\mathrm{e}}^{f\,x}\,{\mathrm{e}}^{e}+1\right)}{3\,a^2\,f^2}-\frac{4\,{\mathrm{e}}^{e+f\,x}\,\left(c+d\,x\right)}{3\,a^2\,f\,\left(3\,{\mathrm{e}}^{e+f\,x}+3\,{\mathrm{e}}^{2\,e+2\,f\,x}+{\mathrm{e}}^{3\,e+3\,f\,x}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + a*cosh(e + f*x))^2,x)
```

```
[Out] (2*d)/(3*a^2*f^2*(exp(e + f*x) + 1)) - (2*(d + c*f + d*f*x))/(3*a^2*f^2*(2*
exp(e + f*x) + exp(2*e + 2*f*x) + 1)) + (2*d*x)/(3*a^2*f) - (2*d*log(exp(f*
x)*exp(e) + 1))/(3*a^2*f^2) - (4*exp(e + f*x)*(c + d*x))/(3*a^2*f*(3*exp(e
+ f*x) + 3*exp(2*e + 2*f*x) + exp(3*e + 3*f*x) + 1))
```

3.119 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+a\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Mathematica [A]
time = 21.27, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\,(a+a\cosh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 2*d^2 + (d^2*f*x*e^(2*e) + (c*d
*f - 2*d^2)*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + (6*c*d*f^2 + d^2*f)*x
*e^e + (3*c^2*f^2 + c*d*f - 4*d^2)*e^e)*e^(f*x))/(a^2*d^3*f^3*x^3 + 3*a^2*c
*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d^3*f^3*x^3*e^(3*e) +
 3*a^2*c*d^2*f^3*x^2*e^(3*e) + 3*a^2*c^2*d*f^3*x*e^(3*e) + a^2*c^3*f^3*e^(3
*e))*e^(3*f*x) + 3*(a^2*d^3*f^3*x^3*e^(2*e) + 3*a^2*c*d^2*f^3*x^2*e^(2*e) +
 3*a^2*c^2*d*f^3*x*e^(2*e) + a^2*c^3*f^3*e^(2*e))*e^(2*f*x) + 3*(a^2*d^3*f^
3*x^3*e^e + 3*a^2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f^3*x*e^e + a^2*c^3*f^3*e
^e)*e^(f*x)) - integrate(2/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 6*d
^3)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*
c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e
+ 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f
*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cosh(f*x + e)^2 + 2*(a^2*d*
x + a^2*c)*cosh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c\cosh^2(e+fx)+2c\cosh(e+fx)+c+dx\cosh^2(e+fx)+2dx\cosh(e+fx)+dx}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Integral(1/(c*cosh(e + f*x)**2 + 2*c*cosh(e + f*x) + c + d*x*cosh(e + f*x)*
*2 + 2*d*x*cosh(e + f*x) + d*x), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\mathrm{cosh}\left(e+f\,x\right)\right)}^2\,\left(c+d\,x\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)),x)
[Out] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)), x)
```

3.120 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Mathematica [A]
time = 21.73, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+a\cosh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 6*d^2 + 2*(d^2*f*x*e^(2*e) + (c
*d*f - 3*d^2)*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + 2*(3*c*d*f^2 + d^2*
f)*x*e^e + (3*c^2*f^2 + 2*c*d*f - 12*d^2)*e^e)*e^(f*x))/(a^2*d^4*f^3*x^4 +
4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f
^3 + (a^2*d^4*f^3*x^4*e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^(3*e) + 6*a^2*c^2*d^2
*f^3*x^2*e^(3*e) + 4*a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^4*f^3*e^(3*e))*e^(3*f*
x) + 3*(a^2*d^4*f^3*x^4*e^(2*e) + 4*a^2*c*d^3*f^3*x^3*e^(2*e) + 6*a^2*c^2*d
^2*f^3*x^2*e^(2*e) + 4*a^2*c^3*d*f^3*x*e^(2*e) + a^2*c^4*f^3*e^(2*e))*e^(2*
f*x) + 3*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3
*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^e)*e^(f*x)) - integrate(4/
3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*d^3)/(a^2*d^5*f^3*x^5 + 5*a
^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*a^2*c^3*d^2*f^3*x^2 + 5*a^2*
c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5*e^e + 5*a^2*c*d^4*f^3*x^4*e^e
+ 10*a^2*c^2*d^3*f^3*x^3*e^e + 10*a^2*c^3*d^2*f^3*x^2*e^e + 5*a^2*c^4*d*f^3
*x*e^e + a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d*
x + a^2*c^2)*cosh(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*cosh
(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{1}{c^2\cosh^2(e+fx)+2c^2\cosh(e+fx)+c^2+2cdx\cosh^2(e+fx)+4cdx\cosh(e+fx)+2cdx+d^2x^2\cosh^2(e+fx)+2d^2x^2\cosh(e+fx)+d^2x^2}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Integral(1/(c**2*cosh(e + f*x)**2 + 2*c**2*cosh(e + f*x) + c**2 + 2*c*d*x*c
osh(e + f*x)**2 + 4*c*d*x*cosh(e + f*x) + 2*c*d*x + d**2*x**2*cosh(e + f*x)
**2 + 2*d**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+a\,\mathrm{cosh}\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2), x)
```

3.121 $\int x^3 \sqrt{a + a\cosh(c + dx)}\, dx$

Optimal. Leaf size=110

$$-\frac{96\sqrt{a+a\cosh(c+dx)}}{d^4}-\frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2}+\frac{48x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3}+\frac{2x^3\sqrt{a}}{}$$

```
[Out] -96*(a+a*cosh(d*x+c))^(1/2)/d^4-12*x^2*(a+a*cosh(d*x+c))^(1/2)/d^2+48*x*(a+
a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*cosh(d*x+c))^(1/2)*
tanh(1/2*d*x+1/2*c)/d
```

Rubi [A]

time = 0.11, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2718}

$$-\frac{96\sqrt{a\cosh(c+dx)+a}}{d^4}+\frac{48x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d^3}-\frac{12x^2\sqrt{a\cosh(c+dx)+a}}{d^2}+\frac{2x^3\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-96*Sqrt[a + a*Cosh[c + d*x]])/d^4 - (12*x^2*Sqrt[a + a*Cosh[c + d*x]])/d^
2 + (48*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d^3 + (2*x^3*Sqrt[
a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^3\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right.\right.$$
$$= \frac{2x^3\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(6\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\right.\right.}{}$$
$$= -\frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x^3\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} +$$
$$= -\frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{48x\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} +$$
$$= -\frac{96\sqrt{a+a\cosh(c+dx)}}{d^4} - \frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{48x\sqrt{a+a\cosh}}{}$$

Mathematica [A]
time = 0.15, size = 53, normalized size = 0.48

$$\frac{2\sqrt{a(1+\cosh(c+dx))}\left(-6(8+d^2x^2)+dx(24+d^2x^2)\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-6*(8 + d^2*x^2) + d*x*(24 + d^2*x^2)*Tanh[
(c + d*x)/2]))/d^4
```

Maple [A]
time = 0.80, size = 108, normalized size = 0.98

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^2 e^{-dx-c}}\,\left(d^3x^3e^{dx+c}-d^3x^3-6d^2x^2e^{dx+c}-6d^2x^2+24dx\,e^{dx+c}-24dx-48\,e^{dx+c}-48\right)}{\left(e^{dx+c}+1\right)d^4}$	108

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^3*x^3*exp(
d*x+c)-d^3*x^3-6*d^2*x^2*exp(d*x+c)-6*d^2*x^2+24*d*x*exp(d*x+c)-24*d*x-48*e
xp(d*x+c)-48)/d^4
```

Maxima [A]
time = 0.48, size = 120, normalized size = 1.09

$$-\frac{\left(\sqrt{2}\,\sqrt{a}\,d^3x^3+6\,\sqrt{2}\,\sqrt{a}\,d^2x^2+24\,\sqrt{2}\,\sqrt{a}\,dx-\left(\sqrt{2}\,\sqrt{a}\,d^3x^3e^c-6\,\sqrt{2}\,\sqrt{a}\,d^2x^2e^c+24\,\sqrt{2}\,\sqrt{a}\,dxe^c-48\,\sqrt{2}\,\sqrt{a}\,e^c\right)e^{(dx)}+48\,\sqrt{2}\,\sqrt{a}\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*d^3*x^3 + 6*sqrt(2)*sqrt(a)*d^2*x^2 + 24*sqrt(2)*sqrt(a)*
d*x - (sqrt(2)*sqrt(a)*d^3*x^3*e^c - 6*sqrt(2)*sqrt(a)*d^2*x^2*e^c + 24*sqr
t(2)*sqrt(a)*d*x*e^c - 48*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 48*sqrt(2)*sqrt(a)
)*e^(-1/2*d*x - 1/2*c)/d^4
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [**A**]
time = 0.40, size = 147, normalized size = 1.34

$$\frac{\sqrt{2}\left(\sqrt{a}\,d^{3}x^{3}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}\,d^{3}x^{3}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-6\sqrt{a}\,d^{2}x^{2}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-6\sqrt{a}\,d^{2}x^{2}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+24\sqrt{a}\,dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-24\sqrt{a}\,dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-48\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-48\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d^3*x^3*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^3*x^3*e^(-1/2*d*x
- 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(-1/
2*d*x - 1/2*c) + 24*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 24*sqrt(a)*d*x*e^(-1/
2*d*x - 1/2*c) - 48*sqrt(a)*e^(1/2*d*x + 1/2*c) - 48*sqrt(a)*e^(-1/2*d*x -
1/2*c))/d^4
```

Mupad [**B**]
time = 0.21, size = 117, normalized size = 1.06

$$-\frac{\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{c+d\,x}}{2}+\frac{{\mathrm{e}}^{-c-d\,x}}{2}\right)}\,\left(\frac{96\,{\mathrm{e}}^{c+d\,x}}{d^4}+\frac{48\,x}{d^3}+\frac{96}{d^4}+\frac{2\,x^3}{d}+\frac{12\,x^2}{d^2}-\frac{2\,x^3\,{\mathrm{e}}^{c+d\,x}}{d}+\frac{12\,x^2\,{\mathrm{e}}^{c+d\,x}}{d^2}-\frac{48\,x\,{\mathrm{e}}^{c+d\,x}}{d^3}\right)}{{\mathrm{e}}^{c+d\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] -((a + a*(exp(c + d*x)/2 + exp(- c - d*x)/2))^(1/2)*((96*exp(c + d*x))/d^4
+ (48*x)/d^3 + 96/d^4 + (2*x^3)/d + (12*x^2)/d^2 - (2*x^3*exp(c + d*x))/d +
 (12*x^2*exp(c + d*x))/d^2 - (48*x*exp(c + d*x))/d^3))/(exp(c + d*x) + 1)
```

3.122 $\int x^2 \sqrt{a + a\cosh(c + dx)}\, dx$

Optimal. Leaf size=88

$$-\frac{8x\sqrt{a + a\cosh(c + dx)}}{d^2} + \frac{16\sqrt{a + a\cosh(c + dx)}\,\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d^3} + \frac{2x^2\sqrt{a + a\cosh(c + dx)}\,\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)}{d}$$

```
[Out] -8*x*(a+a*cosh(d*x+c))^(1/2)/d^2+16*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/
2*c)/d^3+2*x^2*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/2*c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3400, 3377, 2717}

$$\frac{16\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cosh(c + dx) + a}}{d^3} - \frac{8x\sqrt{a\cosh(c + dx) + a}}{d^2} + \frac{2x^2\tanh\left(\frac{c}{2} + \frac{dx}{2}\right)\sqrt{a\cosh(c + dx) + a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-8*x*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (16*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c
/2 + (d*x)/2])/d^3 + (2*x^2*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/
d
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^2\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right.\right.$$

$$= \frac{2x^2\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(4\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\right.\right.}{}$$

$$= -\frac{8x\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x^2\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} + \frac{(8}{}$$

$$= -\frac{8x\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{16\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^2}{}$$

Mathematica [A]
time = 0.11, size = 44, normalized size = 0.50

$$\frac{2\sqrt{a(1+\cosh(c+dx))}\left(-4dx+(8+d^2x^2)\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-4*d*x + (8 + d^2*x^2)*Tanh[(c + d*x)/2]))/
d^3
```

Maple [A]
time = 0.48, size = 86, normalized size = 0.98

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^2 e^{-dx-c}}\,\left(d^2x^2 e^{dx+c}-d^2x^2-4dx\,e^{dx+c}-4dx+8\,e^{dx+c}-8\right)}{\left(e^{dx+c}+1\right)d^3}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^2*x^2*exp(
d*x+c)-d^2*x^2-4*d*x*exp(d*x+c)-4*d*x+8*exp(d*x+c)-8)/d^3
```

Maxima [A]
time = 0.48, size = 90, normalized size = 1.02

$$-\frac{\left(\sqrt{2}\,\sqrt{a}\,d^2x^2+4\,\sqrt{2}\,\sqrt{a}\,dx-\left(\sqrt{2}\,\sqrt{a}\,d^2x^2e^c-4\,\sqrt{2}\,\sqrt{a}\,dxe^c+8\,\sqrt{2}\,\sqrt{a}\,e^c\right)e^{(dx)}+8\,\sqrt{2}\,\sqrt{a}\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*d^2*x^2 + 4*sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d^2*x^
2*e^c - 4*sqrt(2)*sqrt(a)*d*x*e^c + 8*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 8*sqrt
(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^3
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [**A**]
time = 0.41, size = 107, normalized size = 1.22

$$\frac{\sqrt{2}\left(\sqrt{a}\,d^2x^2e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}\,d^2x^2e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-4\,\sqrt{a}\,dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-4\,\sqrt{a}\,dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+8\,\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-8\,\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^2*x^2*e^(-1/2*d*x
- 1/2*c) - 4*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*d*x*e^(-1/2*d*x -
1/2*c) + 8*sqrt(a)*e^(1/2*d*x + 1/2*c) - 8*sqrt(a)*e^(-1/2*d*x - 1/2*c))/d^
3
```

Mupad [**B**]
time = 0.93, size = 95, normalized size = 1.08

$$-\frac{\sqrt{a+a\,\left(\frac{{\mathrm{e}}^{c+d\,x}}{2}+\frac{{\mathrm{e}}^{-c-d\,x}}{2}\right)}\,\left(\frac{8\,x}{d^2}-\frac{16\,{\mathrm{e}}^{c+d\,x}}{d^3}+\frac{16}{d^3}+\frac{2\,x^2}{d}-\frac{2\,x^2\,{\mathrm{e}}^{c+d\,x}}{d}+\frac{8\,x\,{\mathrm{e}}^{c+d\,x}}{d^2}\right)}{{\mathrm{e}}^{c+d\,x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] -((a + a*(exp(c + d*x)/2 + exp(- c - d*x)/2))^(1/2)*((8*x)/d^2 - (16*exp(c
+ d*x))/d^3 + 16/d^3 + (2*x^2)/d - (2*x^2*exp(c + d*x))/d + (8*x*exp(c + d*
x))/d^2))/(exp(c + d*x) + 1)
```

3.123 $\int x\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{4\sqrt{a+a\cosh(c+dx)}}{d^2}+\frac{2x\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

```
[Out] -4*(a+a*cosh(d*x+c))^(1/2)/d^2+2*x*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/2
*c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3400, 3377, 2718}

$$\frac{2x\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{d}-\frac{4\sqrt{a\cosh(c+dx)+a}}{d^2}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (-4*Sqrt[a + a*Cosh[c + d*x]])/d^2 + (2*x*Sqrt[a + a*Cosh[c + d*x]]*Tanh[c/
2 + (d*x)/2])/d
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)\right.$$

$$= \frac{2x\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} - \frac{\left(2\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}(ic\right.\right.}{}$$

$$= -\frac{4\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

Mathematica [A]
time = 0.08, size = 34, normalized size = 0.64

$$\frac{2\sqrt{a(1+\cosh(c+dx))}\left(-2+dx\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cosh[c + d*x]],x]
[Out] (2*Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*Tanh[(c + d*x)/2]))/d^2
```

Maple [A]
time = 0.48, size = 64, normalized size = 1.21

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^2 e^{-dx-c}}\,\left(dx\,e^{dx+c}-dx-2\,e^{dx+c}-2\right)}{\left(e^{dx+c}+1\right)d^2}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d*x*exp(d*x+
c)-d*x-2*exp(d*x+c)-2)/d^2
```

Maxima [A]
time = 0.48, size = 60, normalized size = 1.13

$$-\frac{\left(\sqrt{2}\,\sqrt{a}\,dx-\left(\sqrt{2}\,\sqrt{a}\,dxe^{c}-2\,\sqrt{2}\,\sqrt{a}\,e^{c}\right)e^{(dx)}+2\,\sqrt{2}\,\sqrt{a}\,\right)e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

[Out] -(sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d*x*e^c - 2*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 2*sqrt(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^2

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integrate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [**A**]
time = 0.43, size = 67, normalized size = 1.26

$$\frac{\sqrt{2}\left(\sqrt{a}\,dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}\,dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-2\,\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-2\,\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] sqrt(2)*(sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - sqrt(a)*d*x*e^(-1/2*d*x - 1/2*c) - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c))/d^2
```

Mupad [**B**]
time = 0.91, size = 56, normalized size = 1.06

$$\frac{2\,x\sinh\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d\cosh\left(\frac{c}{2}+\frac{d\,x}{2}\right)}-\frac{4\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] (2*x*sinh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/(d*cosh(c/2 + (d*x)/2)) - (4*(a + a*cosh(c + d*x))^(1/2))/d^2
```

3.124 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx$

Optimal. Leaf size=83

$$\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)+\sqrt{a+a\cosh(c+dx)}\,\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right)\mathrm{Sh}$$

```
[Out] Chi(1/2*d*x)*cosh(1/2*c)*sech(1/2*d*x+1/2*c)*(a+a*cosh(d*x+c))^(1/2)+sech(1
/2*d*x+1/2*c)*Shi(1/2*d*x)*sinh(1/2*c)*(a+a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3400, 3384, 3379, 3382}

$$\cosh\left(\frac{c}{2}\right)\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\sinh\left(\frac{c}{2}\right)\mathrm{Shi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

```
[Out] Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/
2] + Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(
d*x)/2]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\right.}{x}$$

$$= \left(\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\cosh\left(\frac{d}{2}\right.}{x}$$

$$= \cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)+\sqrt{a+a\cosh(c}$$

Mathematica [A]
time = 0.06, size = 54, normalized size = 0.65

$$\sqrt{a(1+\cosh(c+dx))}\,\text{sech}\left(\frac{1}{2}(c+dx)\right)\left(\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x,x]
[Out] Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(Cosh[c/2]*CoshIntegral[(d*x)
/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])
```

Maple [F]
time = 0.77, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cosh(dx+c)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x,x)
[Out] int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x, x)
```

Giac [A]
time = 0.42, size = 32, normalized size = 0.39

$$\frac{1}{2}\sqrt{2}\left(\sqrt{a}\,\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)}+\sqrt{a}\,\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(2)*(sqrt(a)*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*Ei(-1/2*d*x)*e^(-1/2*c
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\cosh\left(c+d\,x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/2)/x,x)
```

```
[Out] int((a + a*cosh(c + d*x))^(1/2)/x, x)
```

3.125 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}d\sqrt{a+a\cosh(c+dx)}\,\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right)+\frac{1}{2}d\cosh\left(\frac{c}{2}\right)\sqrt{a+}$$

```
[Out] -(a+a*cosh(d*x+c))^(1/2)/x+1/2*d*cosh(1/2*c)*sech(1/2*d*x+1/2*c)*Shi(1/2*d*
x)*(a+a*cosh(d*x+c))^(1/2)+1/2*d*Chi(1/2*d*x)*sech(1/2*d*x+1/2*c)*sinh(1/2*
c)*(a+a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3379, 3382}

$$\frac{1}{2}d\sinh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{2}d\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}-\frac{\sqrt{a\cosh(c+dx)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cosh[c + d*x]]/x) + (d*Sqrt[a + a*Cosh[c + d*x]]*CoshIntegral[
(d*x)/2]*Sech[c/2 + (d*x)/2]*Sinh[c/2])/2 + (d*Cosh[c/2]*Sqrt[a + a*Cosh[c
+ d*x]]*Sech[c/2 + (d*x)/2]*SinhIntegral[(d*x)/2])/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}\right.}{x^2}$$
$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}\left(d\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.\right.$$
$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}\left(d\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\right.\right.\right.$$
$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{x}+\frac{1}{2}d\sqrt{a+a\cosh(c+dx)}\,\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right.$$

Mathematica [**A**]
time = 0.11, size = 75, normalized size = 0.68

$$\frac{\sqrt{a(1+\cosh(c+dx))}\left(-2+dx\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\sinh\left(\frac{c}{2}\right)+dx\cosh\left(\frac{c}{2}\right)\mathrm{sech}\left(\frac{1}{2}(c+dx)\right)\mathrm{Shi}\left(\frac{dx}{2}\right)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*CoshIntegral[(d*x)/2]*Sech[(c + d*x)
/2]*Sinh[c/2] + d*x*Cosh[c/2]*Sech[(c + d*x)/2]*SinhIntegral[(d*x)/2]))/(2*
x)
```

Maple [**F**]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cosh(dx+c)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x**2, x)
```

Giac [A]
time = 0.42, size = 68, normalized size = 0.62

$$\frac{\sqrt{2}\left(\sqrt{a}\,dx\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)}-\sqrt{a}\,dx\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)}-2\,\sqrt{a}\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-2\,\sqrt{a}\,e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}\right)}{4\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(sqrt(a)*d*x*Ei(1/2*d*x)*e^(1/2*c) - sqrt(a)*d*x*Ei(-1/2*d*x)*e
^(-1/2*c) - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c))
/x
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\cosh\left(c+d\,x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*cosh(c + d*x))^(1/2)/x^2, x)
```

3.126 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3}\,dx$

Optimal. Leaf size=151

$$-\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}+\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)+\frac{1}{8}d^2\sqrt{a+a\cosh(}$$

```
[Out] -1/2*(a+a*cosh(d*x+c))^(1/2)/x^2+1/8*d^2*Chi(1/2*d*x)*cosh(1/2*c)*sech(1/2*
d*x+1/2*c)*(a+a*cosh(d*x+c))^(1/2)+1/8*d^2*sech(1/2*d*x+1/2*c)*Shi(1/2*d*x)
*sinh(1/2*c)*(a+a*cosh(d*x+c))^(1/2)-1/4*d*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2
*d*x+1/2*c)/x
```

Rubi [A]
time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 3378, 3384, 3379, 3382}

$$\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\mathrm{Chi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}+\frac{1}{8}d^2\sinh\left(\frac{c}{2}\right)\mathrm{Shi}\left(\frac{dx}{2}\right)\mathrm{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}-\frac{\sqrt{a\cosh(c+dx)+a}}{2x^2}-\frac{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cosh[c + d*x]]/x^2 + (d^2*Cosh[c/2]*Sqrt[a + a*Cosh[c + d*x
]]*CoshIntegral[(d*x)/2]*Sech[c/2 + (d*x)/2])/8 + (d^2*Sqrt[a + a*Cosh[c +
d*x]]*Sech[c/2 + (d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/2])/8 - (d*Sqrt[a +
a*Cosh[c + d*x]]*Tanh[c/2 + (d*x)/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3}\,dx = \left(\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}\right.}{x^3}$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}+\frac{1}{4}\left(d\sqrt{a+a\cosh(c+dx)}\,\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\right.\right.$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}-\frac{d\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}+\frac{1}{8}\left(d^2\sqrt{a\right.$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}-\frac{d\sqrt{a+a\cosh(c+dx)}\,\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}+\frac{1}{8}\left(d^2\cos\right.$$

$$= -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2}+\frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\,\text{Chi}\left(\frac{dx}{2}\right)\text{sech}$$

Mathematica [A]
time = 0.18, size = 97, normalized size = 0.64

$$\frac{\sqrt{a(1+\cosh(c+dx))}\left(-4+d^2x^2\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)+d^2x^2\text{sech}\left(\frac{1}{2}(c+dx)\right)\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)-2dx\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

```
[Out] (Sqrt[a*(1 + Cosh[c + d*x])]*(-4 + d^2*x^2*Cosh[c/2]*CoshIntegral[(d*x)/2]*
Sech[(c + d*x)/2] + d^2*x^2*Sech[(c + d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x)/
2] - 2*d*x*Tanh[(c + d*x)/2]))/(8*x^2)
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cosh\left(dx+c\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(d*x + c) + a)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}{x^{3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cosh(c + d*x) + 1))/x**3, x)
```

Giac [A]
time = 0.41, size = 107, normalized size = 0.71

$$\frac{\sqrt{2}\left(\sqrt{a}\, d^{2}x^{2}\mathrm{Ei}\left(\frac{1}{2}\, dx\right) e^{\left(\frac{1}{2}\, c\right)} + \sqrt{a}\, d^{2}x^{2}\mathrm{Ei}\left(-\frac{1}{2}\, dx\right) e^{\left(-\frac{1}{2}\, c\right)} - 2\, \sqrt{a}\, dx e^{\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)} + 2\, \sqrt{a}\, dx e^{\left(-\frac{1}{2}\, dx - \frac{1}{2}\, c\right)} - 4\, \sqrt{a}\, e^{\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)} - 4\, \sqrt{a}\, e^{\left(-\frac{1}{2}\, dx - \frac{1}{2}\, c\right)}\right)}{16\, x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(2)*(sqrt(a)*d^2*x^2*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*d^2*x^2*Ei(-1
/2*d*x)*e^(-1/2*c) - 2*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) + 2*sqrt(a)*d*x*e^(-
1/2*d*x - 1/2*c) - 4*sqrt(a)*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*e^(-1/2*d*x -
1/2*c))/x^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\cosh\left(c+d\,x\right)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cosh(c + d*x))^(1/2)/x^3, x)
```

3.127 $\int x^3 \sqrt{a+a\cosh(x)}\,dx$

Optimal. Leaf size=68

$$-96\sqrt{a+a\cosh(x)}-12x^2\sqrt{a+a\cosh(x)}+48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

```
[Out] -96*(a+a*cosh(x))^(1/2)-12*x^2*(a+a*cosh(x))^(1/2)+48*x*(a+a*cosh(x))^(1/2)
*tanh(1/2*x)+2*x^3*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Rubi [A]

time = 0.09, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2718}

$$2x^3\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-12x^2\sqrt{a\cosh(x)+a}-96\sqrt{a\cosh(x)+a}+48x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -96*Sqrt[a + a*Cosh[x]] - 12*x^2*Sqrt[a + a*Cosh[x]] + 48*x*Sqrt[a + a*Cosh
[x]]*Tanh[x/2] + 2*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = \left(\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int x^3\cosh\left(\frac{x}{2}\right)dx$$
$$= 2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) - \left(6\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int x^2\sinh\left(\frac{x}{2}\right)d$$
$$= -12x^2\sqrt{a+a\cosh(x)} + 2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) + \left(24\sqrt{a+a\cosh(x)}\right.$$
$$= -12x^2\sqrt{a+a\cosh(x)} + 48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) + 2x^3\sqrt{a+a\cosh(x)}$$
$$= -96\sqrt{a+a\cosh(x)} - 12x^2\sqrt{a+a\cosh(x)} + 48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) +$$

Mathematica [A]
time = 0.04, size = 33, normalized size = 0.49

$$2\sqrt{a(1+\cosh(x))}\left(-6(8+x^2)+x(24+x^2)\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*Sqrt[a + a*Cosh[x]],x]
[Out] 2*Sqrt[a*(1 + Cosh[x])]*(-6*(8 + x^2) + x*(24 + x^2)*Tanh[x/2])
```

Maple [A]
time = 0.47, size = 62, normalized size = 0.91

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\,(e^x+1)^2\,e^{-x}}\,(x^3e^x-x^3-6x^2e^x-6x^2+24x\,e^x-24x-48\,e^x-48)}{e^x+1}$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^3*exp(x)-x^3-6*x^2*exp
(x)-6*x^2+24*x*exp(x)-24*x-48*exp(x)-48)
```

Maxima [A]
time = 0.47, size = 88, normalized size = 1.29

$$-\left(\sqrt{2}\,\sqrt{a}\,x^3+6\,\sqrt{2}\,\sqrt{a}\,x^2+24\,\sqrt{2}\,\sqrt{a}\,x-\left(\sqrt{2}\,\sqrt{a}\,x^3-6\,\sqrt{2}\,\sqrt{a}\,x^2+24\,\sqrt{2}\,\sqrt{a}\,x-48\,\sqrt{2}\,\sqrt{a}\right)e^x+48\,\sqrt{2}\,\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*x^3 + 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - (sqr
t(2)*sqrt(a)*x^3 - 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - 48*sqrt(2
)*sqrt(a))*e^x + 48*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}\sqrt{a\left(\cosh\left(x\right)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(x**3*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x^3, x)
```

Mupad [B]
time = 0.91, size = 63, normalized size = 0.93

$$-\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}\left(48\,x+96\,\mathrm{e}^{x}+12\,x^{2}\,\mathrm{e}^{x}-2\,x^{3}\,\mathrm{e}^{x}-48\,x\,\mathrm{e}^{x}+12\,x^{2}+2\,x^{3}+96\right)}{\mathrm{e}^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cosh(x))^(1/2),x)
```

```
[Out] -((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(48*x + 96*exp(x) + 12*x^2*exp(x) -
2*x^3*exp(x) - 48*x*exp(x) + 12*x^2 + 2*x^3 + 96))/(exp(x) + 1)
```

3.128 $\int x^2 \sqrt{a + a\cosh(x)}\, dx$

Optimal. Leaf size=53

$$-8x\sqrt{a + a\cosh(x)} + 16\sqrt{a + a\cosh(x)}\tanh\left(\frac{x}{2}\right) + 2x^2\sqrt{a + a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

```
[Out] -8*x*(a+a*cosh(x))^(1/2)+16*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+2*x^2*(a+a*cosh
(x))^(1/2)*tanh(1/2*x)
```

Rubi [A]
time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3377, 2717}

$$2x^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x) + a} - 8x\sqrt{a\cosh(x) + a} + 16\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x) + a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -8*x*Sqrt[a + a*Cosh[x]] + 16*Sqrt[a + a*Cosh[x]]*Tanh[x/2] + 2*x^2*Sqrt[a
+ a*Cosh[x]]*Tanh[x/2]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x^2\sqrt{a+a\cosh(x)}\,dx = \left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int x^2\cosh\left(\frac{x}{2}\right)\,dx$$
$$= 2x^2\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right) - \left(4\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int x\sinh\left(\frac{x}{2}\right)\,d$$
$$= -8x\sqrt{a+a\cosh(x)} + 2x^2\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right) + \left(8\sqrt{a+a\cosh(x)}\,\mathrm{se}\right.$$
$$= -8x\sqrt{a+a\cosh(x)} + 16\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right) + 2x^2\sqrt{a+a\cosh(x)}\,\mathrm{ta}$$

Mathematica [A]
time = 0.03, size = 31, normalized size = 0.58

$$8\sqrt{a(1+\cosh(x))}\left(-x+\frac{1}{4}(8+x^2)\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] 8*Sqrt[a*(1 + Cosh[x])]*(-x + ((8 + x^2)*Tanh[x/2])/4)
```

Maple [A]
time = 0.41, size = 50, normalized size = 0.94

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\,(\mathrm{e}^{x}+1)^{2}\,\mathrm{e}^{-x}}\,(x^{2}\mathrm{e}^{x}-x^{2}-4x\,\mathrm{e}^{x}-4x+8\,\mathrm{e}^{x}-8)}{\mathrm{e}^{x}+1}$	50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^2*exp(x)-x^2-4*x*exp(x
)-4*x+8*exp(x)-8)
```

Maxima [A]
time = 0.48, size = 66, normalized size = 1.25

$$-\left(\sqrt{2}\,\sqrt{a}\,x^2+4\,\sqrt{2}\,\sqrt{a}\,x-\left(\sqrt{2}\,\sqrt{a}\,x^2-4\,\sqrt{2}\,\sqrt{a}\,x+8\,\sqrt{2}\,\sqrt{a}\right)e^x+8\,\sqrt{2}\,\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*x^2 + 4*sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x^2 - 4*sqrt
(2)*sqrt(a)*x + 8*sqrt(2)*sqrt(a))*e^x + 8*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \sqrt{a\left(\cosh\left(x\right)+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x^2, x)
```

Mupad [B]
time = 0.07, size = 51, normalized size = 0.96

$$-\frac{\sqrt{a+a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}\,\left(8\,x-16\,e^{x}-2\,x^2\,e^{x}+8\,x\,e^{x}+2\,x^2+16\right)}{e^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cosh(x))^(1/2),x)
```

```
[Out] -((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(8*x - 16*exp(x) - 2*x^2*exp(x) + 8*
x*exp(x) + 2*x^2 + 16))/(exp(x) + 1)
```

3.129 $\int x\sqrt{a+a\cosh(x)}\,dx$

Optimal. Leaf size=32

$$-4\sqrt{a+a\cosh(x)}+2x\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right)$$

```
[Out] -4*(a+a*cosh(x))^(1/2)+2*x*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Rubi [A]
time = 0.04, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 3377, 2718}

$$2x\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-4\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] -4*Sqrt[a + a*Cosh[x]] + 2*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x\sqrt{a+a\cosh(x)}\,dx &= \left(\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int x\cosh\left(\frac{x}{2}\right)\,dx\\ &= 2x\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right)-\left(2\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int\sinh\left(\frac{x}{2}\right)\,dx\\ &= -4\sqrt{a+a\cosh(x)}+2x\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.02, size = 22, normalized size = 0.69

$$2\sqrt{a(1+\cosh(x))}\left(-2+x\tanh\left(\frac{x}{2}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[a + a*Cosh[x]],x]
```

```
[Out] 2*Sqrt[a*(1 + Cosh[x])]*(-2 + x*Tanh[x/2])
```

Maple [A]
time = 0.40, size = 38, normalized size = 1.19

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{x}+1\right)^{2}\mathrm{e}^{-x}}\,\left(x\,\mathrm{e}^{x}-x-2\,\mathrm{e}^{x}-2\right)}{\mathrm{e}^{x}+1}$	38

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x*exp(x)-x-2*exp(x)-2)
```

Maxima [A]
time = 0.48, size = 44, normalized size = 1.38

$$-\left(\sqrt{2}\,\sqrt{a}\,x-\left(\sqrt{2}\,\sqrt{a}\,x-2\,\sqrt{2}\,\sqrt{a}\right)e^{x}+2\,\sqrt{2}\,\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x - 2*sqrt(2)*sqrt(a))*e^x + 2*sqrt(
2)*sqrt(a))*e^(-1/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{a\left(\cosh\left(x\right)+1\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))**(1/2),x)
```

```
[Out] Integral(x*sqrt(a*(cosh(x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)*x, x)
```

Mupad [B]
time = 0.88, size = 39, normalized size = 1.22

$$-\frac{\sqrt{a+a\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}\,\left(2\,x+4\,\mathrm{e}^{x}-2\,x\,\mathrm{e}^{x}+4\right)}{\mathrm{e}^{x}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cosh(x))^(1/2),x)
```

```
[Out] -((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(2*x + 4*exp(x) - 2*x*exp(x) + 4))/(
exp(x) + 1)
```

3.130 $\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx$

Optimal. Leaf size=23

$$\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)$$

```
[Out] Chi(1/2*x)*sech(1/2*x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3400, 3382}

$$\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x,x]
```

```
[Out] Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx &= \left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x}\,dx\\ &= \sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.01, size = 23, normalized size = 1.00

$$\sqrt{a(1+\cosh(x))}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x,x]
```

```
[Out] Sqrt[a*(1 + Cosh[x])]*CoshIntegral[x/2]*Sech[x/2]
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a + a\cosh\left(x\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x,x)
```

```
[Out] int((a+a*cosh(x))^(1/2)/x,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(x\right) + 1\right)}}{x}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\sqrt{a+a\cosh\left(x\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(1/2)/x,x)
```

```
[Out] int((a + a*cosh(x))^(1/2)/x, x)
```

3.131 $\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{a+a\cosh(x)}}{x}+\frac{1}{2}\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\text{Shi}\left(\frac{x}{2}\right)$$

```
[Out] -(a+a*cosh(x))^(1/2)/x+1/2*sech(1/2*x)*Shi(1/2*x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3400, 3378, 3379}

$$\frac{1}{2}\text{Shi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x^2,x]
```

```
[Out] -(Sqrt[a + a*Cosh[x]]/x) + (Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[x/2]
)/2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx = \left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{x} + \frac{1}{2}\left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{x} + \frac{1}{2}\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\mathrm{Shi}\left(\frac{x}{2}\right)$$

Mathematica [**A**]
time = 0.04, size = 33, normalized size = 0.79

$$\frac{\sqrt{a(1+\cosh(x))}\left(-2+x\mathrm{sech}\left(\frac{x}{2}\right)\mathrm{Shi}\left(\frac{x}{2}\right)\right)}{2x}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x^2,x]
[Out] (Sqrt[a*(1 + Cosh[x])]*(-2 + x*Sech[x/2]*SinhIntegral[x/2]))/(2*x)
```

Maple [**F**]
time = 0.37, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x^2,x)
[Out] int((a+a*cosh(x))^(1/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(x\right)+1\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{a+a\cosh\left(x\right)}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(1/2)/x^2,x)
```

```
[Out] int((a + a*cosh(x))^(1/2)/x^2, x)
```

3.132 $\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx$

Optimal. Leaf size=67

$$-\frac{\sqrt{a+a\cosh(x)}}{2x^2}+\frac{1}{8}\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)-\frac{\sqrt{a+a\cosh(x)}\,\tanh\left(\frac{x}{2}\right)}{4x}$$

```
[Out] -1/2*(a+a*cosh(x))^(1/2)/x^2+1/8*Chi(1/2*x)*sech(1/2*x)*(a+a*cosh(x))^(1/2)
-1/4*(a+a*cosh(x))^(1/2)*tanh(1/2*x)/x
```

Rubi [A]
time = 0.08, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3378, 3382}

$$\frac{1}{8}\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{\sqrt{a\cosh(x)+a}}{2x^2}-\frac{\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{4x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Cosh[x]]/x^3,x]
```

```
[Out] -1/2*Sqrt[a + a*Cosh[x]]/x^2 + (Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[
x/2])/8 - (Sqrt[a + a*Cosh[x]]*Tanh[x/2])/(4*x)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{x}{2}\right)}{x^3}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} + \frac{1}{4}\left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} - \frac{\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)}{4x} + \frac{1}{8}\left(\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)$$
$$= -\frac{\sqrt{a+a\cosh(x)}}{2x^2} + \frac{1}{8}\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right) - \frac{\sqrt{a+a\cosh(x)}\,\mathrm{ta}}{4x}$$

Mathematica [A]
time = 0.06, size = 44, normalized size = 0.66

$$\frac{\sqrt{a(1+\cosh(x))}\left(-4+x^2\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)-2x\tanh\left(\frac{x}{2}\right)\right)}{8x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Cosh[x]]/x^3,x]
[Out] (Sqrt[a*(1 + Cosh[x])]*(-4 + x^2*CoshIntegral[x/2]*Sech[x/2] - 2*x*Tanh[x/2
]))/(8*x^2)
```

Maple [F]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(1/2)/x^3,x)
[Out] int((a+a*cosh(x))^(1/2)/x^3,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\cosh\left(x\right)+1\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(a*(cosh(x) + 1))/x**3, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{a+a\cosh\left(x\right)}}{x^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(1/2)/x^3,x)
```

```
[Out] int((a + a*cosh(x))^(1/2)/x^3, x)
```

3.133 $\int x^3(a+a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=185

$$-\frac{1280}{9}a\sqrt{a+a\cosh(x)}-16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a-}$$

```
[Out] -1280/9*a*(a+a*cosh(x))^(1/2)-16*a*x^2*(a+a*cosh(x))^(1/2)-64/27*a*cosh(1/2
*x)^2*(a+a*cosh(x))^(1/2)-8/3*a*x^2*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+32/9*
a*x*cosh(1/2*x)*sinh(1/2*x)*(a+a*cosh(x))^(1/2)+4/3*a*x^3*cosh(1/2*x)*sinh(
1/2*x)*(a+a*cosh(x))^(1/2)+640/9*a*x*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+8/3*a*
x^3*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Rubi [A]

time = 0.15, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3400, 3392, 3377, 2718, 3391}

$$\frac{4}{3}ax^3\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax^3\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-16ax^2\sqrt{a\cosh(x)+a}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{1280}{9}a\sqrt{a\cosh(x)+a}+\frac{32}{9}ax\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{640}{9}ax\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (-1280*a*Sqrt[a + a*Cosh[x]])/9 - 16*a*x^2*Sqrt[a + a*Cosh[x]] - (64*a*Cosh
[x/2]^2*Sqrt[a + a*Cosh[x]])/27 - (8*a*x^2*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])
/3 + (32*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/9 + (4*a*x^3*Cosh[x/2
]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (640*a*x*Sqrt[a + a*Cosh[x]]*Tanh[x/2]
)/9 + (8*a*x^3*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
```

```
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$
\begin{aligned}
\int x^3(a+a\cosh(x))^{3/2}\,dx &= \left(2a\sqrt{a+a\cosh(x)}\,\operatorname{sech}\left(\frac{x}{2}\right)\right)\int x^3\cosh^3\left(\frac{x}{2}\right)\,dx \\
&= -\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^3\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right. \\
&= -\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{32}{9}ax \\
&= -16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}-\frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right) \\
&= -\frac{128}{9}a\sqrt{a+a\cosh(x)}-16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cos} \\
&= -\frac{1280}{9}a\sqrt{a+a\cosh(x)}-16ax^2\sqrt{a+a\cosh(x)}-\frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\operatorname{co}}
\end{aligned}
$$

Mathematica [A]
time = 0.21, size = 70, normalized size = 0.38

$$\frac{2}{27}a\sqrt{a(1+\cosh(x))}\left(-2\left(968+117x^2\right)+3x\left(328+15x^2\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(-2\left(8+9x^2\right)+3x\left(8+3x^2\right)\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cosh[x])]*(-2*(968 + 117*x^2) + 3*x*(328 + 15*x^2)*Tanh[x/
2] + Cosh[x]*(-2*(8 + 9*x^2) + 3*x*(8 + 3*x^2)*Tanh[x/2])))/27
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int x^{3}(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^3*(a+a*cosh(x))^(3/2),x)
```

Maxima [A]
time = 0.48, size = 180, normalized size = 0.97

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^{3}+18\sqrt{2}a^{\frac{3}{2}}x^{2}+24\sqrt{2}a^{\frac{3}{2}}x+16\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^{3}-18\sqrt{2}a^{\frac{3}{2}}x^{2}+24\sqrt{2}a^{\frac{3}{2}}x-16\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{2}a^{\frac{3}{2}}x^{3}-6\sqrt{2}a^{\frac{3}{2}}x^{2}+24\sqrt{2}a^{\frac{3}{2}}x-48\sqrt{2}a^{\frac{3}{2}}\right)e^{(2x)}+81\left(\sqrt{2}a^{\frac{3}{2}}x^{3}+6\sqrt{2}a^{\frac{3}{2}}x^{2}+24\sqrt{2}a^{\frac{3}{2}}x+48\sqrt{2}a^{\frac{3}{2}}\right)e^{x}\right)e^{\left(-\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/54*(9*sqrt(2)*a^(3/2)*x^3 + 18*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*
x + 16*sqrt(2)*a^(3/2) - (9*sqrt(2)*a^(3/2)*x^3 - 18*sqrt(2)*a^(3/2)*x^2 +
24*sqrt(2)*a^(3/2)*x - 16*sqrt(2)*a^(3/2))*e^(3*x) - 81*(sqrt(2)*a^(3/2)*x^
3 - 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*x - 48*sqrt(2)*a^(3/2))*e^(2
*x) + 81*(sqrt(2)*a^(3/2)*x^3 + 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*
x + 48*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{3}(a(\cosh(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x**3*(a*(cosh(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.41, size = 192, normalized size = 1.04

$$-\frac{1}{54}\sqrt{2}\left(54\,a^{\frac{3}{2}}x^{3}e^{\left(-\frac{1}{2}\,x\right)}+9\,a^{\frac{3}{2}}x^{3}e^{\left(-\frac{3}{2}\,x\right)}+324\,a^{\frac{3}{2}}x^{2}e^{\left(-\frac{1}{2}\,x\right)}+18\,a^{\frac{3}{2}}x^{2}e^{\left(-\frac{3}{2}\,x\right)}+1296\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)}+24\,a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}\,x\right)}+2592\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}+16\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}-\left(9\,a^{\frac{3}{2}}x^{3}-18\,a^{\frac{3}{2}}x^{2}+24\,a^{\frac{3}{2}}x-16\,a^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}\,x\right)}-81\left(a^{\frac{3}{2}}x^{3}-6\,a^{\frac{3}{2}}x^{2}+24\,a^{\frac{3}{2}}x-48\,a^{\frac{3}{2}}\right)e^{\left(\frac{1}{2}\,x\right)}+27\left(a^{\frac{3}{2}}x^{3}+6\,a^{\frac{3}{2}}x^{2}+24\,a^{\frac{3}{2}}x+48\,a^{\frac{3}{2}}\right)e^{\left(-\frac{1}{2}\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/54*sqrt(2)*(54*a^(3/2)*x^3*e^(-1/2*x) + 9*a^(3/2)*x^3*e^(-3/2*x) + 324*a
^(3/2)*x^2*e^(-1/2*x) + 18*a^(3/2)*x^2*e^(-3/2*x) + 1296*a^(3/2)*x*e^(-1/2*
x) + 24*a^(3/2)*x*e^(-3/2*x) + 2592*a^(3/2)*e^(-1/2*x) + 16*a^(3/2)*e^(-3/2
*x) - (9*a^(3/2)*x^3 - 18*a^(3/2)*x^2 + 24*a^(3/2)*x - 16*a^(3/2))*e^(3/2*x
) - 81*(a^(3/2)*x^3 - 6*a^(3/2)*x^2 + 24*a^(3/2)*x - 48*a^(3/2))*e^(1/2*x)
+ 27*(a^(3/2)*x^3 + 6*a^(3/2)*x^2 + 24*a^(3/2)*x + 48*a^(3/2))*e^(-1/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,{\left(a+a\,\mathrm{cosh}\left(x\right)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + a*cosh(x))^(3/2),x)
```

```
[Out] int(x^3*(a + a*cosh(x))^(3/2), x)
```

3.134 $\int x^2(a+a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=145

$$-\frac{32}{3}ax\sqrt{a+a\cosh(x)}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right)+\frac{22}{9}$$

```
[Out] -32/3*a*x*(a+a*cosh(x))^(1/2)-16/9*a*x*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+4/
3*a*x^2*cosh(1/2*x)*sinh(1/2*x)*(a+a*cosh(x))^(1/2)+224/9*a*(a+a*cosh(x))^(
1/2)*tanh(1/2*x)+8/3*a*x^2*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+32/27*a*sinh(1/2
*x)^2*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Rubi [A]

time = 0.11, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}=0.357$, Rules used = {3400, 3392, 3377, 2717, 2713}

$$\frac{4}{3}ax^2\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax^2\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{32}{3}ax\sqrt{a\cosh(x)+a}+\frac{224}{9}a\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{32}{27}a\sinh^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (-32*a*x*Sqrt[a + a*Cosh[x]])/3 - (16*a*x*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/
9 + (4*a*x^2*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (224*a*Sqrt[a + a
*Cosh[x]]*Tanh[x/2])/9 + (8*a*x^2*Sqrt[a + a*Cosh[x]]*Tanh[x/2])/3 + (32*a*
Sqrt[a + a*Cosh[x]]*Sinh[x/2]^2*Tanh[x/2])/27
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
```

```
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int x^2(a+a\cosh(x))^{3/2}\,dx &= \left(2a\sqrt{a+a\cosh(x)}\,\operatorname{sech}\left(\frac{x}{2}\right)\right)\int x^2\cosh^3\left(\frac{x}{2}\right)\,dx\\ &= -\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right.\\ &= -\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right.\\ &= -\frac{32}{3}ax\sqrt{a+a\cosh(x)}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\\ &= -\frac{32}{3}ax\sqrt{a+a\cosh(x)}-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\end{aligned}$$

Mathematica [A]
time = 0.18, size = 54, normalized size = 0.37

$$\frac{2}{27}a\sqrt{a(1+\cosh(x))}\left(-156x+\left(328+45x^2\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(-12x+\left(8+9x^2\right)\tanh\left(\frac{x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Cosh[x])]*(-156*x + (328 + 45*x^2)*Tanh[x/2] + Cosh[x]*(-1
2*x + (8 + 9*x^2)*Tanh[x/2])))/27
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int x^2(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2*(a+a*cosh(x))^(3/2),x)
```

Maxima [A]
time = 0.49, size = 136, normalized size = 0.94

$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^2+12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^2-12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{2}a^{\frac{3}{2}}x^2-4\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(2x)}+81\left(\sqrt{2}a^{\frac{3}{2}}x^2+4\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{x}\right)e^{\left(-\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/54*(9*sqrt(2)*a^(3/2)*x^2 + 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2) - (
9*sqrt(2)*a^(3/2)*x^2 - 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(3*x) -
 81*(sqrt(2)*a^(3/2)*x^2 - 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(2*x)
 + 81*(sqrt(2)*a^(3/2)*x^2 + 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^x)*
e^(-3/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2(a(\cosh(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x**2*(a*(cosh(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.43, size = 144, normalized size = 0.99

$$-\frac{1}{54}\sqrt{2}\left(54a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}x\right)}+9a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}x\right)}+216a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}x\right)}+12a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}x\right)}+432a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)}+8a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}-\left(9a^{\frac{3}{2}}x^2-12a^{\frac{3}{2}}x+8a^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}x\right)}-81\left(a^{\frac{3}{2}}x^2-4a^{\frac{3}{2}}x+8a^{\frac{3}{2}}\right)e^{\left(\frac{1}{2}x\right)}+27\left(a^{\frac{3}{2}}x^2+4a^{\frac{3}{2}}x+8a^{\frac{3}{2}}\right)e^{\left(-\frac{1}{2}x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/54*sqrt(2)*(54*a^(3/2)*x^2*e^(-1/2*x) + 9*a^(3/2)*x^2*e^(-3/2*x) + 216*a
^(3/2)*x*e^(-1/2*x) + 12*a^(3/2)*x*e^(-3/2*x) + 432*a^(3/2)*e^(-1/2*x) + 8*
a^(3/2)*e^(-3/2*x) - (9*a^(3/2)*x^2 - 12*a^(3/2)*x + 8*a^(3/2))*e^(3/2*x) -
 81*(a^(3/2)*x^2 - 4*a^(3/2)*x + 8*a^(3/2))*e^(1/2*x) + 27*(a^(3/2)*x^2 + 4
*a^(3/2)*x + 8*a^(3/2))*e^(-1/2*x))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2 (a + a\cosh(x))^{3/2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2*(a + a*cosh(x))^(3/2), x)
```

3.135 $\int x(a+a\cosh(x))^{3/2}\,dx$

Optimal. Leaf size=89

$$-\frac{16}{3}a\sqrt{a+a\cosh(x)}-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)+\frac{8}{3}ax\sqrt{}$$

```
[Out] -16/3*a*(a+a*cosh(x))^(1/2)-8/9*a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+4/3*a*x
*cosh(1/2*x)*sinh(1/2*x)*(a+a*cosh(x))^(1/2)+8/3*a*x*(a+a*cosh(x))^(1/2)*ta
nh(1/2*x)
```

Rubi [A]
time = 0.06, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 3391, 3377, 2718}

$$-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{16}{3}a\sqrt{a\cosh(x)+a}+\frac{4}{3}ax\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{8}{3}ax\tanh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (-16*a*Sqrt[a + a*Cosh[x]])/3 - (8*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/9 + (
4*a*x*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/3 + (8*a*x*Sqrt[a + a*Cosh[x
]]*Tanh[x/2])/3
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
```

```
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int x(a+a\cosh(x))^{3/2}\,dx = \left(2a\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int x\cosh^3\left(\frac{x}{2}\right)\,dx$$
$$= -\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right)+\ldots$$
$$= -\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right)+\ldots$$
$$= -\frac{16}{3}a\sqrt{a+a\cosh(x)}-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+\ldots}$$

Mathematica [A]
time = 0.07, size = 56, normalized size = 0.63

$$\frac{1}{9}a\sqrt{a(1+\cosh(x))}\,\text{sech}\left(\frac{x}{2}\right)\left(-54\cosh\left(\frac{x}{2}\right)-2\cosh\left(\frac{3x}{2}\right)+3x\left(9\sinh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + a*Cosh[x])^(3/2),x]
[Out] (a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(-54*Cosh[x/2] - 2*Cosh[(3*x)/2] + 3*x*(
9*Sinh[x/2] + Sinh[(3*x)/2])))/9
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int x(a+a\cosh(x))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a+a*cosh(x))^(3/2),x)
[Out] int(x*(a+a*cosh(x))^(3/2),x)
```

Maxima [A]
time = 0.48, size = 92, normalized size = 1.03

$$-\frac{1}{18}\left(3\sqrt{2}\,a^{\frac{3}{2}}x+2\sqrt{2}\,a^{\frac{3}{2}}-\left(3\sqrt{2}\,a^{\frac{3}{2}}x-2\sqrt{2}\,a^{\frac{3}{2}}\right)e^{(3x)}-27\left(\sqrt{2}\,a^{\frac{3}{2}}x-2\sqrt{2}\,a^{\frac{3}{2}}\right)e^{(2x)}+27\left(\sqrt{2}\,a^{\frac{3}{2}}x+2\sqrt{2}\,a^{\frac{3}{2}}\right)e^{x}\right)e^{\left(-\frac{3}{2}x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] -1/18*(3*sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2) - (3*sqrt(2)*a^(3/2)*x - 2*s
qrt(2)*a^(3/2))*e^(3*x) - 27*(sqrt(2)*a^(3/2)*x - 2*sqrt(2)*a^(3/2))*e^(2*x
) + 27*(sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x(a(\cosh(x)+1))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x*(a*(cosh(x) + 1))**(3/2), x)
```

Giac [A]
time = 0.42, size = 96, normalized size = 1.08

$$-\frac{1}{18}\sqrt{2}\left(18\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)}+3\,a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}\,x\right)}+36\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}+2\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}-\left(3\,a^{\frac{3}{2}}x-2\,a^{\frac{3}{2}}\right)e^{\left(\frac{3}{2}\,x\right)}-27\left(a^{\frac{3}{2}}x-2\,a^{\frac{3}{2}}\right)e^{\left(\frac{1}{2}\,x\right)}+9\left(a^{\frac{3}{2}}x+2\,a^{\frac{3}{2}}\right)e^{\left(-\frac{1}{2}\,x\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] -1/18*sqrt(2)*(18*a^(3/2)*x*e^(-1/2*x) + 3*a^(3/2)*x*e^(-3/2*x) + 36*a^(3/2
)*e^(-1/2*x) + 2*a^(3/2)*e^(-3/2*x) - (3*a^(3/2)*x - 2*a^(3/2))*e^(3/2*x) -
 27*(a^(3/2)*x - 2*a^(3/2))*e^(1/2*x) + 9*(a^(3/2)*x + 2*a^(3/2))*e^(-1/2*x
))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,{\left(a+a\,\cosh(x)\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + a*cosh(x))^(3/2),x)
```

```
[Out] int(x*(a + a*cosh(x))^(3/2), x)
```

3.136 $\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx$

Optimal. Leaf size=55

$$\frac{3}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)$$

```
[Out] 3/2*a*Chi(1/2*x)*sech(1/2*x)*(a+a*cosh(x))^(1/2)+1/2*a*Chi(3/2*x)*sech(1/2*
x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3393, 3382}

$$\frac{3}{2}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{1}{2}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x,x]
```

```
[Out] (3*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2])/2 + (a*Sqrt[a + a*Cos
h[x]]*CoshIntegral[(3*x)/2]*Sech[x/2])/2
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \left(2a\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x}\,dx$$
$$= \left(2a\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \left(\frac{3\cosh\left(\frac{x}{2}\right)}{4x}+\frac{\cosh\left(\frac{3x}{2}\right)}{4x}\right)dx$$
$$= \frac{1}{2}\left(a\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh\left(\frac{3x}{2}\right)}{x}\,dx + \frac{1}{2}\left(3a\sqrt{a+a\cosh(x)}\,\mathrm{sech}\left(\frac{x}{2}\right.\right.$$
$$= \frac{3}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)+\frac{1}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)$$

Mathematica [**A**]
time = 0.02, size = 36, normalized size = 0.65

$$\frac{1}{2}a\sqrt{a(1+\cosh(x))}\left(3\mathrm{Chi}\left(\frac{x}{2}\right)+\mathrm{Chi}\left(\frac{3x}{2}\right)\right)\mathrm{sech}\left(\frac{x}{2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x,x]
```

```
[Out] (a*Sqrt[a*(1 + Cosh[x])]*(3*CoshIntegral[x/2] + CoshIntegral[(3*x)/2])*Sech
[x/2])/2
```

Maple [**F**]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cosh(x))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cosh(x)+1))^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x,x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)/x, x)
```

Giac [A]
time = 0.41, size = 40, normalized size = 0.73

$$\frac{1}{4}\sqrt{2}\left(a^{\frac{3}{2}}\operatorname{Ei}\left(\frac{3}{2}x\right)+3a^{\frac{3}{2}}\operatorname{Ei}\left(\frac{1}{2}x\right)+3a^{\frac{3}{2}}\operatorname{Ei}\left(-\frac{1}{2}x\right)+a^{\frac{3}{2}}\operatorname{Ei}\left(-\frac{3}{2}x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(a^(3/2)*Ei(3/2*x) + 3*a^(3/2)*Ei(1/2*x) + 3*a^(3/2)*Ei(-1/2*x)
 + a^(3/2)*Ei(-3/2*x))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(3/2)/x,x)
```

```
[Out] int((a + a*cosh(x))^(3/2)/x, x)
```

3.137 $\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx$

Optimal. Leaf size=79

$$-\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x}+\frac{3}{4}a\sqrt{a+a\cosh(x)}\,\operatorname{sech}\left(\frac{x}{2}\right)\operatorname{Shi}\left(\frac{x}{2}\right)+\frac{3}{4}a\sqrt{a+a\cosh(x)}\,\operatorname{sech}\left(\frac{x}{2}\right)\operatorname{Shi}\left(\frac{3}{\vphantom{2}}\right.$$

```
[Out] -2*a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)/x+3/4*a*sech(1/2*x)*Shi(1/2*x)*(a+a*
cosh(x))^(1/2)+3/4*a*sech(1/2*x)*Shi(3/2*x)*(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3400, 3394, 3379}

$$\frac{3}{4}a\text{Shi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{3}{4}a\text{Shi}\left(\frac{3x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x^2,x]
```

```
[Out] (-2*a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/
2]*SinhIntegral[x/2])/4 + (3*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*SinhIntegral[(
3*x)/2])/4
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx = \left(2a\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^2}\,dx$$
$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \left(3ia\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int \left(-\frac{i\sinh\left(\frac{}{}\right.}{4x}\right.$$
$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \frac{1}{4}\left(3a\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\right)\int \frac{\sinh\left(\frac{x}{2}\right)}{x}\,d$$
$$= -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \frac{3}{4}a\sqrt{a+a\cosh(x)}\,\text{sech}\left(\frac{x}{2}\right)\text{Shi}\left(\frac{x}{2}\right) + \frac{3}{4}a\sqrt{a}$$

Mathematica [**A**]
time = 0.07, size = 53, normalized size = 0.67

$$-\frac{a\sqrt{a(1+\cosh(x))}\,\text{sech}\left(\frac{x}{2}\right)\left(8\cosh^3\left(\frac{x}{2}\right)-3x\text{Shi}\left(\frac{x}{2}\right)-3x\text{Shi}\left(\frac{3x}{2}\right)\right)}{4x}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x^2,x]
```

```
[Out] -1/4*(a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(8*Cosh[x/2]^3 - 3*x*SinhIntegral[x
/2] - 3*x*SinhIntegral[(3*x)/2]))/x
```

Maple [**F**]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cosh(x))^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x^2,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x^2,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cosh(x) + 1))^{\frac{3}{2}}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x**2,x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)/x**2, x)
```

Giac [A]
time = 0.41, size = 112, normalized size = 1.42

$$\frac{1}{8}\sqrt{2}\left(\frac{3a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{3}{2}x\right) + 3a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{1}{2}x\right) - a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{1}{2}x\right) - 2a^{\frac{3}{2}}e^{\left(\frac{3}{2}x\right)} - 6a^{\frac{3}{2}}e^{\left(\frac{1}{2}x\right)} - 2a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)}}{x} - \frac{2a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{1}{2}x\right) + 3a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{3}{2}x\right) + 4a^{\frac{3}{2}}e^{\left(-\frac{1}{2}x\right)} + 2a^{\frac{3}{2}}e^{\left(-\frac{3}{2}x\right)}}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*((3*a^(3/2)*x*Ei(3/2*x) + 3*a^(3/2)*x*Ei(1/2*x) - a^(3/2)*x*Ei(
-1/2*x) - 2*a^(3/2)*e^(3/2*x) - 6*a^(3/2)*e^(1/2*x) - 2*a^(3/2)*e^(-1/2*x))
/x - (2*a^(3/2)*x*Ei(-1/2*x) + 3*a^(3/2)*x*Ei(-3/2*x) + 4*a^(3/2)*e^(-1/2*x
) + 2*a^(3/2)*e^(-3/2*x))/x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(3/2)/x^2,x)
```

```
[Out] int((a + a*cosh(x))^(3/2)/x^2, x)
```

3.138 $\int \frac{(a+a\cosh(x))^{3/2}}{x^3} dx$

Optimal. Leaf size=109

$$-\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2}+\frac{3}{16}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)+\frac{9}{16}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}$$

```
[Out] -a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)/x^2+3/16*a*Chi(1/2*x)*sech(1/2*x)*(a+a
*cosh(x))^(1/2)+9/16*a*Chi(3/2*x)*sech(1/2*x)*(a+a*cosh(x))^(1/2)-3/2*a*cos
h(1/2*x)*sinh(1/2*x)*(a+a*cosh(x))^(1/2)/x
```

Rubi [A]
time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3400, 3395, 3382, 3393}

$$\frac{3}{16}a\mathrm{Chi}\left(\frac{x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}+\frac{9}{16}a\mathrm{Chi}\left(\frac{3x}{2}\right)\mathrm{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}-\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{x^2}-\frac{3a\sinh\left(\frac{x}{2}\right)\cosh\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}}{2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Cosh[x])^(3/2)/x^3,x]
```

```
[Out] -((a*Cosh[x/2]^2*Sqrt[a + a*Cosh[x]])/x^2) + (3*a*Sqrt[a + a*Cosh[x]]*CoshI
ntegral[x/2]*Sech[x/2])/16 + (9*a*Sqrt[a + a*Cosh[x]]*CoshIntegral[(3*x)/2]
*Sech[x/2])/16 - (3*a*Cosh[x/2]*Sqrt[a + a*Cosh[x]]*Sinh[x/2])/(2*x)
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
```

```
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx = \left(2a\sqrt{a+a\cosh(x)}\,\operatorname{sech}\left(\frac{x}{2}\right)\right)\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^3}\,dx$$

$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3a\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\,\sinh\left(\frac{x}{2}\right)}{2x} - \frac{1}{2}\Big(3a$$

$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a\cos}{}$$

$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} - \frac{3}{2}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) - \frac{3a\cos}{}$$

$$= -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} + \frac{3}{16}a\sqrt{a+a\cosh(x)}\,\mathrm{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right) + \frac{9}{16}a$$

Mathematica [A]
time = 0.05, size = 69, normalized size = 0.63

$$\frac{(a(1+\cosh(x)))^{3/2}\left(3x^2\mathrm{Chi}\left(\frac{x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)+9x^2\mathrm{Chi}\left(\frac{3x}{2}\right)\operatorname{sech}^3\left(\frac{x}{2}\right)-8\left(2+3x\tanh\left(\frac{x}{2}\right)\right)\right)}{32x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Cosh[x])^(3/2)/x^3,x]
```

```
[Out] ((a*(1 + Cosh[x]))^(3/2)*(3*x^2*CoshIntegral[x/2]*Sech[x/2]^3 + 9*x^2*CoshI
ntegral[(3*x)/2]*Sech[x/2]^3 - 8*(2 + 3*x*Tanh[x/2])))/(32*x^2)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cosh(x))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(x))^(3/2)/x^3,x)
```

```
[Out] int((a+a*cosh(x))^(3/2)/x^3,x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(x) + a)^(3/2)/x^3, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\cosh(x) + 1))^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))**(3/2)/x**3,x)
```

```
[Out] Integral((a*(cosh(x) + 1))**(3/2)/x**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(81) = 162.
time = 0.42, size = 170, normalized size = 1.56

$$\frac{1}{32}\sqrt{2}\left(\frac{9\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(\frac{3}{2}\,x\right) + 3\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(\frac{1}{2}\,x\right) + a^{\frac{3}{2}}x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right) - 6\,a^{\frac{3}{2}}xe^{\left(\frac{3}{2}\,x\right)} - 6\,a^{\frac{3}{2}}xe^{\left(\frac{1}{2}\,x\right)} + 2\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)} - 4\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)} - 12\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)} - 4\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}}{x^2} + \frac{2\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right) + 9\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(-\frac{3}{2}\,x\right) + 4\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)} + 6\,a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}\,x\right)} - 8\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)} - 4\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/32*sqrt(2)*((9*a^(3/2)*x^2*Ei(3/2*x) + 3*a^(3/2)*x^2*Ei(1/2*x) + a^(3/2)*
x^2*Ei(-1/2*x) - 6*a^(3/2)*x*e^(3/2*x) - 6*a^(3/2)*x*e^(1/2*x) + 2*a^(3/2)*
```

```
x*e^(-1/2*x) - 4*a^(3/2)*e^(3/2*x) - 12*a^(3/2)*e^(1/2*x) - 4*a^(3/2)*e^(-1
/2*x))/x^2 + (2*a^(3/2)*x^2*Ei(-1/2*x) + 9*a^(3/2)*x^2*Ei(-3/2*x) + 4*a^(3/
2)*x*e^(-1/2*x) + 6*a^(3/2)*x*e^(-3/2*x) - 8*a^(3/2)*e^(-1/2*x) - 4*a^(3/2)
*e^(-3/2*x))/x^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(x))^(3/2)/x^3,x)
```

```
[Out] int((a + a*cosh(x))^(3/2)/x^3, x)
```

3.139 $\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=383

$$\frac{4x^3 \text{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}}-\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}+\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}}{d^2\sqrt{a+a\cosh(c+}}$$

```
[Out] 4*x^3*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(1
/2)-12*I*x^2*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+a*
cosh(d*x+c))^(1/2)+12*I*x^2*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+1/2
*c))/d^2/(a+a*cosh(d*x+c))^(1/2)+48*I*x*cosh(1/2*d*x+1/2*c)*polylog(3,-I*ex
p(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-48*I*x*cosh(1/2*d*x+1/2*c)*po
lylog(3,I*exp(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-96*I*cosh(1/2*d*x
+1/2*c)*polylog(4,-I*exp(1/2*d*x+1/2*c))/d^4/(a+a*cosh(d*x+c))^(1/2)+96*I*c
osh(1/2*d*x+1/2*c)*polylog(4,I*exp(1/2*d*x+1/2*c))/d^4/(a+a*cosh(d*x+c))^(1
/2)
```

Rubi [A]
time = 0.15, antiderivative size = 383, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3400, 4265, 2611, 6744, 2320, 6724}

$$\frac{4x^3\text{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cosh(c+dx)+a}}-\frac{96i\text{Li}_4\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cosh(c+dx)+a}}+\frac{96i\text{Li}_4\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^4\sqrt{a\cosh(c+dx)+a}}+\frac{48ix\text{Li}_3\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cosh(c+dx)+a}}-\frac{48ix\text{Li}_3\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cosh(c+dx)+a}}-\frac{12ix^2\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{12ix^2\text{Li}_2\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x^3*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2
)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((12*I)*x^2*Cosh[c/2 + (d*x)/2]*PolyL
og[2, I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((48*I)*x*Cos
h[c/2 + (d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c
 + d*x]]) - ((48*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/
(d^3*Sqrt[a + a*Cosh[c + d*x]]) - ((96*I)*Cosh[c/2 + (d*x)/2]*PolyLog[4, (-
I)*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]]) + ((96*I)*Cosh[c/2 +
 (d*x)/2]*PolyLog[4, I*E^(c/2 + (d*x)/2)])/(d^4*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x^3\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\,dx}{\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(6i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x^2\log\left(1\right.}{d\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12i}{}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12i}{}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12i}{}$$

$$= \frac{4x^3\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{12i}{}$$

Mathematica [A]
time = 1.11, size = 213, normalized size = 0.56

$$\frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(d^3x^3\log\left(1-ie^{\frac{1}{2}(c+dx)}\right)-d^3x^3\log\left(1+ie^{\frac{1}{2}(c+dx)}\right)-6d^2x^2\text{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)+6d^2x^2\text{PolyLog}\left(2,ie^{\frac{1}{2}(c+dx)}\right)+24dx\text{PolyLog}\left(3,-ie^{\frac{1}{2}(c+dx)}\right)-24dx\text{PolyLog}\left(3,ie^{\frac{1}{2}(c+dx)}\right)-48\text{PolyLog}\left(4,-ie^{\frac{1}{2}(c+dx)}\right)+48\text{PolyLog}\left(4,ie^{\frac{1}{2}(c+dx)}\right)\right)}{d^4\sqrt{a(1+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] ((2*I)*Cosh[(c + d*x)/2]*(d^3*x^3*Log[1 - I*E^((c + d*x)/2)] - d^3*x^3*Log[
1 + I*E^((c + d*x)/2)] - 6*d^2*x^2*PolyLog[2, (-I)*E^((c + d*x)/2)] + 6*d^2
*x^2*PolyLog[2, I*E^((c + d*x)/2)] + 24*d*x*PolyLog[3, (-I)*E^((c + d*x)/2)
] - 24*d*x*PolyLog[3, I*E^((c + d*x)/2)] - 48*PolyLog[4, (-I)*E^((c + d*x)/
2)] + 48*PolyLog[4, I*E^((c + d*x)/2)]))/(d^4*Sqrt[a*(1 + Cosh[c + d*x])])
```

Maple [F]
time = 0.46, size = 0, normalized size = 0.00

$$\int \frac{x^3}{\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d^3*integrate(x^3*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*c
) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3), x) + 12*sqrt(2)*d^2*integrate
(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^(d*
x + c) + sqrt(a)*d^3), x) + 48*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c)/(s
qrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3), x) +
 96*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3)*d
) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^4)) - 2*(sqrt(2)*sqrt(a)*d^3*x^3
*e^(1/2*c) + 6*sqrt(2)*sqrt(a)*d^2*x^2*e^(1/2*c) + 24*sqrt(2)*sqrt(a)*d*x*e
^(1/2*c) + 48*sqrt(2)*sqrt(a)*e^(1/2*c))*e^(1/2*d*x)/(a*d^4*e^(d*x + c) + a
*d^4)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{3}}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**3/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(x^3/(a + a*cosh(c + d*x))^(1/2), x)
```

3.140 $\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=269

$$\frac{4x^2\operatorname{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}}-\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}+\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2\right.}{d^2\sqrt{a+a\cosh(c+dx}}$$

```
[Out] 4*x^2*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(1
/2)-8*I*x*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+a*cos
h(d*x+c))^(1/2)+8*I*x*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+1/2*c))/d
^2/(a+a*cosh(d*x+c))^(1/2)+16*I*cosh(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/2*d*
x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-16*I*cosh(1/2*d*x+1/2*c)*polylog(3,I*
exp(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3400, 4265, 2611, 2320, 6724}

$$\frac{4x^2\operatorname{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cosh(c+dx)+a}}+\frac{16i\operatorname{Li}_3\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cosh(c+dx)+a}}-\frac{16i\operatorname{Li}_3\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3\sqrt{a\cosh(c+dx)+a}}-\frac{8ix\operatorname{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{8ix\operatorname{Li}_2\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x^2*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c
+ d*x]]) - ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])
/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((8*I)*x*Cosh[c/2 + (d*x)/2]*PolyLog[2,
I*E^(c/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]]) + ((16*I)*Cosh[c/2 +
(d*x)/2]*PolyLog[3, (-I)*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[a + a*Cosh[c + d*x]]
) - ((16*I)*Cosh[c/2 + (d*x)/2]*PolyLog[3, I*E^(c/2 + (d*x)/2)])/(d^3*Sqrt[
a + a*Cosh[c + d*x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
```

```
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x^2\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\,dx}{\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(4i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int x\log\left(1\right.}{d\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\,\mathrm{co}}{d^2}$$

$$= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\,\mathrm{co}}{d^2}$$

$$= \frac{4x^2\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\mathrm{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{8ix\,\mathrm{co}}{d^2}$$

Mathematica [A]
time = 1.14, size = 163, normalized size = 0.61

$$\frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(d^2x^2\log\left(1-ie^{\frac{1}{2}(c+dx)}\right)-d^2x^2\log\left(1+ie^{\frac{1}{2}(c+dx)}\right)-4dx\text{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)+4dx\text{PolyLog}\left(2,ie^{\frac{1}{2}(c+dx)}\right)+8\text{PolyLog}\left(3,-ie^{\frac{1}{2}(c+dx)}\right)-8\text{PolyLog}\left(3,ie^{\frac{1}{2}(c+dx)}\right)\right)}{d^3\sqrt{a(1+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] ((2*I)*Cosh[(c + d*x)/2]*(d^2*x^2*Log[1 - I*E^((c + d*x)/2)] - d^2*x^2*Log[
1 + I*E^((c + d*x)/2)] - 4*d*x*PolyLog[2, (-I)*E^((c + d*x)/2)] + 4*d*x*Pol
yLog[2, I*E^((c + d*x)/2)] + 8*PolyLog[3, (-I)*E^((c + d*x)/2)] - 8*PolyLog
[3, I*E^((c + d*x)/2)]))/(d^3*Sqrt[a*(1 + Cosh[c + d*x])])
```

Maple [F]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d^2*integrate(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*c
) + 2*sqrt(a)*d^2*e^(d*x + c) + sqrt(a)*d^2), x) + 8*sqrt(2)*d*integrate(x*
e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*c) + 2*sqrt(a)*d^2*e^(d*x + c
) + sqrt(a)*d^2), x) + 16*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^2*e^(d*x
 + c) + sqrt(a)*d^2)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^3)) - 2*(s
qrt(2)*d^2*x^2*e^(1/2*c) + 4*sqrt(2)*d*x*e^(1/2*c) + 8*sqrt(2)*e^(1/2*c))*e
^(1/2*d*x)/(sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x**2/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(x^2/(a + a*cosh(c + d*x))^(1/2), x)
```

3.141 $\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=157

$$\frac{4x\text{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}}-\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}+\frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}$$

```
[Out] 4*x*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(1/2
)-4*I*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+a*cosh(d*
x+c))^(1/2)+4*I*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+1/2*c))/d^2/(a+
a*cosh(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3400, 4265, 2317, 2438}

$$\frac{4x\text{ArcTan}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a\cosh(c+dx)+a}}-\frac{4i\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}+\frac{4i\text{Li}_2\left(ie^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2\sqrt{a\cosh(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*x*ArcTan[E^(c/2 + (d*x)/2)]*Cosh[c/2 + (d*x)/2])/(d*Sqrt[a + a*Cosh[c +
d*x]]) - ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])/(d^
2*Sqrt[a + a*Cosh[c + d*x]]) + ((4*I)*Cosh[c/2 + (d*x)/2]*PolyLog[2, I*E^(c
/2 + (d*x)/2)])/(d^2*Sqrt[a + a*Cosh[c + d*x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
```

```
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\int x\csc\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\,dx}{\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(2i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\int\log\left(1-\right.}{d\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{\left(4i\sin\left(\frac{1}{2}\left(ic+\frac{\pi}{2}\right)+\frac{\pi}{4}+\frac{idx}{2}\right)\right)\text{Subst}\left(\int\right.}{d^2\sqrt{a+a\cosh(c+dx)}}$$

$$= \frac{4x\tan^{-1}\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Li}_2\left(-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{4i\cosh}{d^2\sqrt{}}$$

Mathematica [A]
time = 0.54, size = 117, normalized size = 0.75

$$\frac{4\cosh\left(\frac{1}{2}(c+dx)\right)\left(dx\text{ArcTan}\left(\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)-i\text{PolyLog}\left(2,-i\left(\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)+i\text{PolyLog}\left(2,i\left(\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)}{d^2\sqrt{a(1+\cosh(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

```
[Out] (4*Cosh[(c + d*x)/2]*(d*x*ArcTan[Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] - I
*PolyLog[2, (-I)*(Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])] + I*PolyLog[2, I*
(Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2])]))/(d^2*Sqrt[a*(1 + Cosh[c + d*x])]
)
```

Maple [F]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d*e^(2*d*x + 2*c) + 2*
sqrt(a)*d*e^(d*x + c) + sqrt(a)*d), x) + 4*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((s
qrt(a)*d*e^(d*x + c) + sqrt(a)*d)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)
*d^2)) - 2*(sqrt(2)*sqrt(a)*d*x*e^(1/2*c) + 2*sqrt(2)*sqrt(a)*e^(1/2*c))*e^
(1/2*d*x)/(a*d^2*e^(d*x + c) + a*d^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(x/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(x/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*cosh(c + d*x))^(1/2),x)
```

```
[Out] int(x/(a + a*cosh(c + d*x))^(1/2), x)
```

3.142 $\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x\sqrt{a+a\cosh(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
[Out] Defer[Int][1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Mathematica [A]
time = 1.96, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
[Out] Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Maple [A]
time = 0.43, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
[Out] int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cosh(d*x + c) + a)/(a*x*cosh(d*x + c) + a*x), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))**(1/2),x)
[Out] Integral(1/(x*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x\,\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*cosh(c + d*x))^(1/2)),x)
```

```
[Out] int(1/(x*(a + a*cosh(c + d*x))^(1/2)), x)
```

3.143 $\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{1}{x^2\sqrt{a+a\cosh(c+dx)}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Rubi [A]
time = 0.07, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
[Out] Defer[Int][1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Mathematica [A]
time = 1.07, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
[Out] Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Maple [A]
time = 0.44, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{a+a\cosh\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

```
[Out] int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(d*x + c) + a)/(a*x^2*cosh(d*x + c) + a*x^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \sqrt{a \left(\cosh \left(c+dx\right)+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(x**2*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{1}{x^2 \sqrt{a + a \cosh(c + d x)}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*cosh(c + d*x))^(1/2)),x)
[Out] int(1/(x^2*(a + a*cosh(c + d*x))^(1/2)), x)
```

3.144 $\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=402

$$\frac{3x^2}{a\sqrt{a+a\cosh(x)}}-\frac{24x\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x^3\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{24i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right.}{a\sqrt{a+a\cosh(x)}}$$

```
[Out] 3*x^2/a/(a+a*cosh(x))^(1/2)-24*x*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh
(x))^(1/2)+x^3*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(1/2)+24*I*co
sh(1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)-3*I*x^2*cosh(1/2*x
)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)-24*I*cosh(1/2*x)*polylog(2
,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+3*I*x^2*cosh(1/2*x)*polylog(2,I*exp(1/
2*x))/a/(a+a*cosh(x))^(1/2)+12*I*x*cosh(1/2*x)*polylog(3,-I*exp(1/2*x))/a/(
a+a*cosh(x))^(1/2)-12*I*x*cosh(1/2*x)*polylog(3,I*exp(1/2*x))/a/(a+a*cosh(x
))^(1/2)-24*I*cosh(1/2*x)*polylog(4,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+24
*I*cosh(1/2*x)*polylog(4,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+1/2*x^3*tanh(1
/2*x)/a/(a+a*cosh(x))^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 9, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {3400, 4271, 4265, 2317, 2438, 2611, 6744, 2320, 6724}

$$\frac{x^3\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{24x\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{3ix^2\text{Li}_2\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{3ix^2\text{Li}_2\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{12ix\text{Li}_3\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{12ix\text{Li}_3\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{24i\text{Li}_2\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{24i\text{Li}_2\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{24i\text{Li}_4\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{24i\text{Li}_4\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{x^3\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a\cosh(x)+a}}+\frac{3x^2}{a\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (3*x^2)/(a*Sqrt[a + a*Cosh[x]]) - (24*x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[
a + a*Cosh[x]]) + (x^3*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) +
 ((24*I)*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((3*
I)*x^2*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I
)*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((3*I)*x^2*Cos
h[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((12*I)*x*Cosh[x/2]
*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((12*I)*x*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((24*I)*Cosh[x/2]*PolyLog[4,
 (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((24*I)*Cosh[x/2]*PolyLog[4, I*E^
(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^3*Tanh[x/2])/(2*a*Sqrt[a + a*Cosh[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
```

```
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\int x^3\text{sech}^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cosh(x)}}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right)\int x^3\text{sech}\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cosh(x)}} - \frac{(6\cosh}{a\sqrt{}}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{}{2a}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{3i}{}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24}{}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24}{}$$

$$= \frac{3x^2}{a\sqrt{a+a\cosh(x)}} - \frac{24x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^3\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{24}{}$$

Mathematica [**A**]
time = 1.93, size = 716, normalized size = 1.78

Antiderivative was successfully verified.

```
[In] Integrate[x^3/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] ((-1/8*I)*Cosh[x/2]*((48*I)*x^2*Cosh[x/2] + 7*Pi^4*Cosh[x/2]^2 + (4*I)*Pi^3
*x*Cosh[x/2]^2 + 6*Pi^2*x^2*Cosh[x/2]^2 - (4*I)*Pi*x^3*Cosh[x/2]^2 - x^4*Co
sh[x/2]^2 - 192*x*Cosh[x/2]^2*Log[1 - I/E^(x/2)] + (8*I)*Pi^3*Cosh[x/2]^2*L
og[1 + I/E^(x/2)] + 192*x*Cosh[x/2]^2*Log[1 + I/E^(x/2)] + 24*Pi^2*x*Cosh[x
/2]^2*Log[1 + I/E^(x/2)] - (24*I)*Pi*x^2*Cosh[x/2]^2*Log[1 + I/E^(x/2)] - 8
*x^3*Cosh[x/2]^2*Log[1 + I/E^(x/2)] - 24*Pi^2*x*Cosh[x/2]^2*Log[1 - I*E^(x/
2)] + (24*I)*Pi*x^2*Cosh[x/2]^2*Log[1 - I*E^(x/2)] - (8*I)*Pi^3*Cosh[x/2]^2
*Log[1 + I*E^(x/2)] + 8*x^3*Cosh[x/2]^2*Log[1 + I*E^(x/2)] + (8*I)*Pi^3*Cos
h[x/2]^2*Log[Tan[(Pi + I*x)/4]] - 48*(8 + Pi^2 - (2*I)*Pi*x - x^2)*Cosh[x/2
]^2*PolyLog[2, (-I)/E^(x/2)] + 384*Cosh[x/2]^2*PolyLog[2, I/E^(x/2)] + 48*x
^2*Cosh[x/2]^2*PolyLog[2, (-I)*E^(x/2)] - 48*Pi^2*Cosh[x/2]^2*PolyLog[2, I*
E^(x/2)] + (96*I)*Pi*x*Cosh[x/2]^2*PolyLog[2, I*E^(x/2)] + (192*I)*Pi*Cosh[
x/2]^2*PolyLog[3, (-I)/E^(x/2)] + 192*x*Cosh[x/2]^2*PolyLog[3, (-I)/E^(x/2)
] - 192*x*Cosh[x/2]^2*PolyLog[3, (-I)*E^(x/2)] - (192*I)*Pi*Cosh[x/2]^2*Pol
yLog[3, I*E^(x/2)] + 384*Cosh[x/2]^2*PolyLog[4, (-I)/E^(x/2)] + 384*Cosh[x/
2]^2*PolyLog[4, (-I)*E^(x/2)] + (8*I)*x^3*Sinh[x/2]))/(a*(1 + Cosh[x]))^(3/
2)
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a + a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^3/(a+a*cosh(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 8/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 + 36*sqrt(2)*integrate(1/9*x^3*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3
*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 72*sqrt(2)*integra
te(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*
x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 96*sqrt(2)*integrate(1/9*x*e^(3/2*x)/(a
^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^
```

```
(3/2)), x) - 4/27*(9*sqrt(2)*sqrt(a)*x^3 + 18*sqrt(2)*sqrt(a)*x^2 + 24*sqrt
(2)*sqrt(a)*x + 16*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2*e^(2*x)
+ 3*a^2*e^x + a^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(a*cosh(x) + a)*x^3/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(a+a*cosh(x))**(3/2),x)
[Out] Integral(x**3/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="giac")
[Out] integrate(x^3/(a*cosh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(a + a*cosh(x))^(3/2),x)
[Out] int(x^3/(a + a*cosh(x))^(3/2), x)
```

3.145 $\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=248

$$\frac{2x}{a\sqrt{a+a\cosh(x)}}+\frac{x^2\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{4\text{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{2ix\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/\right.}}{a\sqrt{a+a\cosh(x)}}$$

```
[Out] 2*x/a/(a+a*cosh(x))^(1/2)+x^2*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x)
)^(1/2)-4*arctan(sinh(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(1/2)-2*I*x*cosh(
1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+2*I*x*cosh(1/2*x)*pol
ylog(2,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+4*I*cosh(1/2*x)*polylog(3,-I*exp
(1/2*x))/a/(a+a*cosh(x))^(1/2)-4*I*cosh(1/2*x)*polylog(3,I*exp(1/2*x))/a/(a
+a*cosh(x))^(1/2)+1/2*x^2*tanh(1/2*x)/a/(a+a*cosh(x))^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3400, 4271, 3855, 4265, 2611, 2320, 6724}

$$\frac{x^2\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{4\cosh\left(\frac{x}{2}\right)\text{ArcTan}\left(\sinh\left(\frac{x}{2}\right)\right)}{a\sqrt{a\cosh(x)+a}}-\frac{2ix\text{Li}_2\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{2ix\text{Li}_2\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{4i\text{Li}_3\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{4i\text{Li}_3\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{x^2\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a\cosh(x)+a}}+\frac{2x}{a\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*x)/(a*Sqrt[a + a*Cosh[x]]) + (x^2*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a +
 a*Cosh[x]]) - (4*ArcTan[Sinh[x/2]]*Cosh[x/2])/(a*Sqrt[a + a*Cosh[x]]) - ((
2*I)*x*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((2*I)
*x*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + ((4*I)*Cosh[x
/2]*PolyLog[3, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) - ((4*I)*Cosh[x/2]*Po
lyLog[3, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x^2*Tanh[x/2])/(2*a*Sqrt[a
+ a*Cosh[x]])
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
```

```
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4271

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (Dist[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2))), Int[(c + d*x)
^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Dist[b^2*((n - 2)/(n - 1)), Int
[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x] - Simp[b^2*d*m*(c + d*x)^(m -
 1)*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /; FreeQ[{b, c, d
, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\int x^2\text{sech}^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cosh(x)}}$$
$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}} + \frac{\cosh\left(\frac{x}{2}\right)\int x^2\text{sech}\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cosh(x)}} - \frac{(2\cosh(}{a\sqrt{a}}$$
$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{}{2a}$$
$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2i}{}$$
$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2i}{}$$
$$= \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\tan^{-1}\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2i}{}$$

Mathematica [**A**]
time = 0.69, size = 214, normalized size = 0.86

$$\frac{\cosh\left(\frac{x}{2}\right)\left(4x\cosh\left(\frac{x}{2}\right) - 16\text{ArcTan}\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\cosh^2\left(\frac{x}{2}\right) + 2x^2\text{ArcTan}\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\cosh^2\left(\frac{x}{2}\right) - 4ix\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-i\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\right) + 4ix\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(2,i\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\right) + 8i\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,-i\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\right) - 8i\cosh^2\left(\frac{x}{2}\right)\text{PolyLog}\left(3,i\left(\cosh\left(\frac{x}{2}\right)+\sinh\left(\frac{x}{2}\right)\right)\right) + x^2\sinh\left(\frac{x}{2}\right)\right)}{(a(1+\cosh(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (Cosh[x/2]*(4*x*Cosh[x/2] - 16*ArcTan[Cosh[x/2] + Sinh[x/2]]*Cosh[x/2]^2 +
2*x^2*ArcTan[Cosh[x/2] + Sinh[x/2]]*Cosh[x/2]^2 - (4*I)*x*Cosh[x/2]^2*PolyL
og[2, (-I)*(Cosh[x/2] + Sinh[x/2])] + (4*I)*x*Cosh[x/2]^2*PolyLog[2, I*(Cos
h[x/2] + Sinh[x/2])] + (8*I)*Cosh[x/2]^2*PolyLog[3, (-I)*(Cosh[x/2] + Sinh[
x/2])] - (8*I)*Cosh[x/2]^2*PolyLog[3, I*(Cosh[x/2] + Sinh[x/2])] + x^2*Sinh
[x/2]))/(a*(1 + Cosh[x]))^(3/2)
```

Maple [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a+a\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2/(a+a*cosh(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 4/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
 + 36*sqrt(2)*integrate(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3
*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 48*sqrt(2)*integra
te(1/9*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*x)
 + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/27*(9*sqrt(2)*x^2 + 12*sqrt(2)*x + 8*sq
rt(2))*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(
3/2))
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)*x^2/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\left(a\left(\cosh\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x**2/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(a*cosh(x) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{(a + a\cosh(x))^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(a + a*cosh(x))^(3/2),x)
```

```
[Out] int(x^2/(a + a*cosh(x))^(3/2), x)
```

3.146 $\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=140

$$\frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{i\cosh\left(\frac{x}{2}\right)\text{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}$$

```
[Out] 1/a/(a+a*cosh(x))^(1/2)+x*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(1
/2)-I*cosh(1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+I*cosh(1/2
*x)*polylog(2,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+1/2*x*tanh(1/2*x)/a/(a+a*
cosh(x))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3400, 4270, 4265, 2317, 2438}

$$\frac{x\text{ArcTan}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}-\frac{i\text{Li}_2\left(-ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{i\text{Li}_2\left(ie^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a\cosh(x)+a}}+\frac{1}{a\sqrt{a\cosh(x)+a}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a\cosh(x)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[x/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] 1/(a*Sqrt[a + a*Cosh[x]]) + (x*ArcTan[E^(x/2)]*Cosh[x/2])/(a*Sqrt[a + a*Cos
h[x]]) - (I*Cosh[x/2]*PolyLog[2, (-I)*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (
I*Cosh[x/2]*PolyLog[2, I*E^(x/2)])/(a*Sqrt[a + a*Cosh[x]]) + (x*Tanh[x/2])/
(2*a*Sqrt[a + a*Cosh[x]])
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3400

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Dist[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n])), Int[(c + d*x)^m*Sin[e/2 + a*
(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rule 4265

```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Dist[d*(m/(f*fz*I)), Int[(c + d*x)^(m - 1)*Log[1
 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Dist[d*(m/(f*fz*I)), Int[(c +
d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c,
d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Rule 4270

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (Dist[b^2*((n - 2)/(n - 1)), Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2),
x], x] - Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x]) /
; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx &= \frac{\cosh\left(\frac{x}{2}\right)\int x\operatorname{sech}^3\left(\frac{x}{2}\right)\,dx}{2a\sqrt{a+a\cosh(x)}}\\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}+\frac{\cosh\left(\frac{x}{2}\right)\int x\operatorname{sech}\left(\frac{x}{2}\right)\,dx}{4a\sqrt{a+a\cosh(x)}}\\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}-\frac{(i\cosh\left(\frac{x}{2}\right)}{2a\sqrt{}}\\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}-\frac{(i\cosh\left(\frac{x}{2}\right)}{}\\
&= \frac{1}{a\sqrt{a+a\cosh(x)}}+\frac{x\tan^{-1}\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{i\cosh\left(\frac{x}{2}\right)\operatorname{Li}_2\left(-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{i\cosh\left(\frac{x}{2}\right.}{a\sqrt{a+}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 137, normalized size = 0.98

$$\frac{2\cosh^2\left(\frac{x}{2}\right)}{(a(1+\cosh(x)))^{3/2}}+\frac{2\cosh^3\left(\frac{x}{2}\right)\left(-\frac{1}{2}ix\left(\log\left(1-ie^{-x/2}\right)-\log\left(1+ie^{-x/2}\right)\right)-i\left(\operatorname{PolyLog}\left(2,-ie^{-x/2}\right)-\operatorname{PolyLog}\left(2,ie^{-x/2}\right)\right)\right)}{(a(1+\cosh(x)))^{3/2}}+\frac{x\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)}{(a(1+\cosh(x)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/(a + a*Cosh[x])^(3/2),x]
```

```
[Out] (2*Cosh[x/2]^2)/(a*(1 + Cosh[x]))^(3/2) + (2*Cosh[x/2]^3*((-1/2*I)*x*(Log[1
 - I/E^(x/2)] - Log[1 + I/E^(x/2)]) - I*(PolyLog[2, (-I)/E^(x/2)] - PolyLog
```

```
[2, I/E^(x/2)])))/(a*(1 + Cosh[x]))^(3/2) + (x*Cosh[x/2]*Sinh[x/2])/(a*(1 +
 Cosh[x]))^(3/2)
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \frac{x}{(a + a\cosh(x))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(x/(a+a*cosh(x))^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] 1/9*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) + 3
*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2))
+ 12*sqrt(2)*integrate(1/3*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x)
 + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/9*(3*sqrt(2)*sqrt(a
)*x + 2*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x
 + a^2)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)*x/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(a(\cosh(x) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(x/(a*(cosh(x) + 1))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x/(a*cosh(x) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{{\left(a+a\,\cosh\left(x\right)\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(a + a*cosh(x))^(3/2),x)
```

```
[Out] int(x/(a + a*cosh(x))^(3/2), x)
```

3.147 $\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{x(a+a\cosh(x))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x/(a+a*cosh(x))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x*(a + a*Cosh[x])^(3/2)),x]
[Out] Defer[Int][1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Mathematica [A]
time = 6.46, size = 0, normalized size = 0.00

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x*(a + a*Cosh[x])^(3/2)),x]
[Out] Integrate[1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a+a\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(1/x/(a+a*cosh(x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)/(a^2*x*cosh(x)^2 + 2*a^2*x*cosh(x) + a^2*x), x
)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(a\left(\cosh\left(x\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(1/(x*(a*(cosh(x) + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x\left(a+a\cosh\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + a*cosh(x))^(3/2)),x)
[Out] int(1/(x*(a + a*cosh(x))^(3/2)), x)
```

3.148 $\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\frac{1}{x^2(a+a\cosh(x))^{3/2}},x\right)$$

```
[Out] Unintegrable(1/x^2/(a+a*cosh(x))^(3/2),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
[Out] Defer[Int][1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Rubi steps

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Mathematica [A]
time = 7.88, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
[Out] Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Maple [A]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\left(a+a\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(a+a*cosh(x))^(3/2),x)
```

```
[Out] int(1/x^2/(a+a*cosh(x))^(3/2),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*cosh(x) + a)/(a^2*x^2*cosh(x)^2 + 2*a^2*x^2*cosh(x) + a^2*x
^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(a \left(\cosh \left(x\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(a+a*cosh(x))**(3/2),x)
```

```
[Out] Integral(1/(x**2*(a*(cosh(x) + 1))**(3/2)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{x^2 \left(a + a\cosh\left(x\right)\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + a*cosh(x))^(3/2)),x)
```

```
[Out] int(1/(x^2*(a + a*cosh(x))^(3/2)), x)
```

3.149 $\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$

Optimal. Leaf size=21

$$\text{Int}\left(\frac{\sqrt[3]{a+a\cosh(c+dx)}}{x},x\right)$$

```
[Out] Unintegrable((a+a*cosh(d*x+c))^(1/3)/x,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

```
[Out] Defer[Int][(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Rubi steps

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Mathematica [A]
time = 1.95, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

```
[Out] Integrate[(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Maple [A]
time = 0.38, size = 0, normalized size = 0.00

$$\int \frac{(a+a\cosh(dx+c))^{\frac{1}{3}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

```
[Out] int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

```
[Out] integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt[3]{a\left(\cosh\left(c+dx\right)+1\right)}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))**(1/3)/x,x)
```

```
[Out] Integral((a*(cosh(c + d*x) + 1))**(1/3)/x, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.05

$$\int \frac{(a + a\cosh(c + d\,x))^{1/3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(c + d*x))^(1/3)/x,x)
[Out] int((a + a*cosh(c + d*x))^(1/3)/x, x)
```

3.150 $\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$

Optimal. Leaf size=23

$$\text{Int}((c+dx)^m(a+a\cosh(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
[Out] Defer[Int][(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx = \int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Mathematica [A]
time = 4.35, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
[Out] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+a\cosh\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a + a\cosh(e + f\,x))^n\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^n*(c + d*x)^m,x)
[Out] int((a + a*cosh(e + f*x))^n*(c + d*x)^m, x)
```

3.151 $\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx$

Optimal. Leaf size=402

$$\frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3\,2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m}{}$$

```
[Out] 5/2*a^3*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*a^3*exp(3*e-3*c*f/d)*(d*x+c)^m*G
AMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3*2^(-3-m)*a^3*exp(2*e-2*c*f/
d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*a^3*exp(e-
c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-15/8*a^3*exp(
-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a
^3*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
-1/8*3^(-1-m)*a^3*exp(-3*e+3*c*f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((
f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.41, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3399, 3393, 3388, 2212}

$$\frac{a^3 3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3a^3 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{15a^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}-\frac{15a^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}-\frac{3a^3 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}-\frac{a^3 3^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}+\frac{5a^3(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

```
[Out] (5*a^3*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (3^(-1 - m)*a^3*E^(3*e - (3*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^
m) + (3*2^(-3 - m)*a^3*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(
c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (15*a^3*E^(e - (c*f)/d)*(c + d*x
)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (15*a^
3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(8*f*((f*(c +
 d*x))/d)^m) - (3*2^(-3 - m)*a^3*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 +
 m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 - m)*a^3*E^(-3*e +
 (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8*f*((f*(c + d*x)
)/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx &= \left(8a^3\right)\int (c+dx)^m\sin^6\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)dx\\ &= \left(8a^3\right)\int \left(\frac{5}{16}(c+dx)^m+\frac{15}{32}(c+dx)^m\cosh(e+fx)+\frac{3}{16}(c+dx)^{m}\right.\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{4}a^3\int (c+dx)^m\cosh(3e+3fx)\,dx+\frac{1}{2}\left(3a^3\right)\int\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{8}a^3\int e^{-i(3ie+3ifx)}(c+dx)^m\,dx+\frac{1}{8}a^3\int e^{i(3ie+3i}\\ &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.}{8f}\end{aligned}$$

Mathematica [A]
time = 1.64, size = 429, normalized size = 1.07

$$-\frac{2^{-6-m}3^{-1-m}a^3e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))^3\left(-2^m de^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)-3^{2+m}de^{5e+\frac{cf}{d}}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)-5\,2^m3^{2+m}de^{4e+\frac{2cf}{d}}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)+5\,2^m3^{2+m}de^{2e+\frac{4cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)+3^{2+m}de^{e+\frac{5cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)+2^me^{\frac{3cf}{d}}\left(-20\,3^{1+m}e^{3e}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m+de^{\frac{3cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)\right)\right)\operatorname{sech}^6\left(\frac{1}{2}(e+fx)\right)}{df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

```
[Out] -((2^(-6 - m)*3^(-1 - m)*a^3*(c + d*x)^m*(1 + Cosh[e + f*x])^3*(-(2^m*d*E^(
6*e)*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d]) - 3^(2 +
 m)*d*E^(5*e + (c*f)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x
))/d] - 5*2^m*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*G
amma[1 + m, -((f*(c + d*x))/d)] + 5*2^m*3^(2 + m)*d*E^(2*e + (4*c*f)/d)*(1
+ m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] + 3^(2 + m)*d*E^(
e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c + d*x))/
d] + 2^m*E^((3*c*f)/d)*(-20*3^(1 + m)*E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)
^2)/d^2))^m + d*E^((3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (
3*f*(c + d*x))/d]))*Sech[(e + f*x)/2]^6)/(d*E^(3*(e + (c*f)/d))*f*(1 + m)*(
-((f^2*(c + d*x)^2)/d^2))^m))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m (a+a\cosh (fx+e))^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

Maxima [A]
time = 0.14, size = 381, normalized size = 0.95

$$-\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{3cf}{d}-3e\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{3cf}{d}+3e\right)}E_{-m}\left(-\frac{3(dx+c)f}{d}\right)}{d}\right)a^3-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{2cf}{d}-2e\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{2cf}{d}+2e\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2(dx+c)^{m+1}}{d(m+1)}\right)a^3-\frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^3+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="maxima")
```

```
[Out] -1/8*((d*x + c)^(m + 1)*e^(3*c*f/d - 3*e)*exp_integral_e(-m, 3*(d*x + c)*f/
d)/d + 3*(d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/
d + 3*(d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d
 + (d*x + c)^(m + 1)*e^(-3*c*f/d + 3*e)*exp_integral_e(-m, -3*(d*x + c)*f/d
)/d)*a^3 - 3/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d
*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2
*(d*x + c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^3 - 3/2*((d*x + c)^(
m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*x + c)^(m + 1
)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^3 + (d*x + c)^(m +
 1)*a^3/(d*(m + 1))
```

Fricas [A]
time = 0.14, size = 783, normalized size = 1.95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*((a^3*d*m + a^3*d)*cosh((d*m*log(3*f/d) - 3*c*f + 3*d*cosh(1) + 3*d*s
inh(1))/d)*gamma(m + 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*cosh((d*m*
log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(1))/d)*gamma(m + 1, 2*(d*f*x +
c*f)/d) + 45*(a^3*d*m + a^3*d)*cosh((d*m*log(f/d) - c*f + d*cosh(1) + d*sin
h(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(a^3*d*m + a^3*d)*cosh((d*m*log
(-f/d) + c*f - d*cosh(1) - d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) - 9
*(a^3*d*m + a^3*d)*cosh((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1
))/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (a^3*d*m + a^3*d)*cosh((d*m*log(-3
*f/d) + 3*c*f - 3*d*cosh(1) - 3*d*sinh(1))/d)*gamma(m + 1, -3*(d*f*x + c*f)
/d) - (a^3*d*m + a^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/d)*sinh((d*m*log(3*f/d
) - 3*c*f + 3*d*cosh(1) + 3*d*sinh(1))/d) - 9*(a^3*d*m + a^3*d)*gamma(m + 1
, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(
1))/d) - 45*(a^3*d*m + a^3*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f
/d) - c*f + d*cosh(1) + d*sinh(1))/d) + 45*(a^3*d*m + a^3*d)*gamma(m + 1, -
(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d*cosh(1) - d*sinh(1))/d) + 9*
(a^3*d*m + a^3*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) +
2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) + (a^3*d*m + a^3*d)*gamma(m + 1, -3*(
d*f*x + c*f)/d)*sinh((d*m*log(-3*f/d) + 3*c*f - 3*d*cosh(1) - 3*d*sinh(1))/
d) - 60*(a^3*d*f*x + a^3*c*f)*cosh(m*log(d*x + c)) - 60*(a^3*d*f*x + a^3*c*
f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\cosh(e + f\,x))^3\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^3*(c + d*x)^m,x)
[Out] int((a + a*cosh(e + f*x))^3*(c + d*x)^m, x)
```

3.152 $\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx$

Optimal. Leaf size=263

$$\frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+}{d}\right.}{}$$

```
[Out] 3/2*a^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*a^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA
(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a^2*exp(e-c*f/d)*(d*x+c)^m*GAMMA(
1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a^2*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+
m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*a^2*exp(-2*e+2*c*f/d)*(d*x+c)^m
*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]

time = 0.24, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3399, 3393, 3388, 2212}

$$\frac{a^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}-\frac{a^2e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}-\frac{a^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{3a^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] (3*a^2*(c + d*x)^(1 + m))/(2*d*(1 + m)) + (2^(-3 - m)*a^2*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m)
 + (a^2*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-
((f*(c + d*x))/d))^m) - (a^2*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (2^(-3 - m)*a^2*E^(-2*e + (2*c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3399

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Dist[(2*a)^n, Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b))) +
 f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^2
, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx &= \left(4a^2\right)\int (c+dx)^m\sin^4\left(\frac{1}{2}(ie+\pi)+\frac{ifx}{2}\right)\,dx \\
&= \left(4a^2\right)\int \left(\frac{3}{8}(c+dx)^m+\frac{1}{2}(c+dx)^m\cosh(e+fx)+\frac{1}{8}(c+dx)^m\operatorname{cosh}\right. \\
&= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{2}a^2\int (c+dx)^m\cosh(2e+2fx)\,dx+\left(2a^2\right)\int (c \\
&= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{1}{4}a^2\int e^{-i(2ie+2ifx)}(c+dx)^m\,dx+\frac{1}{4}a^2\int e^{i(2ie+2if} \\
&= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.}{f}
\end{aligned}$$

Mathematica [A]
time = 0.78, size = 302, normalized size = 1.15

$$-\frac{2^{-5-m}a^2e^{-2\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))^2\left(-32^{2+m}e^{2\left(e+\frac{cf}{d}\right)}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m-de^{4e}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)-2^{3+m}de^{3e+\frac{cf}{d}}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)+2^{3+m}de^{e+\frac{3cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)+de^{\frac{4cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)\right)\operatorname{sech}^4\left(\frac{1}{2}(e+fx)\right)}{df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] -((2^(-5 - m)*a^2*(c + d*x)^m*(1 + Cosh[e + f*x])^2*(-3*2^(2 + m)*E^(2*(e +
 (c*f)/d))*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(4*e)*(1 + m)*(f*
(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^(3 + m)*d*E^(3*e + (c*f)/
d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, -((f*(c + d*x))/d)] + 2^(3 + m)*d*E
^(e + (3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/
d] + d*E^((4*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c +
```

```
d*x))/d])*Sech[(e + f*x)/2]^4)/(d*E^(2*(e + (c*f)/d))*f*(1 + m)*(-((f^2*(c
+ d*x)^2)/d^2))^m))
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m \left(a+a\cosh\left(fx+e\right)\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.08, size = 213, normalized size = 0.81

$$-\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{2cf}{d}-2e\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{2cf}{d}+2e\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2(dx+c)^{m+1}}{d(m+1)}\right)a^2-\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2+\frac{(dx+c)^{m+1}a^2}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -1/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c)*f/
d)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x + c)
*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^2 - ((d*x + c)^(m + 1)*e^(c*f/
d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(-c*f/d +
e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2 + (d*x + c)^(m + 1)*a^2/(d*(m
+ 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. 2(261) = 522.
time = 0.10, size = 539, normalized size = 2.05

$$-\frac{(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh(1)+2d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)+8(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-8(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh(1)-2d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{2(dfx+cf)}{d}\right)-(a^2dm+a^2d)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh(1)+2d\sinh(1)}{d}\right)-8(a^2dm+a^2d)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)+8(a^2dm+a^2d)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)+(a^2dm+a^2d)\Gamma\left(m+1,-\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh(1)-2d\sinh(1)}{d}\right)-12(a^2dfx+a^2cf)\cosh\left(m\log\left(dx+c\right)\right)-12(a^2dfx+a^2cf)\sinh\left(m\log\left(dx+c\right)\right)}{8(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((a^2*d*m + a^2*d)*cosh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*si
nh(1))/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 8*(a^2*d*m + a^2*d)*cosh((d*m*l
og(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8
*(a^2*d*m + a^2*d)*cosh((d*m*log(-f/d) + c*f - d*cosh(1) - d*sinh(1))/d)*ga
mma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*cosh((d*m*log(-2*f/d) + 2*
c*f - 2*d*cosh(1) - 2*d*sinh(1))/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (a^2
*d*m + a^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) - 2*c*f
+ 2*d*cosh(1) + 2*d*sinh(1))/d) - 8*(a^2*d*m + a^2*d)*gamma(m + 1, (d*f*x +
```

```
c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d) + 8*(a^2*d*m
+ a^2*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d*cosh(
1) - d*sinh(1))/d) + (a^2*d*m + a^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sin
h((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) - 12*(a^2*d*f*x
+ a^2*c*f)*cosh(m*log(d*x + c)) - 12*(a^2*d*f*x + a^2*c*f)*sinh(m*log(d*x +
c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a + a\cosh(e + f\,x))^2\,(c + d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))^2*(c + d*x)^m,x)
```

```
[Out] int((a + a*cosh(e + f*x))^2*(c + d*x)^m, x)
```

3.153 $\int (c+dx)^m(a+a\cosh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{ae^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+\right.}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)+1/2*a*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d
)/f/((-f*(d*x+c)/d)^m)-1/2*a*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3388, 2212}

$$\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{ae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (a*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (a*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+a\cosh(e+fx))\,dx &= \int \left(a(c+dx)^m + a(c+dx)^m\cosh(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} + a\int (c+dx)^m\cosh(e+fx)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{1}{2}a\int e^{-i(ie+ifx)}(c+dx)^m\,dx + \frac{1}{2}a\int e^{i(ie+ifx)}(c+dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)} + \frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 189, normalized size = 1.44

$$-\frac{ae^{-e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))\left(-2e^{e+\frac{cf}{d}}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m-de^{2e}(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)+de^{\frac{2cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)\right)\operatorname{sech}^2\left(\frac{1}{2}(e+fx)\right)}{4df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
[Out] -1/4*(a*E^(-e - (c*f)/d)*(c + d*x)^m*(1 + Cosh[e + f*x])*(-2*E^(e + (c*f)/d
)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(2*e)*(1 + m)*(f*(c/d + x)
)^m*Gamma[1 + m, -((f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c + d
*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d])*Sech[(e + f*x)/2]^2)/(d*f*(1 + m)
*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int (dx+c)^m\,(a+a\cosh(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
[Out] int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.06, size = 102, normalized size = 0.78

$$-\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d +
 (d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a +
 (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.
time = 0.10, size = 271, normalized size = 2.07

$$-\frac{(adm+ad)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-(adm+ad)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(adm+ad)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)+(adm+ad)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)-2\,(adfx+acf)\cosh\left(m\log\left(dx+c\right)\right)-2\,(adfx+acf)\sinh\left(m\log\left(dx+c\right)\right)}{2\,(dfm+df)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((a*d*m + a*d)*cosh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*ga
mma(m + 1, (d*f*x + c*f)/d) - (a*d*m + a*d)*cosh((d*m*log(-f/d) + c*f - d*c
osh(1) - d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a*d*m + a*d)*gamma
(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/
d) + (a*d*m + a*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f
 - d*cosh(1) - d*sinh(1))/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c)) - 2
*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+a*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((a*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a+a\cosh\left(e+f\,x\right)\right)\,{\left(c+d\,x\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + a*cosh(e + f*x))*(c + d*x)^m,x)
[Out] int((a + a*cosh(e + f*x))*(c + d*x)^m, x)
```

3.154 $\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a+a\cosh(e+fx)},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx=\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Mathematica [A]
time = 2.94, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+a\cosh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\cosh(e+fx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**m/(cosh(e + f*x) + 1), x)/a
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{a+a\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cosh(e + f*x)),x)
[Out] int((c + d*x)^m/(a + a*cosh(e + f*x)), x)
```

3.155 $\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2} \, dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{(a+a\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2} \, dx = \int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2} \, dx$$

Mathematica [A]
time = 6.08, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2} \, dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{\left(a+a\cosh\left(fx+e\right)\right)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
[Out] int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m/(a^2*cosh(f*x + e)^2 + 2*a^2*cosh(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{(c+dx)^m}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+a*cosh(f*x+e))**2,x)
[Out] Integral((c + d*x)**m/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x)/a**2
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+a\cosh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + a*cosh(e + f*x))^2,x)
[Out] int((c + d*x)^m/(a + a*cosh(e + f*x))^2, x)
```

3.156 $\int (c+dx)^3(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=89

$$\frac{a(c+dx)^4}{4d}-\frac{6bd^3\cosh(e+fx)}{f^4}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3}+\frac{b(c+dx)^3\sinh(e+fx)}{f}$$

```
[Out] 1/4*a*(d*x+c)^4/d-6*b*d^3*cosh(f*x+e)/f^4-3*b*d*(d*x+c)^2*cosh(f*x+e)/f^2+6
*b*d^2*(d*x+c)*sinh(f*x+e)/f^3+b*(d*x+c)^3*sinh(f*x+e)/f
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{b(c+dx)^3\sinh(e+fx)}{f}-\frac{6bd^3\cosh(e+fx)}{f^4}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^4)/(4*d) - (6*b*d^3*Cosh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*Co
sh[e + f*x])/f^2 + (6*b*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (b*(c + d*x)^3*S
inh[e + f*x])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int (c+dx)^3(a+b\cosh(e+fx))\,dx &= \int \left(a(c+dx)^3+b(c+dx)^3\cosh(e+fx)\right)\,dx \\
&= \frac{a(c+dx)^4}{4d}+b\int (c+dx)^3\cosh(e+fx)\,dx \\
&= \frac{a(c+dx)^4}{4d}+\frac{b(c+dx)^3\sinh(e+fx)}{f}-\frac{(3bd)\int (c+dx)^2\sinh(e+fx)}{f} \\
&= \frac{a(c+dx)^4}{4d}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{b(c+dx)^3\sinh(e+fx)}{f}+ \\
&= \frac{a(c+dx)^4}{4d}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3} \\
&= \frac{a(c+dx)^4}{4d}-\frac{6bd^3\cosh(e+fx)}{f^4}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{6bd^2(}{}
\end{aligned}
$$

Mathematica [A]
time = 0.29, size = 123, normalized size = 1.38

$$\frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-\frac{3bd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^4}+\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*b*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + (b*(c + d*x)*(c^2*f^2 +
2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(87) = 174$.
time = 0.84, size = 482, normalized size = 5.42

method	result
risch	$\frac{a\,d^3x^4}{4}+ac\,d^2x^3+\frac{3a\,c^2d\,x^2}{2}+c^3ax+\frac{a\,c^4}{4d}+\frac{b\left(d^3f^3x^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2x-3c\right.}{2f^4}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+e)\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((fx+e)\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/4*d^3/f^3*a*(f*x+e)^4+d^3/f^3*b*((f*x+e)^3*sinh(f*x+e)-3*(f*x+e)^2*c
osh(f*x+e)+6*(f*x+e)*sinh(f*x+e)-6*cosh(f*x+e))-d^3/f^3*e*a*(f*x+e)^3-3*d^3
/f^3*e*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))+d^2/f^
2*c*a*(f*x+e)^3+3*d^2/f^2*c*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+
2*sinh(f*x+e))+3/2*d^3/f^3*e^2*a*(f*x+e)^2+3*d^3/f^3*e^2*b*((f*x+e)*sinh(f*
x+e)-cosh(f*x+e))-3*d^2/f^2*e*c*a*(f*x+e)^2-6*d^2/f^2*e*c*b*((f*x+e)*sinh(f
*x+e)-cosh(f*x+e))+3/2*d/f*c^2*a*(f*x+e)^2+3*d/f*c^2*b*((f*x+e)*sinh(f*x+e)
-cosh(f*x+e))-d^3/f^3*e^3*a*(f*x+e)-d^3/f^3*e^3*b*sinh(f*x+e)+3*d^2/f^2*e^2
*c*a*(f*x+e)+3*d^2/f^2*e^2*c*b*sinh(f*x+e)-3*d/f*e*c^2*a*(f*x+e)-3*d/f*e*c^
2*b*sinh(f*x+e)+c^3*a*(f*x+e)+b*c^3*sinh(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(91) = 182$.
time = 0.28, size = 250, normalized size = 2.81

$$\frac{1}{4}ad^3x^4 + acd^2x^3 + \frac{3}{2}ac^2dx^2 + ac^3x + \frac{3}{2}bc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{3}{2}bcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{1}{2}bd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right) + \frac{bc^3\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*x
*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2*x
^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e
)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*
x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*sinh(f
*x + e)/f
```

Fricas [A]
time = 0.38, size = 174, normalized size = 1.96

$$\frac{ad^3f^4x^4 + 4\,acd^2f^4x^3 + 6\,ac^2df^4x^2 + 4\,ac^3f^4x - 12\,(bd^3f^2x^2 + 2\,bcd^2f^2x + bc^2df^2 + 2\,bd^3)\cosh(fx + \cosh(1) + \sinh(1)) + 4\,(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + bc^3f^3 + 6\,bcd^2f + 3\,(bc^2df^3 + 2\,bd^3f)x)\sinh(fx + \cosh(1) + \sinh(1))}{4\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 + 2*b*d^3)*cosh(f*x + c
osh(1) + sinh(1)) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*
c*d^2*f + 3*(b*c^2*d*f^3 + 2*b*d^3*f)*x)*sinh(f*x + cosh(1) + sinh(1)))/f^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.
time = 0.28, size = 264, normalized size = 2.97

$$\begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\sinh(e+fx)}{f} + \frac{3bc^2dx\sinh(e+fx)}{f} - \frac{3bc^2d\cosh(e+fx)}{f^2} + \frac{3bcd^2x^2\sinh(e+fx)}{f} - \frac{6bcd^2x\cosh(e+fx)}{f^2} + \frac{6bcd^2\sinh(e+fx)}{f^3} + \frac{bd^3x^3\sinh(e+fx)}{f} - \frac{3bd^3x^2\cosh(e+fx)}{f^2} + \frac{6bd^3x\sinh(e+fx)}{f^3} - \frac{6bd^3\cosh(e+fx)}{f^4} & \text{for}\: f \neq 0 \\ (a + b\cosh(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 + b
*c**3*sinh(e + f*x)/f + 3*b*c**2*d*x*sinh(e + f*x)/f - 3*b*c**2*d*cosh(e +
f*x)/f**2 + 3*b*c*d**2*x**2*sinh(e + f*x)/f - 6*b*c*d**2*x*cosh(e + f*x)/f*
*2 + 6*b*c*d**2*sinh(e + f*x)/f**3 + b*d**3*x**3*sinh(e + f*x)/f - 3*b*d**3
*x**2*cosh(e + f*x)/f**2 + 6*b*d**3*x*sinh(e + f*x)/f**3 - 6*b*d**3*cosh(e
+ f*x)/f**4, Ne(f, 0)), ((a + b*cosh(e))*(c**3*x + 3*c**2*d*x**2/2 + c*d**2
*x**3 + d**3*x**4/4), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) =$ 174.
time = 0.41, size = 258, normalized size = 2.90

$$\frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x - 3\,bd^3f^2x^2 + bc^3f^3 - 6\,bcd^2f^2x - 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f - 6\,bd^3)e^{(fx+e)}}{2\,f^4} - \frac{(bd^3f^3x^3 + 3\,bcd^2f^3x^2 + 3\,bc^2df^3x + 3\,bd^3f^2x^2 + bc^3f^3 + 6\,bcd^2f^2x + 3\,bc^2df^2 + 6\,bd^3fx + 6\,bcd^2f + 6\,bd^3)e^{(-fx-e)}}{2\,f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x^
3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6*b
*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f*x
+ e)/f^4 - 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*d
^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*
b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

Mupad [B]
time = 0.99, size = 187, normalized size = 2.10

$$\frac{\sinh(e+f\,x)\,\left(b\,c^3\,f^2+6\,b\,c\,d^2\right)}{f^3} - \frac{3\cosh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{3\,x\sinh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{3\,b\,d^3\,x^2\cosh(e+f\,x)}{f^2} + \frac{b\,d^3\,x^3\sinh(e+f\,x)}{f} - \frac{6\,b\,c\,d^2\,x\cosh(e+f\,x)}{f^2} + \frac{3\,b\,c\,d^2\,x^2\sinh(e+f\,x)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x)^3,x)
```

```
[Out] (sinh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*cosh(e + f*x)*(2*b*d^3 + b
*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*sinh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (3*b*d^3*x^2*cosh(e +
 f*x))/f^2 + (b*d^3*x^3*sinh(e + f*x))/f - (6*b*c*d^2*x*cosh(e + f*x))/f^2
+ (3*b*c*d^2*x^2*sinh(e + f*x))/f
```

3.157 $\int (c+dx)^2(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=67

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{2bd^2\sinh(e+fx)}{f^3}+\frac{b(c+dx)^2\sinh(e+fx)}{f}$$

```
[Out] 1/3*a*(d*x+c)^3/d-2*b*d*(d*x+c)*cosh(f*x+e)/f^2+2*b*d^2*sinh(f*x+e)/f^3+b*(
d*x+c)^2*sinh(f*x+e)/f
```

Rubi [A]
time = 0.06, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3377, 2717}

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\frac{2bd^2\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^3)/(3*d) - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*b*d^2*Sinh
[e + f*x])/f^3 + (b*(c + d*x)^2*Sinh[e + f*x])/f
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^2(a+b\cosh(e+fx))\,dx &= \int \left(a(c+dx)^2+b(c+dx)^2\cosh(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^3}{3d}+b\int (c+dx)^2\cosh(e+fx)\,dx\\ &= \frac{a(c+dx)^3}{3d}+\frac{b(c+dx)^2\sinh(e+fx)}{f}-\frac{(2bd)\int (c+dx)\sinh(e+fx)\,}{f}\\ &= \frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\ (\\ &= \frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{2bd^2\sinh(e+fx)}{f^3}+\frac{b(c+d}{}\end{aligned}$$

Mathematica [A]
time = 0.21, size = 83, normalized size = 1.24

$$\frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 +
 (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(65) = 130.
time = 0.77, size = 240, normalized size = 3.58

method	result
risch	$\frac{a\,d^2x^3}{3}+adc\,x^2+a\,c^2x+\frac{a\,c^3}{3d}+\frac{b\left(d^2x^2f^2+2cd\,f^2x+c^2f^2-2d^2fx-2cdf+2d^2\right)\mathrm{e}^{fx+e}}{2f^3}-\frac{b\left(d^2x^2f^2+2cd\,f^2x+\right.}{}$
derivativedivides	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\sinh(fx+e)-\cosh(fx+e}{f^2}}{}$
default	$\frac{\frac{d^2a(fx+e)^3}{3f^2}+\frac{d^2b\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}-\frac{d^2ea(fx+e)^2}{f^2}-\frac{2d^2eb((fx+e)\sinh(fx+e)-\cosh(fx+e}{f^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a*(f*x+e)^3+d^2/f^2*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cos
h(f*x+e)+2*sinh(f*x+e))-d^2/f^2*e*a*(f*x+e)^2-2*d^2/f^2*e*b*((f*x+e)*sinh(f
*x+e)-cosh(f*x+e))+d/f*c*a*(f*x+e)^2+2*d/f*c*b*((f*x+e)*sinh(f*x+e)-cosh(f*
```

```
x+e))+d^2/f^2*e^2*a*(f*x+e)+d^2/f^2*e^2*b*sinh(f*x+e)-2*d/f*e*c*a*(f*x+e)-2
*d/f*e*c*b*sinh(f*x+e)+a*c^2*(f*x+e)+b*c^2*sinh(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.
time = 0.27, size = 149, normalized size = 2.22

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}bd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{bc^2\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="maxima")
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
(f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*
e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*sinh(f*x + e)
/f
```

Fricas [A]
time = 0.37, size = 108, normalized size = 1.61

$$\frac{ad^2f^3x^3 + 3\,acdf^3x^2 + 3\,ac^2f^3x - 6\,(bd^2fx + bcdf)\cosh(fx + \cosh(1) + \sinh(1)) + 3\,(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2)\sinh(fx + \cosh(1) + \sinh(1))}{3\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="fricas")
[Out] 1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(b*d^2*f*x + b*c*d
*f)*cosh(f*x + cosh(1) + sinh(1)) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^
2*f^2 + 2*b*d^2)*sinh(f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.
time = 0.17, size = 151, normalized size = 2.25

$$\begin{cases} ac^2x + acdx^2 + \frac{ad^2x^3}{3} + \frac{bc^2\sinh(e+fx)}{f} + \frac{2bcdx\sinh(e+fx)}{f} - \frac{2bcd\cosh(e+fx)}{f^2} + \frac{bd^2x^2\sinh(e+fx)}{f} - \frac{2bd^2x\cosh(e+fx)}{f^2} + \frac{2bd^2\sinh(e+fx)}{f^3} & \text{for } f \neq 0 \\ (a + b\cosh(e))\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cosh(f*x+e)),x)
[Out] Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*sinh(e + f*x)/f +
 2*b*c*d*x*sinh(e + f*x)/f - 2*b*c*d*cosh(e + f*x)/f**2 + b*d**2*x**2*sinh(
e + f*x)/f - 2*b*d**2*x*cosh(e + f*x)/f**2 + 2*b*d**2*sinh(e + f*x)/f**3, N
e(f, 0)), ((a + b*cosh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.
time = 0.40, size = 146, normalized size = 2.18

$$\frac{1}{3}ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2)e^{(fx+e)}}{2\,f^3} - \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2fx + 2\,bcdf + 2\,bd^2)e^{(-fx-e)}}{2\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 - 1/2*(b*d^2
*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2)*e
^(-f*x - e)/f^3
```

Mupad [B]
time = 0.95, size = 110, normalized size = 1.64

$$\frac{a\,d^2\,x^3}{3}+\frac{\mathrm{sinh}\left(e+f\,x\right)\,\left(b\,c^2\,f^2+2\,b\,d^2\right)}{f^3}+a\,c^2\,x+a\,c\,d\,x^2-\frac{2\,b\,d^2\,x\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{b\,d^2\,x^2\,\mathrm{sinh}\left(e+f\,x\right)}{f}-\frac{2\,b\,c\,d\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{2\,b\,c\,d\,x\,\mathrm{sinh}\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x)^2,x)
```

```
[Out] (a*d^2*x^3)/3 + (sinh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*d
*x^2 - (2*b*d^2*x*cosh(e + f*x))/f^2 + (b*d^2*x^2*sinh(e + f*x))/f - (2*b*c
*d*cosh(e + f*x))/f^2 + (2*b*c*d*x*sinh(e + f*x))/f
```

3.158 $\int (c+dx)(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=45

$$\frac{a(c+dx)^2}{2d}-\frac{bd\cosh(e+fx)}{f^2}+\frac{b(c+dx)\sinh(e+fx)}{f}$$

```
[Out] 1/2*a*(d*x+c)^2/d-b*d*cosh(f*x+e)/f^2+b*(d*x+c)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}=0.188$, Rules used = {3398, 3377, 2718}

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{bd\cosh(e+fx)}{f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^2)/(2*d) - (b*d*Cosh[e + f*x])/f^2 + (b*(c + d*x)*Sinh[e + f*x
])/f
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+b\cosh(e+fx))\,dx &= \int (a(c+dx)+b(c+dx)\cosh(e+fx))\,dx \\ &= \frac{a(c+dx)^2}{2d}+b\int (c+dx)\cosh(e+fx)\,dx \\ &= \frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{(bd)\int \sinh(e+fx)\,dx}{f} \\ &= \frac{a(c+dx)^2}{2d}-\frac{bd\cosh(e+fx)}{f^2}+\frac{b(c+dx)\sinh(e+fx)}{f}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.02

$$\frac{-2bd\cosh(e+fx)+f(afx(2c+dx)+2b(c+dx)\sinh(e+fx))}{2f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (-2*b*d*Cosh[e + f*x] + f*(a*f*x*(2*c + d*x) + 2*b*(c + d*x)*Sinh[e + f*x])
)/(2*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.
time = 0.79, size = 91, normalized size = 2.02

method	result	size
risch	$\frac{ad\,x^2}{2}+acx+\frac{b(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}-\frac{b(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$	60
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\sinh(fx+e)}{f}+ac(fx+e)+bc\sinh(fx+e)}{f}$	91
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\sinh(fx+e)}{f}+ac(fx+e)+bc\sinh(fx+e)}{f}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a*(f*x+e)^2+d/f*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-d/f*e*a*(f
*x+e)-d/f*e*b*sinh(f*x+e)+a*c*(f*x+e)+b*c*sinh(f*x+e))
```

Maxima [A]
time = 0.26, size = 70, normalized size = 1.56

$$\frac{1}{2}\,adx^2+acx+\frac{1}{2}\,bd\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{bc\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-
f*x - e)/f^2) + b*c*sinh(f*x + e)/f
```

Fricas [A]
time = 0.36, size = 57, normalized size = 1.27

$$\frac{adf^2x^2 + 2\,acf^2x - 2\,bd\cosh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right) + 2\left(bdfx + bcf\right)\sinh\left(fx + \cosh\left(1\right) + \sinh\left(1\right)\right)}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*cosh(f*x + cosh(1) + sinh(1)) + 2*(b
*d*f*x + b*c*f)*sinh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.09, size = 68, normalized size = 1.51

$$\begin{cases} acx + \frac{adx^2}{2} + \frac{bc\sinh\left(e+fx\right)}{f} + \frac{bdx\sinh\left(e+fx\right)}{f} - \frac{bd\cosh\left(e+fx\right)}{f^2} & \text{for}\ f \neq 0 \\ \left(a + b\cosh\left(e\right)\right)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x)
```

```
[Out] Piecewise((a*c*x + a*d*x**2/2 + b*c*sinh(e + f*x)/f + b*d*x*sinh(e + f*x)/f
 - b*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a + b*cosh(e))*(c*x + d*x**2/2), Tr
ue))
```

Giac [A]
time = 0.42, size = 64, normalized size = 1.42

$$\frac{1}{2}\,adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2\,f^2} - \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] 1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 - 1/2*(b*
d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

Mupad [B]
time = 0.08, size = 49, normalized size = 1.09

$$\frac{f\,\left(b\,c\,\mathrm{sinh}(e+f\,x)+b\,d\,x\,\mathrm{sinh}(e+f\,x)\right)-b\,d\,\mathrm{cosh}(e+f\,x)}{f^2}+a\,c\,x+\frac{a\,d\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x),x)
```

```
[Out] (f*(b*c*sinh(e + f*x) + b*d*x*sinh(e + f*x)) - b*d*cosh(e + f*x))/f^2 + a*c
*x + (a*d*x^2)/2
```

3.159 $\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx$

Optimal. Leaf size=64

$$\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

```
[Out] b*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d+a*ln(d*x+c)/d-b*Shi(c*f/d+f*x)*sinh(-e+c*
f/d)/d
```

Rubi [A]
time = 0.09, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3398, 3384, 3379, 3382}

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

```
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
(b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx &= \int \left(\frac{a}{c+dx}+\frac{b\cosh(e+fx)}{c+dx}\right)dx\\ &= \frac{a\log(c+dx)}{d}+b\int \frac{\cosh(e+fx)}{c+dx}\,dx\\ &= \frac{a\log(c+dx)}{d}+\left(b\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx+\left(b\sinh\left(e-\frac{cf}{d}\right)\right)\\ &= \frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 57, normalized size = 0.89

$$\frac{b\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+a\log(c+dx)+b\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x),x]
[Out] (b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + a*Log[c + d*x] + b*Sinh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Maple [A]
time = 0.87, size = 94, normalized size = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}-\frac{b\,e^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{b\,e^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
[Out] a*ln(d*x+c)/d-1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp(
-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 72, normalized size = 1.12

$$-\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(-c*f/d + e)*e
xp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Fricas [A]
time = 0.39, size = 122, normalized size = 1.91

$$\frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\,a\log\left(dx+c\right)+\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/2*((b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1
) - d*sinh(1))/d) + 2*a*log(d*x + c) + (b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*
x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{a+b\cosh\left(e+fx\right)}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x)
```

```
[Out] Integral((a + b*cosh(e + f*x))/(c + d*x), x)
```

Giac [A]
time = 0.40, size = 67, normalized size = 1.05

$$\frac{b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/2*(b*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + b*Ei(-(d*f*x + c*f)/d)*e^(-e + c
*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int\frac{a+b\cosh(e+f\,x)}{c+d\,x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))/(c + d*x),x)
```

```
[Out] int((a + b*cosh(e + f*x))/(c + d*x), x)
```

3.160 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal. Leaf size=87

$$-\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{bf\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a/d/(d*x+c)-b*cosh(f*x+e)/d/(d*x+c)+b*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2-
b*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3398, 3378, 3384, 3379, 3382}

$$-\frac{a}{d(c+dx)}+\frac{bf\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\cosh(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
```

```
[Out] -(a/(d*(c + d*x))) - (b*Cosh[e + f*x])/(d*(c + d*x)) + (b*f*CoshIntegral[(c
*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (b*f*Cosh[e - (c*f)/d]*SinhIntegral[(
c*f)/d + f*x])/d^2
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx &= \int \left(\frac{a}{(c+dx)^2}+\frac{b\cosh(e+fx)}{(c+dx)^2}\right)dx\\ &= -\frac{a}{d(c+dx)}+b\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{(bf)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d}\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{\left(bf\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\sinh\left(\frac{cf}{d}+fx\right)}{c+dx}\,dx}{d}+\frac{\left(bf\sinh\right.}{}\\ &= -\frac{a}{d(c+dx)}-\frac{b\cosh(e+fx)}{d(c+dx)}+\frac{bf\mathrm{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.25, size = 71, normalized size = 0.82

$$\frac{-\frac{d(a+b\cosh(e+fx))}{c+dx}+bf\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+bf\cosh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
[Out] (-((d*(a + b*Cosh[e + f*x]))/(c + d*x)) + b*f*CoshIntegral[f*(c/d + x)]*Sin
h[e - (c*f)/d] + b*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Maple [A]
time = 0.88, size = 149, normalized size = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}-\frac{fb\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}+\frac{fb\,\mathrm{e}^{\frac{cf-de}{d}}\operatorname{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{bf\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{bf\,\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{expIntegral}\left(1,-fx-e-\right.}{2d^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a/d/(d*x+c)-1/2*f*b*exp(-f*x-e)/d/(d*f*x+c*f)+1/2*f*b/d^2*exp((c*f-d*e)/d)
*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*b*f/d^2*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Maxima [A]
time = 0.30, size = 89, normalized size = 1.02

$$-\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^(-
c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x + c*
d)
```

Fricas [A]
time = 0.35, size = 178, normalized size = 2.05

$$-\frac{2\,bd\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{2\left(d^3x+cd^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*d*cosh(f*x + cosh(1) + sinh(1)) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((
d*f*x + c*f)/d) - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*co
sh(1) - d*sinh(1))/d) - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d) + (b*d*f*x +
 b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d))/(d^3*
x + c*d^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 631 vs. 2(90) = 180.
time = 0.43, size = 631, normalized size = 7.25

$$\frac{1}{2}b\left(\frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\text{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-def^2\text{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}+cf^3\text{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)}-df^2e^{\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^2}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^5e+cd^4f\right)f}-\frac{\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\text{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}-def^2\text{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+cf^3\text{Ei}\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(-\frac{de-cf}{d}\right)}+df^2e^{\left(-\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)}{d}\right)}\right)d^2}{\left((dx+c)d^4\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-d^5e+cd^4f\right)f}\right)-\frac{a}{(dx+c)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*b*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*
e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*f^
2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e
 - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
+ c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
+ c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d^5
*e + c*d^4*f)*f) - ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-(
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*
f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x
 + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x + c)*(d*e/(d
*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\cosh(e+f\,x)}{{(c+d\,x)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))/(c + d*x)^2,x)
```

```
[Out] int((a + b*cosh(e + f*x))/(c + d*x)^2, x)
```

3.161 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal. Leaf size=123

$$-\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}\right.}{2d^3}$$

```
[Out] -1/2*a/d/(d*x+c)^2+1/2*b*f^2*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^3-1/2*b*cosh(f
*x+e)/d/(d*x+c)^2-1/2*b*f^2*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-1/2*b*f*sinh(
f*x+e)/d^2/(d*x+c)
```

Rubi [A]
time = 0.15, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3398, 3378, 3384, 3379, 3382}

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] -1/2*a/(d*(c + d*x)^2) - (b*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (b*f^2*Cosh[
e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (b*f*Sinh[e + f*x])/(2*
d^2*(c + d*x)) + (b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d
^3)
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx &= \int \left(\frac{a}{(c+dx)^3}+\frac{b\cosh(e+fx)}{(c+dx)^3}\right)dx\\ &= -\frac{a}{2d(c+dx)^2}+b\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx\\ &= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{(bf)\int \frac{\sinh(e+fx)}{(c+dx)^2}\,dx}{2d}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{(bf^2)\int \frac{\cosh(e+fx)}{c+dx}\,dx}{2d^2}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{\left(bf^2\cosh\left(e-\frac{cf}{d}\right)\right)\int \frac{\cosh}{}}{2d^2}\\ &= -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{bf\sinh(e}{2d^2(c+}\end{aligned}$$

Mathematica [A]
time = 0.37, size = 95, normalized size = 0.77

$$\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(ad+bd\cosh(e+fx)+bf(c+dx)\sinh(e+fx))}{(c+dx)^2}+bf^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```

```
[Out] (b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a*d + b*d*Cosh[e +
 f*x] + b*f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + b*f^2*Sinh[e - (c*f)/d]
*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.
time = 0.90, size = 296, normalized size = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2}+\frac{f^3 b\,\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 b\,\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 b\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIn}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a/d/(d*x+c)^2+1/4*f^3*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2
)*x+1/4*f^3*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2*b
*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*b/d^3*exp((c*f-d*e
)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*b*f^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4*b*f
^2/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*b*f^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)
```

Maxima [A]
time = 0.31, size = 100, normalized size = 0.81

$$-\frac{1}{2}\,b\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2 d}\right)-\frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/2*b*(e^(c*f/d - e)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e^
(-c*f/d + e)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(d^
3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(119) = 238$.
time = 0.35, size = 293, normalized size = 2.38

$$-\frac{2\,bd^2\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+2\,ad^2-\left((bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+2\left(bd^2fx+bcdf\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)-\left((bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)}{4\,(d^5x^2+2\,cd^4x+c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(2*b*d^2*cosh(f*x + cosh(1) + sinh(1)) + 2*a*d^2 - ((b*d^2*f^2*x^2 + 2
*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^
2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/
```

```
d) + 2*(b*d^2*f*x + b*c*d*f)*sinh(f*x + cosh(1) + sinh(1)) - ((b*d^2*f^2*x^
2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (b*d^2*f^2*x^2 + 2*b*c
*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh
(1))/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(115) = 230.
time = 0.41, size = 316, normalized size = 2.57

$$\frac{bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 2\,bcdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + bc^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + bc^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} - bd^2fxe^{(fx+e)} + bd^2fxe^{(-fx-e)} - bcdfe^{(fx+e)} + bcdfe^{(-fx-e)} - bd^2e^{(fx+e)} - bd^2e^{(-fx-e)} - 2\,ad^2}{4\,(d^5x^2 + 2\,cd^4x + c^2d^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/4*(b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + b*d^2*f^2*x^2*Ei(-(d
*f*x + c*f)/d)*e^(-e + c*f/d) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*
f/d) + 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b*c^2*f^2*Ei((d*
f*x + c*f)/d)*e^(e - c*f/d) + b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d)
 - b*d^2*f*x*e^(f*x + e) + b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e) + b
*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) - b*d^2*e^(-f*x - e) - 2*a*d^2)/(d^
5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{a+b\cosh(e+f\,x)}{{(c+d\,x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))/(c + d*x)^3,x)
```

```
[Out] int((a + b*cosh(e + f*x))/(c + d*x)^3, x)
```

3.162 $\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$

Optimal. Leaf size=250

$$\frac{3b^2cd^2x}{4f^2}+\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{12abd^3\cosh(e+fx)}{f^4}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3b^2d^3\cos}{8}$$

```
[Out] 3/4*b^2*c*d^2*x/f^2+3/8*b^2*d^3*x^2/f^2+1/4*a^2*(d*x+c)^4/d+1/8*b^2*(d*x+c)
^4/d-12*a*b*d^3*cosh(f*x+e)/f^4-6*a*b*d*(d*x+c)^2*cosh(f*x+e)/f^2-3/8*b^2*d
^3*cosh(f*x+e)^2/f^4-3/4*b^2*d*(d*x+c)^2*cosh(f*x+e)^2/f^2+12*a*b*d^2*(d*x+
c)*sinh(f*x+e)/f^3+2*a*b*(d*x+c)^3*sinh(f*x+e)/f+3/4*b^2*d^2*(d*x+c)*cosh(f
*x+e)*sinh(f*x+e)/f^3+1/2*b^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f
```

Rubi [A]

time = 0.20, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3398, 3377, 2718, 3392, 32, 3391}

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}-\frac{12abd^3\cosh(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{3b^2cd^2x}{4f^2}-\frac{3b^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{b^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\cosh^2(e+fx)}{8f^4}+\frac{3b^2d^3x^2}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (3*b^2*c*d^2*x)/(4*f^2) + (3*b^2*d^3*x^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d)
 + (b^2*(c + d*x)^4)/(8*d) - (12*a*b*d^3*Cosh[e + f*x])/f^4 - (6*a*b*d*(c +
 d*x)^2*Cosh[e + f*x])/f^2 - (3*b^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*b^2*d
*(c + d*x)^2*Cosh[e + f*x]^2)/(4*f^2) + (12*a*b*d^2*(c + d*x)*Sinh[e + f*x]
)/f^3 + (2*a*b*(c + d*x)^3*Sinh[e + f*x])/f + (3*b^2*d^2*(c + d*x)*Cosh[e +
 f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x]
)/(2*f)
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)^3+2ab(c+dx)^3\cosh(e+fx)+b^2(c+dx)^3\cosh^2(e+\right.\\
&= \frac{a^2(c+dx)^4}{4d}+(2ab)\int (c+dx)^3\cosh(e+fx)\,dx+b^2\int (c+dx)^3\,\mathrm{co}\\
&= \frac{a^2(c+dx)^4}{4d}-\frac{3b^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)^3\sinh(e+}{f}\\
&= \frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}-\frac{3b^2d^3\,\mathrm{co}}{}\\
&= \frac{3b^2cd^2x}{4f^2}+\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{6abd(c+dx)^2\,\mathrm{co}}{f^2}\\
&= \frac{3b^2cd^2x}{4f^2}+\frac{3b^2d^3x^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}+\frac{b^2(c+dx)^4}{8d}-\frac{12abd^3\cosh(e+}{f^4}
\end{aligned}$$

Mathematica [A]
time = 0.81, size = 232, normalized size = 0.93

$$\frac{-96abd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh(2(e+fx))+2f\left((2a^2+b^2)f^3x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)+16ab(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\sinh(e+fx)+b^2(c+dx)\left(2c^2f^2+4cdf^2x+d^2(3+2f^2x^2)\right)\sinh(2(e+fx))\right)}{16f^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (-96*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*b^
2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*f
*((2*a^2 + b^2)*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*a*b*
(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + b^2*(
c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)])
)/(16*f^4)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1060 vs. $2(234) = 468$.
time = 0.95, size = 1061, normalized size = 4.24

method	result
risch	$\frac{a^2 d^3 x^4}{4}+\frac{d^3 b^2 x^4}{8}+a^2 c\, d^2 x^3+\frac{d^2 b^2 c\, x^3}{2}+\frac{3 a^2 c^2 d\, x^2}{2}+\frac{3 d\, b^2 c^2 x^2}{4}+c^3 a^2 x+\frac{b^2 c^3 x}{2}+\frac{a^2 c^4}{4 d}+\frac{b^2 c^4}{8 d}+$
derivativedivides	Expression too large to display
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(-6*d^3/f^3*e*a*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+e)*cosh(f*x+e)+2*sinh(f
*x+e))-3*d/f*e*c^2*b^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)+6*d/f*c^
2*a*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))-3*d/f*e*c^2*a^2*(f*x+e)+3*d^2/f^2*e
^2*c*a^2*(f*x+e)+d^2/f^2*c*a^2*(f*x+e)^3+b^2*c^3*(1/2*sinh(f*x+e)*cosh(f*x+
e)+1/2*f*x+1/2*e)+c^3*a^2*(f*x+e)+2*c^3*a*b*sinh(f*x+e)+d^3/f^3*b^2*(1/2*(f
*x+e)^3*cosh(f*x+e)*sinh(f*x+e)+1/8*(f*x+e)^4-3/4*(f*x+e)^2*cosh(f*x+e)^2+3
/4*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+3/8*(f*x+e)^2-3/8*cosh(f*x+e)^2)-12*d^2/
f^2*e*c*a*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+1/4*d^3/f^3*a^2*(f*x+e)^4-6*d
/f*e*c^2*a*b*sinh(f*x+e)+6*d^2/f^2*e^2*c*a*b*sinh(f*x+e)+2*d^3/f^3*a*b*((f*
x+e)^3*sinh(f*x+e)-3*(f*x+e)^2*cosh(f*x+e)+6*(f*x+e)*sinh(f*x+e)-6*cosh(f*x
+e))+3*d/f*c^2*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*c
osh(f*x+e)^2)+3*d^3/f^3*e^2*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f
*x+e)^2-1/4*cosh(f*x+e)^2)-d^3/f^3*e^3*b^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2
*f*x+1/2*e)+3*d^2/f^2*c*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x
+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)-
3*d^3/f^3*e*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f*x+e)+1/6*(f*x+e)^3-1/2*(f
*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1/4*f*x+1/4*e)-d^3/f^3*e*a^
2*(f*x+e)^3-d^3/f^3*e^3*a^2*(f*x+e)+3/2*d^3/f^3*e^2*a^2*(f*x+e)^2+3/2*d/f*c
^2*a^2*(f*x+e)^2-3*d^2/f^2*e*c*a^2*(f*x+e)^2+3*d^2/f^2*e^2*c*b^2*(1/2*sinh(
f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)+6*d^2/f^2*c*a*b*((f*x+e)^2*sinh(f*x+e)-2*
(f*x+e)*cosh(f*x+e)+2*sinh(f*x+e))+6*d^3/f^3*e^2*a*b*((f*x+e)*sinh(f*x+e)-c
```

```
osh(f*x+e))-6*d^2/f^2*e*c*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x
+e)^2-1/4*cosh(f*x+e)^2)-2*d^3/f^3*e^3*a*b*sinh(f*x+e))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 550 vs. $2(244) = 488$.
time = 0.30, size = 550, normalized size = 2.20

$$\frac{1}{4}a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{16}\left(4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2c^2d + \frac{1}{16}\left(8x^3 + \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)})e^{(2fx)}}{f^3} - \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)b^2cd^2 + \frac{1}{32}\left(4x^4 + \frac{(4f^3x^3e^{(2e)} - 6f^2x^2e^{(2e)} + 6fxe^{(2e)} - 3e^{(2e)})e^{(2fx)}}{f^4} - \frac{(4f^3x^3+6f^2x^2+6fx+3)e^{(-2fx-2e)}}{f^4}\right)b^2d^3 + \frac{1}{8}b^2c^3\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^3x + 3abc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + 3abcd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right) + abd^3\left(\frac{(f^3x^3e^e - 3f^2x^2e^e + 6fxe^e - 6e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3+3f^2x^2+6fx+6)e^{(-fx-e)}}{f^4}\right) + \frac{2abc^3\sinh(fx+e)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x*
e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c^
2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*
x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 + 1/32*(
4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(2*e))
*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)/f^4)*
b^2*d^3 + 1/8*b^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*
c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f
^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2
 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6
*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x -
 e)/f^4) + 2*a*b*c^3*sinh(f*x + e)/f
```

Fricas [A]
time = 0.36, size = 424, normalized size = 1.70

$$\frac{2(2a^2+b^2)d^3f^4x^4 + 8(2a^2+b^2)cd^2f^4x^3 + 12(2a^2+b^2)c^2df^4x^2 + 8(2a^2+b^2)c^3f^4x - 3(2b^2d^3f^2x^2 + 4b^2cd^2f^2x + 2b^2c^2df^2 + b^2d^3)\cosh(fx+\cosh(1)+\sinh(1))^2 - 3(2b^2d^3f^2x^2 + 4b^2cd^2f^2x + 2b^2c^2df^2 + b^2d^3)\sinh(fx+\cosh(1)+\sinh(1))^2 - 96(abd^3f^2x^2 + 2abcd^2f^2x + abc^2df^2 + 2abd^3)\cosh(fx+\cosh(1)+\sinh(1)) + 4(8abd^3f^3x^3 + 24abcd^2f^3x^2 + 8abc^3f^3 + 48abcd^2f + 24(abc^2df^3 + 2abd^3f)x + (2b^2d^3f^3x^3 + 6b^2cd^2f^3x^2 + 2b^2c^3f^3 + 3b^2cd^2f + 3(2b^2c^2df^3 + b^2d^3f)x)\cosh(fx+\cosh(1)+\sinh(1)))\sinh(fx+\cosh(1)+\sinh(1))}{16f^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(2*a^2 + b^2)*d^3*f^4*x^4 + 8*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 12*(2*a
^2 + b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 + b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^2
+ 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + cosh(1) + sinh(
1))^2 - 3*(2*b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^
3)*sinh(f*x + cosh(1) + sinh(1))^2 - 96*(a*b*d^3*f^2*x^2 + 2*a*b*c*d^2*f^2*
x + a*b*c^2*d*f^2 + 2*a*b*d^3)*cosh(f*x + cosh(1) + sinh(1)) + 4*(8*a*b*d^3
*f^3*x^3 + 24*a*b*c*d^2*f^3*x^2 + 8*a*b*c^3*f^3 + 48*a*b*c*d^2*f + 24*(a*b*
c^2*d*f^3 + 2*a*b*d^3*f)*x + (2*b^2*d^3*f^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b
^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x +
cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(255) = 510$.

time = 0.51, size = 779, normalized size = 3.12

$$\begin{cases} a^2c^3x + \frac{3a^2c^2dx^2}{2} + a^2cd^2x^3 + \frac{a^2d^3x^4}{4} + \frac{2abc^3\sinh(e+fx)}{f} + \frac{6abc^2dx\sinh(e+fx)}{f} - \frac{6abc^2d\cosh(e+fx)}{f^2} + \frac{6abcd^2x^2\sinh(e+fx)}{f} - \frac{12abcd^2x\cosh(e+fx)}{f^2} + \frac{12abcd^2\sinh(e+fx)}{f^3} + \frac{2abd^3x^3\sinh(e+fx)}{f} - \frac{6abd^3x^2\cosh(e+fx)}{f^2} + \frac{12abd^3x\sinh(e+fx)}{f^3} - \frac{12abd^3\cosh(e+fx)}{f^4} - \frac{b^2c^3x\sinh^2(e+fx)}{2} + \frac{b^2c^3x\cosh^2(e+fx)}{2} + \frac{b^2c^3\sinh(e+fx)\cosh(e+fx)}{2f} - \frac{3b^2c^2dx^2\sinh^2(e+fx)}{4} + \frac{3b^2c^2dx^2\cosh^2(e+fx)}{4} + \frac{3b^2c^2dx\sinh(e+fx)\cosh(e+fx)}{2f} - \frac{3b^2c^2d\cosh^2(e+fx)}{4f^2} - \frac{b^2cd^2x^3\sinh^2(e+fx)}{2} + \frac{b^2cd^2x^3\cosh^2(e+fx)}{2} + \frac{3b^2cd^2x^2\sinh(e+fx)\cosh(e+fx)}{2f} - \frac{3b^2cd^2x\sinh^2(e+fx)}{4f^2} - \frac{3b^2cd^2x\cosh^2(e+fx)}{4f^2} + \frac{3b^2cd^2\sinh(e+fx)\cosh(e+fx)}{4f^3} - \frac{b^2d^3x^4\sinh^2(e+fx)}{8} + \frac{b^2d^3x^4\cosh^2(e+fx)}{8} + \frac{b^2d^3x^3\sinh(e+fx)\cosh(e+fx)}{2f} - \frac{3b^2d^3x^2\sinh^2(e+fx)}{8f^2} - \frac{3b^2d^3x^2\cosh^2(e+fx)}{8f^2} + \frac{3b^2d^3x\sinh(e+fx)\cosh(e+fx)}{4f^3} - \frac{3b^2d^3\cosh^2(e+fx)}{8f^4} & \text{for } f \neq 0 \\ (a+b\cosh(e))^2\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d**
3*x**4/4 + 2*a*b*c**3*sinh(e + f*x)/f + 6*a*b*c**2*d*x*sinh(e + f*x)/f - 6*
a*b*c**2*d*cosh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*sinh(e + f*x)/f - 12*a*b*
c*d**2*x*cosh(e + f*x)/f**2 + 12*a*b*c*d**2*sinh(e + f*x)/f**3 + 2*a*b*d**3
*x**3*sinh(e + f*x)/f - 6*a*b*d**3*x**2*cosh(e + f*x)/f**2 + 12*a*b*d**3*x*
sinh(e + f*x)/f**3 - 12*a*b*d**3*cosh(e + f*x)/f**4 - b**2*c**3*x*sinh(e +
f*x)**2/2 + b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)*cosh(e
 + f*x)/(2*f) - 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*b**2*c**2*d*x**2*
cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*
b**2*c**2*d*cosh(e + f*x)**2/(4*f**2) - b**2*c*d**2*x**3*sinh(e + f*x)**2/2
 + b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3*b**2*c*d
**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)
/(4*f**3) - b**2*d**3*x**4*sinh(e + f*x)**2/8 + b**2*d**3*x**4*cosh(e + f*x
)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*d**3*x**
2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) +
3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**2*d**3*cosh(e + f
*x)**2/(8*f**4), Ne(f, 0)), ((a + b*cosh(e))**2*(c**3*x + 3*c**2*d*x**2/2 +
 c*d**2*x**3 + d**3*x**4/4), True))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(234) = 468.
time = 0.42, size = 599, normalized size = 2.40

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/2
*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x + 1/32*(4*b^
2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^2*x
^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3*a*b*
c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 - 6*a*b
*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a*b*d^3)
*e^(f*x + e)/f^4 - (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3
*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c^2*d*f^2
```

```
+ 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1/32*(4*b^2
*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2*d^3*f^2*x^
2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f*x +
6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B]
time = 2.64, size = 481, normalized size = 1.92

$$a^2\,c^3\,x+\frac{b^2\,c^3\,x}{2}+\frac{a^2\,d^3\,x^4}{4}+\frac{b^2\,d^3\,x^4}{8}+\frac{3\,a^2\,c^2\,d\,x^2}{2}+a^2\,c\,d^2\,x^3+\frac{3\,b^2\,c^2\,d\,x^2}{4}+\frac{b^2\,c\,d^2\,x^3}{2}-\frac{3\,b^2\,d^3\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{16\,f^4}+\frac{b^2\,c^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{12\,a\,b\,d^3\,\mathrm{cosh}\left(e+f\,x\right)}{f^4}+\frac{2\,a\,b\,c^3\,\mathrm{sinh}\left(e+f\,x\right)}{f}-\frac{3\,b^2\,d^3\,x^2\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{8\,f^2}+\frac{b^2\,d^3\,x^3\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{3\,b^2\,c^2\,d\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{8\,f^2}+\frac{3\,b^2\,c\,d^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{8\,f^3}+\frac{3\,b^2\,d^3\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{8\,f^3}-\frac{3\,b^2\,c\,d^2\,x\,\mathrm{cosh}\left(2\,e+2\,f\,x\right)}{4\,f^2}+\frac{3\,b^2\,c^2\,d\,x\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{6\,a\,b\,c^2\,d\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{12\,a\,b\,c\,d^2\,\mathrm{sinh}\left(e+f\,x\right)}{f^3}+\frac{12\,a\,b\,d^3\,x\,\mathrm{sinh}\left(e+f\,x\right)}{f^3}+\frac{3\,b^2\,c\,d^2\,x^2\,\mathrm{sinh}\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{6\,a\,b\,d^3\,x^2\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{2\,a\,b\,d^3\,x^3\,\mathrm{sinh}\left(e+f\,x\right)}{f}+\frac{6\,a\,b\,c\,d^2\,x^2\,\mathrm{sinh}\left(e+f\,x\right)}{f}-\frac{12\,a\,b\,c\,d^2\,x\,\mathrm{cosh}\left(e+f\,x\right)}{f^2}+\frac{6\,a\,b\,c^2\,d\,x\,\mathrm{sinh}\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2*(c + d*x)^3,x)
```

```
[Out] a^2*c^3*x + (b^2*c^3*x)/2 + (a^2*d^3*x^4)/4 + (b^2*d^3*x^4)/8 + (3*a^2*c^2*
d*x^2)/2 + a^2*c*d^2*x^3 + (3*b^2*c^2*d*x^2)/4 + (b^2*c*d^2*x^3)/2 - (3*b^2
*d^3*cosh(2*e + 2*f*x))/(16*f^4) + (b^2*c^3*sinh(2*e + 2*f*x))/(4*f) - (12*
a*b*d^3*cosh(e + f*x))/f^4 + (2*a*b*c^3*sinh(e + f*x))/f - (3*b^2*d^3*x^2*c
osh(2*e + 2*f*x))/(8*f^2) + (b^2*d^3*x^3*sinh(2*e + 2*f*x))/(4*f) - (3*b^2*
c^2*d*cosh(2*e + 2*f*x))/(8*f^2) + (3*b^2*c*d^2*sinh(2*e + 2*f*x))/(8*f^3)
+ (3*b^2*d^3*x*sinh(2*e + 2*f*x))/(8*f^3) - (3*b^2*c*d^2*x*cosh(2*e + 2*f*x
))/(4*f^2) + (3*b^2*c^2*d*x*sinh(2*e + 2*f*x))/(4*f) - (6*a*b*c^2*d*cosh(e
+ f*x))/f^2 + (12*a*b*c*d^2*sinh(e + f*x))/f^3 + (12*a*b*d^3*x*sinh(e + f*x
))/f^3 + (3*b^2*c*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (6*a*b*d^3*x^2*cosh(e
+ f*x))/f^2 + (2*a*b*d^3*x^3*sinh(e + f*x))/f + (6*a*b*c*d^2*x^2*sinh(e + f
*x))/f - (12*a*b*c*d^2*x*cosh(e + f*x))/f^2 + (6*a*b*c^2*d*x*sinh(e + f*x))
/f
```

3.163 $\int (c+dx)^2(a+b\cosh (e+fx))^2\,dx$

Optimal. Leaf size=182

$$\frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}-\frac{4abd(c+dx)\cosh (e+fx)}{f^2}-\frac{b^2d(c+dx)\cosh ^2(e+fx)}{2f^2}+\frac{4abd^2\sinh (e+f}{f^3}$$

```
[Out] 1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d+1/6*b^2*(d*x+c)^3/d-4*a*b*d*(d*x+c)*c
osh(f*x+e)/f^2-1/2*b^2*d*(d*x+c)*cosh(f*x+e)^2/f^2+4*a*b*d^2*sinh(f*x+e)/f^
3+2*a*b*(d*x+c)^2*sinh(f*x+e)/f+1/4*b^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2
*b^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x+e)/f
```

Rubi [A]
time = 0.13, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3377, 2717, 3392, 32, 2715, 8}

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\cosh (e+fx)}{f^2}+\frac{2ab(c+dx)^2\sinh (e+fx)}{f}+\frac{4abd^2\sinh (e+fx)}{f^3}-\frac{b^2d(c+dx)\cosh ^2(e+fx)}{2f^2}+\frac{b^2(c+dx)^2\sinh (e+fx)\cosh (e+fx)}{2f}+\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sinh (e+fx)\cosh (e+fx)}{4f^3}+\frac{b^2d^2x}{4f^2}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) - (
4*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (b^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2
*f^2) + (4*a*b*d^2*Sinh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Sinh[e + f*x])/f
 + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^2*Cosh[e
+ f*x]*Sinh[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3392

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist
[b^2*((n - 1)/n), Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Dist[d
^2*m*((m - 1)/(f^2*n^2)), Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x]
- Simp[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x]) /;
FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^2+2ab(c+dx)^2\cosh(e+fx)+b^2(c+dx)^2\cosh^2(e+\right.$$
$$= \frac{a^2(c+dx)^3}{3d}+(2ab)\int (c+dx)^2\cosh(e+fx)\,dx+b^2\int (c+dx)^2\,\mathrm{co}$$
$$= \frac{a^2(c+dx)^3}{3d}-\frac{b^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{2ab(c+dx)^2\sinh(e+f}{f}$$
$$= \frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}-\frac{4abd(c+dx)\cosh(e+fx)}{f^2}-\frac{b^2d(c+}{}$$
$$= \frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d}-\frac{4abd(c+dx)\cosh(e+fx)}{f^2}-$$

Mathematica [A]
time = 0.63, size = 252, normalized size = 1.38

$$\frac{1}{24}\left(24a^2c^2x+12b^2c^2x+24a^2cdx^2+12b^2cdx^2+8a^2d^2x^3+4b^2d^2x^3-\frac{96abd(c+dx)\cosh(e+fx)}{f^2}-\frac{6b^2d(c+dx)\cosh(2(e+fx))}{f^2}+\frac{96abd^2\sinh(e+fx)}{f^3}+\frac{48abc^2\sinh(e+fx)}{f}+\frac{96abcdx\sinh(e+fx)}{f}+\frac{48abd^2x^2\sinh(e+fx)}{f}+\frac{3b^2d^2\sinh(2(e+fx))}{f^3}+\frac{6b^2c^2\sinh(2(e+fx))}{f}+\frac{12b^2cdx\sinh(2(e+fx))}{f}+\frac{6b^2d^2x^2\sinh(2(e+fx))}{f}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (24*a^2*c^2*x + 12*b^2*c^2*x + 24*a^2*c*d*x^2 + 12*b^2*c*d*x^2 + 8*a^2*d^2*
x^3 + 4*b^2*d^2*x^3 - (96*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (6*b^2*d*(c
+ d*x)*Cosh[2*(e + f*x)])/f^2 + (96*a*b*d^2*Sinh[e + f*x])/f^3 + (48*a*b*c^
2*Sinh[e + f*x])/f + (96*a*b*c*d*x*Sinh[e + f*x])/f + (48*a*b*d^2*x^2*Sinh[
e + f*x])/f + (3*b^2*d^2*Sinh[2*(e + f*x)])/f^3 + (6*b^2*c^2*Sinh[2*(e + f*
x)])/f + (12*b^2*c*d*x*Sinh[2*(e + f*x)])/f + (6*b^2*d^2*x^2*Sinh[2*(e + f*
x)])/f)/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(170) = 340$.
time = 0.93, size = 535, normalized size = 2.94

method	result
risch	$\frac{a^2d^2x^3}{3}+\frac{d^2b^2x^3}{6}+a^2cd\,x^2+\frac{d\,b^2cx^2}{2}+a^2c^2x+\frac{b^2c^2x}{2}+\frac{c^3a^2}{3d}+\frac{b^2c^3}{6d}+\frac{b^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-\right.}{16f^3}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}\right.}{}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}\right.}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/3*d^2/f^2*a^2*(f*x+e)^3+2*d^2/f^2*a*b*((f*x+e)^2*sinh(f*x+e)-2*(f*x+
e)*cosh(f*x+e)+2*sinh(f*x+e))+d^2/f^2*b^2*(1/2*(f*x+e)^2*cosh(f*x+e)*sinh(f
*x+e)+1/6*(f*x+e)^3-1/2*(f*x+e)*cosh(f*x+e)^2+1/4*sinh(f*x+e)*cosh(f*x+e)+1
/4*f*x+1/4*e)-d^2/f^2*e*a^2*(f*x+e)^2-4*d^2/f^2*e*a*b*((f*x+e)*sinh(f*x+e)-
cosh(f*x+e))-2*d^2/f^2*e*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+
e)^2-1/4*cosh(f*x+e)^2)+d/f*c*a^2*(f*x+e)^2+4*d/f*c*a*b*((f*x+e)*sinh(f*x+e
)-cosh(f*x+e))+2*d/f*c*b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)
^2-1/4*cosh(f*x+e)^2)+d^2/f^2*e^2*a^2*(f*x+e)+2*d^2/f^2*e^2*a*b*sinh(f*x+e)
+d^2/f^2*e^2*b^2*(1/2*sinh(f*x+e)*cosh(f*x+e)+1/2*f*x+1/2*e)-2*d/f*e*c*a^2*
(f*x+e)-4*d/f*e*c*a*b*sinh(f*x+e)-2*d/f*e*c*b^2*(1/2*sinh(f*x+e)*cosh(f*x+e
)+1/2*f*x+1/2*e)+a^2*c^2*(f*x+e)+2*c^2*a*b*sinh(f*x+e)+b^2*c^2*(1/2*sinh(f*
x+e)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.29, size = 341, normalized size = 1.87

$$\frac{1}{3}\,a^2d^2x^3+a^2cdx^2+\frac{1}{8}\left(4\,x^2+\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}-\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2cd+\frac{1}{48}\left(8\,x^3+\frac{3\left(2\,f^2x^2e^{(2\,e)}-2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(2\,fx)}}{f^3}-\frac{3\left(2\,f^2x^2+2\,fx+1\right)e^{(-2\,fx-2\,e)}}{f^3}\right)b^2d^2+\frac{1}{8}\,b^2c^2\left(4\,x+\frac{e^{(2\,fx+2\,e)}}{f}-\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2c^2x+2\,abcd\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+abd^2\left(\frac{(f^2x^2e^e-2\,fxe^e+2\,e^e)e^{(fx)}}{f^3}-\frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right)+\frac{2\,abc^2\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2
*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d + 1/48*(8*x^3 + 3*(2*
f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2 + 2
*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 + 1/8*b^2*c^2*(4*x + e^(2*f*x + 2*e
)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(f*x)/
f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e
^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^2*sinh(
f*x + e)/f
```

Fricas [A]
time = 0.37, size = 255, normalized size = 1.40

$$\frac{2\,(2\,a^2+b^2)d^2f^3x^3+6\,(2\,a^2+b^2)cdf^3x^2+6\,(2\,a^2+b^2)c^2f^3x-3\,(b^2d^2fx+b^2cdf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-3\,(b^2d^2fx+b^2cdf)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-48\,(abd^2fx+abcdf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+3\,(8\,abd^2f^2x^2+16\,abcdf^2x+8\,abc^2f^2+16\,abd^2+(2\,b^2d^2f^2x^2+4\,b^2cdf^2x+2\,b^2c^2f^2+b^2d^2)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right))\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{12\,f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 + 6*(2*a^2
+ b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + cosh(1) + sinh(1)
)^2 - 3*(b^2*d^2*f*x + b^2*c*d*f)*sinh(f*x + cosh(1) + sinh(1))^2 - 48*(a*b
*d^2*f*x + a*b*c*d*f)*cosh(f*x + cosh(1) + sinh(1)) + 3*(8*a*b*d^2*f^2*x^2
+ 16*a*b*c*d*f^2*x + 8*a*b*c^2*f^2 + 16*a*b*d^2 + (2*b^2*d^2*f^2*x^2 + 4*b^
2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh(f*x + cosh(1) + sinh(1)))*sinh(
f*x + cosh(1) + sinh(1)))/f^3
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.
time = 0.32, size = 456, normalized size = 2.51

$$\begin{cases} a^2c^2x+a^2cdx^2+\frac{a^2d^2x^3}{3}+\frac{2abc^2\sinh\left(e+fx\right)}{f}+\frac{4abcdx\sinh\left(e+fx\right)}{f}-\frac{4abcd\cosh\left(e+fx\right)}{f^2}+\frac{2abd^2x^2\sinh\left(e+fx\right)}{f}-\frac{4abd^2x\cosh\left(e+fx\right)}{f^2}+\frac{4abd^2\sinh\left(e+fx\right)}{f^3}-\frac{b^2c^2x\sinh^2\left(e+fx\right)}{2}+\frac{b^2c^2x\cosh^2\left(e+fx\right)}{2}+\frac{b^2c^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2cdx^2\sinh^2\left(e+fx\right)}{2}+\frac{b^2cdx^2\cosh^2\left(e+fx\right)}{2}+\frac{b^2cdx\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{f}-\frac{b^2cd\cosh^2\left(e+fx\right)}{2f^2}-\frac{b^2d^2x^3\sinh^2\left(e+fx\right)}{6}+\frac{b^2d^2x^3\cosh^2\left(e+fx\right)}{6}+\frac{b^2d^2x^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2d^2x\sinh^2\left(e+fx\right)}{4f^2}-\frac{b^2d^2x\cosh^2\left(e+fx\right)}{4f^2}+\frac{b^2d^2\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{4f^3} & \text{for}\: f \neq 0 \\ \left(a+b\cosh\left(e\right)\right)^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*sinh
(e + f*x)/f + 4*a*b*c*d*x*sinh(e + f*x)/f - 4*a*b*c*d*cosh(e + f*x)/f**2 +
2*a*b*d**2*x**2*sinh(e + f*x)/f - 4*a*b*d**2*x*cosh(e + f*x)/f**2 + 4*a*b*d
**2*sinh(e + f*x)/f**3 - b**2*c**2*x*sinh(e + f*x)**2/2 + b**2*c**2*x*cosh(
e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*c*d*x**2
*sinh(e + f*x)**2/2 + b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*sinh(e
+ f*x)*cosh(e + f*x)/f - b**2*c*d*cosh(e + f*x)**2/(2*f**2) - b**2*d**2*x**
3*sinh(e + f*x)**2/6 + b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**2*x**2*s
```

```
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4*f**2) -
b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*cosh(e + f*
x)/(4*f**3), Ne(f, 0)), ((a + b*cosh(e))**2*(c**2*x + c*d*x**2 + d**2*x**3/
3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(170) = 340.
time = 0.42, size = 345, normalized size = 1.90

$$\frac{1}{3}a^2d^2x^3 + \frac{1}{6}b^2d^2x^3 + a^2cdx^2 + \frac{1}{2}b^2cdx^2 + a^2c^2x + \frac{1}{2}b^2c^2x + \frac{(2b^2d^2f^2x^2 + 4b^2cdf^2x + 2b^2c^2f^2 - 2b^2d^2fx - 2b^2cdf + b^2d^2)e^{(2fx+2e)}}{16f^3} + \frac{(abd^2f^2x^2 + 2abcdf^2x + abc^2f^2 - 2abd^2fx - 2abcdf + 2abd^2)e^{(fx+e)}}{f^3} - \frac{(abd^2f^2x^2 + 2abcdf^2x + abc^2f^2 + 2abd^2fx + 2abcdf + 2abd^2)e^{(-fx-e)}}{f^3} - \frac{(2b^2d^2f^2x^2 + 4b^2cdf^2x + 2b^2c^2f^2 + 2b^2d^2fx + 2b^2cdf + b^2d^2)e^{(-2fx-2e)}}{16f^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^2
*x + 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*
f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*d^2
*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f + 2*
a*b*d^2)*e^(f*x + e)/f^3 - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2
 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(2*b^2*
d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^2*c*d*f
 + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [B]
time = 1.33, size = 281, normalized size = 1.54

$$a^2c^2x + \frac{b^2c^2x}{2} + \frac{a^2d^2x^3}{3} + \frac{b^2d^2x^3}{6} + \frac{b^2c^2\sinh(2e+2fx)}{4f} + \frac{b^2d^2\sinh(2e+2fx)}{8f^3} + a^2cdx^2 + \frac{b^2cdx^2}{2} + \frac{2abc^2\sinh(e+fx)}{f} + \frac{4abd^2\sinh(e+fx)}{f^3} + \frac{b^2d^2x^2\sinh(2e+2fx)}{4f} - \frac{b^2cd\cosh(2e+2fx)}{4f^2} - \frac{b^2d^2x\cosh(2e+2fx)}{4f^2} - \frac{4abcd\cosh(e+fx)}{f^2} - \frac{4abd^2x\cosh(e+fx)}{f^2} + \frac{2abd^2x^2\sinh(e+fx)}{f} + \frac{b^2cdx\sinh(2e+2fx)}{2f} + \frac{4abcdx\sinh(e+fx)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2*(c + d*x)^2,x)
```

```
[Out] a^2*c^2*x + (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 + (b^2*d^2*x^3)/6 + (b^2*c^2*si
nh(2*e + 2*f*x))/(4*f) + (b^2*d^2*sinh(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^2
+ (b^2*c*d*x^2)/2 + (2*a*b*c^2*sinh(e + f*x))/f + (4*a*b*d^2*sinh(e + f*x))
/f^3 + (b^2*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (b^2*c*d*cosh(2*e + 2*f*x))/
(4*f^2) - (b^2*d^2*x*cosh(2*e + 2*f*x))/(4*f^2) - (4*a*b*c*d*cosh(e + f*x))
/f^2 - (4*a*b*d^2*x*cosh(e + f*x))/f^2 + (2*a*b*d^2*x^2*sinh(e + f*x))/f +
(b^2*c*d*x*sinh(2*e + 2*f*x))/(2*f) + (4*a*b*c*d*x*sinh(e + f*x))/f
```

3.164 $\int (c+dx)(a+b\cosh(e+fx))^2\,dx$

Optimal. Leaf size=116

$$\frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2abd\cosh(e+fx)}{f^2}-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)\sinh(e+fx)}{f}+\frac{b^2(c+dx}{}$$

```
[Out] 1/2*b^2*c*x+1/4*b^2*d*x^2+1/2*a^2*(d*x+c)^2/d-2*a*b*d*cosh(f*x+e)/f^2-1/4*b
^2*d*cosh(f*x+e)^2/f^2+2*a*b*(d*x+c)*sinh(f*x+e)/f+1/2*b^2*(d*x+c)*cosh(f*x
+e)*sinh(f*x+e)/f
```

Rubi [A]

time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {3398, 3377, 2718, 3391}

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\sinh(e+fx)}{f}-\frac{2abd\cosh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{1}{2}b^2cx-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{1}{4}b^2dx^2$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (b^2*c*x)/2 + (b^2*d*x^2)/4 + (a^2*(c + d*x)^2)/(2*d) - (2*a*b*d*Cosh[e + f
*x])/f^2 - (b^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a*b*(c + d*x)*Sinh[e + f*x]
)/f + (b^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)(a+b\cosh(e+fx))^2\,dx &= \int \left(a^2(c+dx)+2ab(c+dx)\cosh(e+fx)+b^2(c+dx)\cosh^2(e+fx)\right. \\ &= \frac{a^2(c+dx)^2}{2d}+(2ab)\int (c+dx)\cosh(e+fx)\,dx+b^2\int (c+dx)\cosh^2(\\ &= \frac{a^2(c+dx)^2}{2d}-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)\sinh(e+fx)}{f}+\frac{b^2(c+ \\ &= \frac{1}{2}b^2cx+\frac{1}{4}b^2dx^2+\frac{a^2(c+dx)^2}{2d}-\frac{2abd\cosh(e+fx)}{f^2}-\frac{b^2d\cosh^2(e+f}{4f^2}\end{aligned}$$

Mathematica [A]
time = 0.51, size = 96, normalized size = 0.83

$$-\frac{2(2a^2+b^2)(e+fx)(-2cf+d(e-fx))+16abd\cosh(e+fx)+b^2d\cosh(2(e+fx))-16abf(c+dx)\sinh(e+fx)-2b^2f(c+dx)\sinh(2(e+fx))}{8f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -1/8*(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*d*Cosh[e +
f*x] + b^2*d*Cosh[2*(e + f*x)] - 16*a*b*f*(c + d*x)*Sinh[e + f*x] - 2*b^2*f
*(c + d*x)*Sinh[2*(e + f*x)])/f^2
```

Maple [A]
time = 0.97, size = 208, normalized size = 1.79

method	result
risch	$\frac{d\,a^2x^2}{2}+a^2cx+\frac{b^2d\,x^2}{4}+\frac{b^2cx}{2}+\frac{b^2(2dxf+2cf-d)e^{2fx+2e}}{16f^2}+\frac{ab(dxf+cf-d)e^{fx+e}}{f^2}-\frac{ab(dxf+cf+d)e^{-fx-e}}{f^2}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{f}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\left(\cosh^2(fx+e)\right)}{4}\right)}{f}-\frac{de\,a^2(}{f}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/f*(1/2*d/f*a^2*(f*x+e)^2+2*d/f*a*b*((f*x+e)*sinh(f*x+e)-cosh(f*x+e))+d/f*
b^2*(1/2*(f*x+e)*cosh(f*x+e)*sinh(f*x+e)+1/4*(f*x+e)^2-1/4*cosh(f*x+e)^2)-d
/f*e*a^2*(f*x+e)-2*d/f*e*a*b*sinh(f*x+e)-d/f*e*b^2*(1/2*sinh(f*x+e)*cosh(f*
x+e)+1/2*f*x+1/2*e)+a^2*c*(f*x+e)+2*a*b*c*sinh(f*x+e)+b^2*c*(1/2*sinh(f*x+e
)*cosh(f*x+e)+1/2*f*x+1/2*e))
```

Maxima [A]
time = 0.28, size = 174, normalized size = 1.50

$$\frac{1}{2}\,a^2dx^2+\frac{1}{16}\left(4\,x^2+\frac{\left(2\,fxe^{(2\,e)}-e^{(2\,e)}\right)e^{(2\,fx)}}{f^2}-\frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2d+\frac{1}{8}\,b^2c\left(4\,x+\frac{e^{(2\,fx+2\,e)}}{f}-\frac{e^{(-2\,fx-2\,e)}}{f}\right)+a^2cx+abd\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)+\frac{2\,abc\sinh\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*
f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d + 1/8*b^2*c*(4*x + e^(2*f*x + 2*e)/f -
 e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x
+ 1)*e^(-f*x - e)/f^2) + 2*a*b*c*sinh(f*x + e)/f
```

Fricas [A]
time = 0.43, size = 137, normalized size = 1.18

$$\frac{2\,(2\,a^2+b^2)df^2x^2+4\,(2\,a^2+b^2)cf^2x-b^2d\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-b^2d\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)^2-16\,abd\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)+4\,\left(4\,abdfx+4\,abcf+(b^2dfx+b^2cf)\cosh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)\right)\sinh\left(fx+\cosh\left(1\right)+\sinh\left(1\right)\right)}{8\,f^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(2*(2*a^2 + b^2)*d*f^2*x^2 + 4*(2*a^2 + b^2)*c*f^2*x - b^2*d*cosh(f*x +
 cosh(1) + sinh(1))^2 - b^2*d*sinh(f*x + cosh(1) + sinh(1))^2 - 16*a*b*d*co
sh(f*x + cosh(1) + sinh(1)) + 4*(4*a*b*d*f*x + 4*a*b*c*f + (b^2*d*f*x + b^2
*c*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/f^2
```

Sympy [A]
time = 0.16, size = 219, normalized size = 1.89

$$\begin{cases} a^2cx+\frac{a^2dx^2}{2}+\frac{2abc\sinh\left(e+fx\right)}{f}+\frac{2abdx\sinh\left(e+fx\right)}{f}-\frac{2abd\cosh\left(e+fx\right)}{f^2}-\frac{b^2cx\sinh^2\left(e+fx\right)}{2}+\frac{b^2cx\cosh^2\left(e+fx\right)}{2}+\frac{b^2c\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2dx^2\sinh^2\left(e+fx\right)}{4}+\frac{b^2dx^2\cosh^2\left(e+fx\right)}{4}+\frac{b^2dx\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}-\frac{b^2d\cosh^2\left(e+fx\right)}{4f^2} & \text{for}\ f\neq 0 \\ \left(a+b\cosh\left(e\right)\right)^2\left(cx+\frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*sinh(e + f*x)/f + 2*a*b*d*x*s
inh(e + f*x)/f - 2*a*b*d*cosh(e + f*x)/f**2 - b**2*c*x*sinh(e + f*x)**2/2 +
 b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b
**2*d*x**2*sinh(e + f*x)**2/4 + b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d*x*s
inh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*cosh(e + f*x)**2/(4*f**2), Ne(f,
0)), ((a + b*cosh(e))**2*(c*x + d*x**2/2), True))
```

Giac [A]
time = 0.41, size = 160, normalized size = 1.38

$$\frac{1}{2}\,a^{2}dx^{2}+\frac{1}{4}\,b^{2}dx^{2}+a^{2}cx+\frac{1}{2}\,b^{2}cx+\frac{(2\,b^{2}dfx+2\,b^{2}cf-b^{2}d)e^{(2\,fx+2\,e)}}{16\,f^{2}}+\frac{(abdfx+abcf-abd)e^{(fx+e)}}{f^{2}}-\frac{(abdfx+abcf+abd)e^{(-fx-e)}}{f^{2}}-\frac{(2\,b^{2}dfx+2\,b^{2}cf+b^{2}d)e^{(-2\,fx-2\,e)}}{16\,f^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] 1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x +
 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e^(
f*x + e)/f^2 - (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*b^2
*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B]
time = 0.15, size = 135, normalized size = 1.16

$$\frac{a^{2}\,d\,x^{2}}{2}+\frac{b^{2}\,d\,x^{2}}{4}+a^{2}\,c\,x+\frac{b^{2}\,c\,x}{2}-\frac{b^{2}\,d\,\cosh(e+f\,x)^{2}}{4\,f^{2}}+\frac{b^{2}\,c\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}-\frac{2\,a\,b\,d\cosh(e+f\,x)}{f^{2}}+\frac{2\,a\,b\,c\sinh(e+f\,x)}{f}+\frac{2\,a\,b\,d\,x\sinh(e+f\,x)}{f}+\frac{b^{2}\,d\,x\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2*(c + d*x),x)
```

```
[Out] (a^2*d*x^2)/2 + (b^2*d*x^2)/4 + a^2*c*x + (b^2*c*x)/2 - (b^2*d*cosh(e + f*x
)^2)/(4*f^2) + (b^2*c*cosh(e + f*x)*sinh(e + f*x))/(2*f) - (2*a*b*d*cosh(e
+ f*x))/f^2 + (2*a*b*c*sinh(e + f*x))/f + (2*a*b*d*x*sinh(e + f*x))/f + (b^
2*d*x*cosh(e + f*x)*sinh(e + f*x))/(2*f)
```

3.165 $\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx$

Optimal. Leaf size=156

$$\frac{2ab\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\,\mathrm{s}}{}$$

```
[Out] 1/2*b^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d+2*a*b*Chi(c*f/d+f*x)*cosh(-
e+c*f/d)/d+a^2*ln(d*x+c)/d+1/2*b^2*ln(d*x+c)/d-1/2*b^2*Shi(2*c*f/d+2*f*x)*s
inh(-2*e+2*c*f/d)/d-2*a*b*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d
```

Rubi [A]
time = 0.23, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3398, 3384, 3379, 3382, 3393}

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\operatorname{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{b^2\log(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (2*a*b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (b^2*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log[c + d*x])/d + (
b^2*Log[c + d*x])/(2*d) + (2*a*b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f
*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*d)
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx &= \int \left(\frac{a^2}{c+dx}+\frac{2ab\cosh(e+fx)}{c+dx}+\frac{b^2\cosh^2(e+fx)}{c+dx}\right)dx\\
&= \frac{a^2\log(c+dx)}{d}+(2ab)\int \frac{\cosh(e+fx)}{c+dx}\,dx+b^2\int \frac{\cosh^2(e+fx)}{c+dx}\,dx\\
&= \frac{a^2\log(c+dx)}{d}+b^2\int \left(\frac{1}{2(c+dx)}+\frac{\cosh(2e+2fx)}{2(c+dx)}\right)dx+\left(2ab\cosh\left(e-\right.\right.\\
&= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\sinh}{}\\
&= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a^2\log(c+dx)}{d}+\frac{b^2\log(c+dx)}{2d}+\frac{2ab\sinh}{}\\
&= \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d}+\frac{a^2\log}{}
\end{aligned}$$

Mathematica [A]
time = 0.17, size = 133, normalized size = 0.85

$$\frac{4ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)+b^2\log(c+dx)+4ab\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

```
[Out] (4*a*b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + b^2*Cosh[2*e - (2*c*f)
/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c + d*x] + b^2*Log[c + d*x]
 + 4*a*b*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Maple [A]
time = 4.14, size = 202, normalized size = 1.29

method	result
risch	$-\frac{ab\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{d}-\frac{ab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{a^2\ln(dx+c)}{d}+\frac{b^2\ln(dx+c)}{2d}-\frac{b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

```
[Out] -a*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a*b/d*exp(-(c*f-d*e)/d)*Ei(
1,-f*x-e-(c*f-d*e)/d)+a^2*ln(d*x+c)/d+1/2*b^2*ln(d*x+c)/d-1/4*b^2/d*exp(-2*
(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/4*b^2/d*exp(2*(c*f-d*e)/d)*Ei
(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 152, normalized size = 0.97

$$-\frac{1}{4}\,b^2\left(\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_1\left(-\frac{2\,(dx+c)f}{d}\right)}{d}-\frac{2\,\log\left(dx+c\right)}{d}\right)-ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^2\log\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(2*c*f/d - 2*e)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(-2*c*
f/d + 2*e)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a*b*
(e^(c*f/d - e)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(-c*f/d + e)*exp_inte
gral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Fricas [A]
time = 0.38, size = 253, normalized size = 1.62

$$\frac{4\,\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)+b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)+2\,(2\,a^2+b^2)\log\left(dx+c\right)+4\,\left(ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)+\left(b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)-b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\sinh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

```
[Out] 1/4*(4*(a*b*Ei((d*f*x + c*f)/d) + a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*
cosh(1) - d*sinh(1))/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c
*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1))/d) + 2*(2*a^2 + b^2)*log(d*x
+ c) + 4*(a*b*Ei((d*f*x + c*f)/d) - a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f -
d*cosh(1) - d*sinh(1))/d) + (b^2*Ei(2*(d*f*x + c*f)/d) - b^2*Ei(-2*(d*f*x +
 c*f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a+b\cosh\left(e+fx\right)\right)^2}{c+dx}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c),x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x), x)
```

Giac [A]
time = 0.42, size = 144, normalized size = 0.92

$$\frac{b^{2}\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}+4\,ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4\,ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+b^{2}\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2\,e+\frac{2\,cf}{d}\right)}+4\,a^{2}\log\left(dx+c\right)+2\,b^{2}\log\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

```
[Out] 1/4*(b^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*Ei((d*f*x + c*f)/d
)*e^(e - c*f/d) + 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b^2*Ei(-2*(d*
f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*log(d*x + c) + 2*b^2*log(d*x + c))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(a+b\cosh\left(e+f\,x\right)\right)}^2}{c+d\,x} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2/(c + d*x),x)
```

```
[Out] int((a + b*cosh(e + f*x))^2/(c + d*x), x)
```

3.166 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{2abf\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

```
[Out] -a^2/d/(d*x+c)-2*a*b*cosh(f*x+e)/d/(d*x+c)-b^2*cosh(f*x+e)^2/d/(d*x+c)+2*a*
b*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2+b^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+
2*f*x)/d^2-b^2*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*a*b*f*Chi(c*f/
d+f*x)*sinh(-e+c*f/d)/d^2
```

Rubi [A]
time = 0.26, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3398, 3378, 3384, 3379, 3382, 3394, 12}

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\cosh(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] -(a^2/(d*(c + d*x))) - (2*a*b*Cosh[e + f*x])/(d*(c + d*x)) - (b^2*Cosh[e +
f*x]^2)/(d*(c + d*x)) + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (
2*c*f)/d])/d^2 + (2*a*b*f*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^
2 + (2*a*b*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*f*Co
sh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3394

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Dist[f*(n/(d*(m + 1
))), Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n -
 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &&
 LtQ[m, -1]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx &= \int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\cosh(e+fx)}{(c+dx)^2}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^2}\right)dx \\
&= -\frac{a^2}{d(c+dx)}+(2ab)\int \frac{\cosh(e+fx)}{(c+dx)^2}\,dx+b^2\int \frac{\cosh^2(e+fx)}{(c+dx)^2}\,dx \\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{(2abf)\int \frac{\sinh(e+fx)}{c+dx}\,dx}{d} \\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{(b^2f)\int \frac{\sinh(2e+2fx)}{c+dx}\,dx}{d} \\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{2abf\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sin}{d^2} \\
&= -\frac{a^2}{d(c+dx)}-\frac{2ab\cosh(e+fx)}{d(c+dx)}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}+\frac{b^2f\,\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sin}{d^2}
\end{aligned}
$$

Mathematica [A]

time = 0.50, size = 233, normalized size = 1.27

$$
\frac{-2a^2d-b^2d-4abd\cosh(e+fx)-b^2d\cosh(2(e+fx))+2b^2f(c+dx)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+4abf(c+dx)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right)+4abcf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+4abdfx\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)+2b^2cf\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)+2b^2dfx\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2f(c+dx)}{d}\right)}{2d^2(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

```
[Out] (-2*a^2*d - b^2*d - 4*a*b*d*Cosh[e + f*x] - b^2*d*Cosh[2*(e + f*x)] + 2*b^2
*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*a*b*
f*(c + d*x)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*a*b*c*f*Cosh[e
- (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Cosh[e - (c*f)/d]*SinhIn
tegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
+ d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))
/d])/(2*d^2*(c + d*x))
```

Maple [A]

time = 4.21, size = 319, normalized size = 1.74

method	result
risch	$-\frac{fab\,e^{-fx-e}}{d(dxf+cf)}+\frac{fab\,e^{\frac{cf-de}{d}}\,\text{expIntegral}\left(1,fx+e+\frac{cf-de}{d}\right)}{d^2}-\frac{abf\,e^{fx+e}}{d^2\left(\frac{cf}{d}+fx\right)}-\frac{abf\,e^{-\frac{cf-de}{d}}\,\text{expIntegral}\left(1,-fx-e-\frac{cf-de}{d}\right)}{d^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)+f*a*b/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f
-d*e)/d)-a*b*f/d^2*exp(f*x+e)/(c*f/d+f*x)-a*b*f/d^2*exp(-(c*f-d*e)/d)*Ei(1,
-f*x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)-1/2*b^2/(d*x+c)/d-1/4*f*b^2*exp(-2*f*x-2*
e)/d/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e
)/d)-1/4*f*b^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*b^2/d^2*exp(-2*(c*f-d*e
)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Maxima [A]
time = 0.32, size = 185, normalized size = 1.01

$$-\frac{1}{4}\,b^2\left(\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_2\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)d}+\frac{2}{d^2x+cd}\right)-ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right)-\frac{a^2}{d^2x+cd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(e^(2*c*f/d - 2*e)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*d
) + e^(-2*c*f/d + 2*e)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d) +
2/(d^2*x + c*d)) - a*b*(e^(c*f/d - e)*exp_integral_e(2, (d*x + c)*f/d)/((d*
x + c)*d) + e^(-c*f/d + e)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d))
 - a^2/(d^2*x + c*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 540 vs. 2(192) = 384.
time = 0.38, size = 540, normalized size = 2.95

$$-\frac{b^2d\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2+4\,abd\cosh\left(fx+\cosh(1)+\sinh(1)\right)+\left(b^2d+(b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)\right)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2+(2\,a^2+b^2)d-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(abdfx+abcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(b^2dfx+b^2cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)-2\left((abdfx+abcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+(abdfx+abcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\frac{cf-d\cosh(1)-d\sinh(1)}{d}\right)-\left((b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(b^2dfx+b^2cf)\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2+(b^2dfx+b^2cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\sinh\left(-\frac{2\,(cf-d\cosh(1)-d\sinh(1))}{d}\right)}{2\left((d^3x+cd^2)\cosh\left(fx+\cosh(1)+\sinh(1)\right)^2-(d^3x+cd^2)\sinh\left(fx+\cosh(1)+\sinh(1)\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(b^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + 4*a*b*d*cosh(f*x + cosh(1) +
sinh(1)) + (b^2*d + (b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(-2*(c*
f - d*cosh(1) - d*sinh(1))/d))*sinh(f*x + cosh(1) + sinh(1))^2 + (2*a^2 + b
^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*d*f*x + a*b*c*f
)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((b^2*d*f*
x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d
*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh(1
))/d) - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) + (a*b*d*f*x + a*b*c*f
)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) - d*sinh(1))/d) - ((b^2*d*f*
x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d
*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(1))^2 + (b^
2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(c*f - d*cosh(1) - d*sin
h(1))/d))/((d^3*x + c*d^2)*cosh(f*x + cosh(1) + sinh(1))^2 - (d^3*x + c*d^2
)*sinh(f*x + cosh(1) + sinh(1))^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+b\cosh{(e+fx)})^2}{(c+dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c)**2,x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(186) = 372$.
time = 0.50, size = 1135, normalized size = 6.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) -
 2*b^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(2*(d*e - c*f)/d) + 2*b^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a*b*(d
*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(((d*x + c
)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) + 4
*a*b*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d
)*e^((d*e - c*f)/d) - 4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f
^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) + 4*a*b*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c)
 + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*a*b*c*f^3*Ei(-((d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 2*(d*x
+ c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)*(d*e/(d*x
 + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 2*b^2*d*e
*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e
^(-2*(d*e - c*f)/d) - 2*b^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - b^2*d*f^2*e^(2*(d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^2*e^((d*x + c)*(d*e/(d*
x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^2*e^(-(d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f)/d) - b^2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*
x + c) + f)/d) - 4*a^2*d*f^2 - 2*b^2*d*f^2)*d^2/(((d*x + c)*d^4*(d*e/(d*x +
 c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(a+b\cosh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2/(c + d*x)^2,x)
```

```
[Out] int((a + b*cosh(e + f*x))^2/(c + d*x)^2, x)
```

3.167 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal. Leaf size=242

$$-\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^3}+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}$$

```
[Out] -1/2*a^2/d/(d*x+c)^2+b^2*f^2*Chi(2*c*f/d+2*f*x)*cosh(-2*e+2*c*f/d)/d^3+a*b*
f^2*Chi(c*f/d+f*x)*cosh(-e+c*f/d)/d^3-a*b*cosh(f*x+e)/d/(d*x+c)^2-1/2*b^2*c
osh(f*x+e)^2/d/(d*x+c)^2-b^2*f^2*Shi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^3-
a*b*f^2*Shi(c*f/d+f*x)*sinh(-e+c*f/d)/d^3-a*b*f*sinh(f*x+e)/d^2/(d*x+c)-b^2
*f*cosh(f*x+e)*sinh(f*x+e)/d^2/(d*x+c)
```

Rubi [A]

time = 0.32, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3398, 3378, 3384, 3379, 3382, 3395, 31, 3393}

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}+\frac{b^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{b^2f\sinh(e+fx)\cosh(e+fx)}{d^2(c+dx)}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/2*a^2/(d*(c + d*x)^2) - (a*b*Cosh[e + f*x])/(d*(c + d*x)^2) - (b^2*Cosh[
e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[(c*
f)/d + f*x])/d^3 + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d +
2*f*x])/d^3 - (a*b*f*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*f*Cosh[e + f*x]*
Sinh[e + f*x])/(d^2*(c + d*x)) + (a*b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c
*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d +
 2*f*x])/d^3
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
 + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

```
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
 NeQ[d*e - c*f, 0]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3395

```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (Dist[b
^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)^(m + 2)*(b*Sin[e +
f*x])^(n - 2), x], x] - Dist[f^2*(n^2/(d^2*(m + 1)*(m + 2))), Int[(c + d*x)
^(m + 2)*(b*Sin[e + f*x])^n, x], x] - Simp[b*f*n*(c + d*x)^(m + 2)*Cos[e +
f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)*(m + 2))), x]) /; FreeQ[{b, c,
d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx &= \int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\cosh(e+fx)}{(c+dx)^3}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^3}\right)dx \\
&= -\frac{a^2}{2d(c+dx)^2}+(2ab)\int \frac{\cosh(e+fx)}{(c+dx)^3}\,dx+b^2\int \frac{\cosh^2(e+fx)}{(c+dx)^3}\,dx \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{b^2 f\cosh(e+fx)\sin}{d^2(c+dx)} \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{b^2f^2\log(c+dx)}{d^3}- \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}- \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{C}}{d^3} \\
&= -\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{C}}{d^3}
\end{aligned}
$$

Mathematica [A]
time = 0.81, size = 394, normalized size = 1.63

$$
-\frac{2a^2d^2+b^2d^2+4abd^2\cosh(e+fx)+b^2d^2\cosh(2(e+fx))-4abf^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-4b^2f^2(c+dx)^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)+4abcdf\sinh(e+fx)+4abd^2fx\sinh(e+fx)+2b^2cdf\sinh(2(e+fx))+2b^2d^2fx\sinh(2(e+fx))-4abc^2f^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-8abcdf^2x\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-4abd^2f^2x^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(f\left(\frac{c}{d}+x\right)\right)-4b^2c^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)-8b^2cdf^2x\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)-4b^2d^2f^2x^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2f(c+dx)}{d}\right)}{4d^3(c+dx)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

```
[Out] -1/4*(2*a^2*d^2 + b^2*d^2 + 4*a*b*d^2*Cosh[e + f*x] + b^2*d^2*Cosh[2*(e + f
*x)] - 4*a*b*f^2*(c + d*x)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] -
4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d]
 + 4*a*b*c*d*f*Sinh[e + f*x] + 4*a*b*d^2*f*x*Sinh[e + f*x] + 2*b^2*c*d*f*Si
nh[2*(e + f*x)] + 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] - 4*a*b*c^2*f^2*Sinh[e -
(c*f)/d]*SinhIntegral[f*(c/d + x)] - 8*a*b*c*d*f^2*x*Sinh[e - (c*f)/d]*Sinh
Integral[f*(c/d + x)] - 4*a*b*d^2*f^2*x^2*Sinh[e - (c*f)/d]*SinhIntegral[f*
(c/d + x)] - 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x
))/d] - 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/
d] - 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d
])/(d^3*(c + d*x)^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(242) = 484$.
time = 4.08, size = 626, normalized size = 2.59

method	result
risch	$\frac{f^{3}ab\,\mathrm{e}^{-fx-e}x}{2d(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}+\frac{f^{3}ab\,\mathrm{e}^{-fx-e}c}{2d^{2}(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}-\frac{f^{2}ab\,\mathrm{e}^{-fx-e}}{2d(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2})}-\frac{f^{2}ab\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}\left(1,fx+e+\right.}{2d^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*cosh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*f^3*a*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*f^3*a*b*e
xp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*f^2*a*b*exp(-f*x-e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*f^2*a*b/d^3*exp((c*f-d*e)/d)*Ei(1,f
*x+e+(c*f-d*e)/d)-1/2*b/d^3*a*f^2*exp(f*x+e)/(c*f/d+f*x)^2-1/2*b/d^3*a*f^2*
exp(f*x+e)/(c*f/d+f*x)-1/2*b/d^3*a*f^2*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d
*e)/d)-1/2*a^2/d/(d*x+c)^2-1/4*b^2/(d*x+c)^2/d+1/4*f^3*b^2*exp(-2*f*x-2*e)/
d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b^2*exp(-2*f*x-2*e)/d^2/(d^2*
f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/8*f^2*b^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2
*c*d*f^2*x+c^2*f^2)-1/2*f^2*b^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*
f-d*e)/d)-1/8*b^2*f^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*b^2*f^2/d^3*exp(
2*f*x+2*e)/(c*f/d+f*x)-1/2*b^2*f^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-
2*(c*f-d*e)/d)
```

Maxima [A]
time = 0.35, size = 205, normalized size = 0.85

$$-\frac{1}{4}\,b^{2}\left(\frac{1}{d^{3}x^{2}+2\,cd^{2}x+c^{2}d}+\frac{e^{\left(\frac{2\,cf}{d}-2\,e\right)}E_{3}\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^{2}d}+\frac{e^{\left(-\frac{2\,cf}{d}+2\,e\right)}E_{3}\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^{2}d}\right)-ab\left(\frac{e^{\left(\frac{cf}{d}-e\right)}E_{3}\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^{2}d}+\frac{e^{\left(-\frac{cf}{d}+e\right)}E_{3}\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^{2}d}\right)-\frac{a^{2}}{2\,(d^{3}x^{2}+2\,cd^{2}x+c^{2}d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(2*c*f/d - 2*e)*exp_integral_
e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(-2*c*f/d + 2*e)*exp_integral_e(3
, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a*b*(e^(c*f/d - e)*exp_integral_e(3,
 (d*x + c)*f/d)/((d*x + c)^2*d) + e^(-c*f/d + e)*exp_integral_e(3, -(d*x +
c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(251) = 502$.
time = 0.38, size = 830, normalized size = 3.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] -1/4*(b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + 4*a*b*d^2*cosh(f*x + cosh(1
) + sinh(1)) + (2*a^2 + b^2)*d^2 + (b^2*d^2 + 2*(b^2*d^2*f^2*x^2 + 2*b^2*c*
d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*cosh(-2*(c*f - d*cosh(1) - d*s
inh(1))/d))*sinh(f*x + cosh(1) + sinh(1))^2 - 2*((a*b*d^2*f^2*x^2 + 2*a*b*c
*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*
f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(c*f - d*cosh(1) - d*sinh(
1))/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x +
 c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 + (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2
*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(c*f - d*cosh(1) - d*sinh
(1))/d) + 4*(a*b*d^2*f*x + a*b*c*d*f + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x +
 cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)) - 2*((a*b*d^2*f^2*x^2 +
2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei((d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a
*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*sinh(-(c*f - d*cosh(1) -
d*sinh(1))/d) - 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(
d*f*x + c*f)/d)*cosh(f*x + cosh(1) + sinh(1))^2 - (b^2*d^2*f^2*x^2 + 2*b^2*
c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d)*sinh(f*x + cosh(1) + sinh(1)
)^2 - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)
/d))*sinh(-2*(c*f - d*cosh(1) - d*sinh(1))/d))/((d^5*x^2 + 2*c*d^4*x + c^2*
d^3)*cosh(f*x + cosh(1) + sinh(1))^2 - (d^5*x^2 + 2*c*d^4*x + c^2*d^3)*sinh
(f*x + cosh(1) + sinh(1))^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + b\cosh{(e + fx)})^2}{(c + dx)^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))**2/(d*x+c)**3,x)
```

```
[Out] Integral((a + b*cosh(e + f*x))**2/(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 678 vs. 2(242) = 484.
time = 0.43, size = 678, normalized size = 2.80

$$\frac{4 b^2 d^2 f^2 x^2 \mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right) e^{\left(2e-\frac{2cf}{d}\right)} + 4abd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 4abd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 4b^2d^2f^2x^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)} + 8b^2cdf^2x\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)} + 8abcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 8abcdf^2x\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 8b^2cdf^2x\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)} + 4b^2c^2f^2\mathrm{Ei}\left(\frac{2(dfx+cf)}{d}\right)e^{\left(2e-\frac{2cf}{d}\right)} + 4abc^2f^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 4abc^2f^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + 4b^2c^2f^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(-2e+\frac{2cf}{d}\right)} - 2b^2d^2fxe^{(2fx+2e)} - 4abd^2fxe^{(fx+e)} + 4abd^2fxe^{(-fx-e)} + 2b^2d^2fxe^{(-2fx-2e)} - 2b^2cdfe^{(2fx+2e)} - 4abcdfe^{(fx+e)} + 4abcdfe^{(-fx-e)} + 2b^2cdfe^{(-2fx-2e)} - b^2d^2e^{(2fx+2e)} - 4abd^2e^{(fx+e)} - 4abd^2e^{(-fx-e)} - b^2d^2e^{(-2fx-2e)} - 4a^2d^2 - 2b^2d^2}{8\left(d^5x^2 + 2cd^4x + c^2d^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*d^2*
f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a*b*d^2*f^2*x^2*Ei(-(d*f*x +
c*f)/d)*e^(-e + c*f/d) + 4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
 2*c*f/d) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8*a*b
*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*a*b*c*d*f^2*x*Ei(-(d*f*x +
```

```
 c*f)/d)*e^(-e + c*f/d) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*c^
2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a*b*c^2*f^2*Ei(-(d*f*x + c*f)/d
)*e^(-e + c*f/d) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d)
- 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) + 4*a*b*d^2*f*x
*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(2*f*x + 2*e
) - 4*a*b*c*d*f*e^(f*x + e) + 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*c*d*f*e^(-2*
f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) - 4*a*b*d^2*e^
(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 - 2*b^2*d^2)/(d^5*x^2 + 2
*c*d^4*x + c^2*d^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(a+b\cosh(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2/(c + d*x)^3,x)
```

```
[Out] int((a + b*cosh(e + f*x))^2/(c + d*x)^3, x)
```

3.168 $\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=436

$$\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{3d(c+dx)^2\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,f^2}$$

```
[Out] (d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)-(d*x+c)^
3*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)+3*d*(d*x+c)^2*po
lylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-3*d*(d*x+c)^
2*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-6*d^2*(d
*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f^3/(a^2-b^2)^(1/2)+6*d^
2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f^3/(a^2-b^2)^(1/2)+
6*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f^4/(a^2-b^2)^(1/2)-6*d^
3*polylog(4,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f^4/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 436, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3401, 2296, 2221, 2611, 6744, 2320, 6724}

$$-\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{6d^2(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{3d(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}}-\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}+\frac{6d^3\text{Li}_4\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^4\sqrt{a^2-b^2}}-\frac{6d^3\text{Li}_4\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^4\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2
 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (3*d*(c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (6*d^2*(c + d*x)*Po
lyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) +
 (6*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sq
rt[a^2 - b^2]*f^3) + (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^
2]))])/(Sqrt[a^2 - b^2]*f^4) - (6*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqr
t[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^4)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^3}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^3}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - (3d)\int$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 3d(c$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 3d(c$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 3d(c$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 3d(c$$

Mathematica [A]

time = 1.03, size = 384, normalized size = 0.88

$$\frac{(c+dx)^3\log\left(1+\frac{b(\cosh(e+fx)+\sinh(e+fx))}{a-\sqrt{a^2-b^2}}\right)-(c+dx)^3\log\left(1+\frac{b(\cosh(e+fx)+\sinh(e+fx))}{a+\sqrt{a^2-b^2}}\right)+\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,\frac{b(\cosh(e+fx)+\sinh(e+fx))}{-a+\sqrt{a^2-b^2}}\right)-2df(c+dx)\text{PolyLog}\left(3,\frac{b(\cosh(e+fx)+\sinh(e+fx))}{-a+\sqrt{a^2-b^2}}\right)+2d^2\text{PolyLog}\left(4,\frac{b(\cosh(e+fx)+\sinh(e+fx))}{-a+\sqrt{a^2-b^2}}\right)\right)}{f^3}-\frac{3d\left(f^2(c+dx)^2\text{PolyLog}\left(2,-\frac{b(\cosh(e+fx)+\sinh(e+fx))}{a+\sqrt{a^2-b^2}}\right)-2df(c+dx)\text{PolyLog}\left(3,-\frac{b(\cosh(e+fx)+\sinh(e+fx))}{a+\sqrt{a^2-b^2}}\right)+2d^2\text{PolyLog}\left(4,-\frac{b(\cosh(e+fx)+\sinh(e+fx))}{a+\sqrt{a^2-b^2}}\right)\right)}{f^3}}{\sqrt{a^2-b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^3*Log[1 + (b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a - Sqrt[a^2 - b^
2])] - (c + d*x)^3*Log[1 + (b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + Sqrt[a^
2 - b^2])] + (3*d*(f^2*(c + d*x)^2*PolyLog[2, (b*(Cosh[e + f*x] + Sinh[e +
f*x]))/(-a + Sqrt[a^2 - b^2])] - 2*d*f*(c + d*x)*PolyLog[3, (b*(Cosh[e + f*
x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])] + 2*d^2*PolyLog[4, (b*(Cosh[e
+ f*x] + Sinh[e + f*x]))/(-a + Sqrt[a^2 - b^2])]))/f^3 - (3*d*(f^2*(c + d*x
)^2*PolyLog[2, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + Sqrt[a^2 - b^2]))
```

```
] - 2*d*f*(c + d*x)*PolyLog[3, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/(a + S
qrt[a^2 - b^2]))] + 2*d^2*PolyLog[4, -((b*(Cosh[e + f*x] + Sinh[e + f*x]))/
(a + Sqrt[a^2 - b^2]))]))/f^3)/(Sqrt[a^2 - b^2]*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{a+b\cosh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1420 vs. 2(402) = 804.
time = 0.38, size = 1420, normalized size = 3.26

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (6*b*d^3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + cosh(1) + sinh(1))
 + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*s
inh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*d^3*sqrt((a^2
 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + co
sh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1)
+ sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) + 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x
+ b*c^2*d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)
) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*
sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*(b*d^3
```

```
*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*c
osh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*
x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/
b^2) + b)/b + 1) - (b*c^3*f^3 - 3*b*c^2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)
^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh
(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sq
rt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x +
cosh(1) + sinh(1)) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + (b*c^3*f^3 - 3*b*c^
2*d*f^2*cosh(1) + 3*b*c*d^2*f*cosh(1)^2 - b*d^3*cosh(1)^3 - b*d^3*sinh(1)^3
 + 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 - 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*c
osh(1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x +
 cosh(1) + sinh(1)) + 2*b*sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 - b
^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*
b*c^2*d*f^2*cosh(1) - 3*b*c*d^2*f*cosh(1)^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(
1)^3 - 3*(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 + 3*(b*c^2*d*f^2 - 2*b*c*d^2
*f*cosh(1) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*
x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + co
sh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) +
 b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3*b*c^2*d*f
^2*cosh(1) - 3*b*c*d^2*f*cosh(1)^2 + b*d^3*cosh(1)^3 + b*d^3*sinh(1)^3 - 3*
(b*c*d^2*f - b*d^3*cosh(1))*sinh(1)^2 + 3*(b*c^2*d*f^2 - 2*b*c*d^2*f*cosh(1
) + b*d^3*cosh(1)^2)*sinh(1))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + cosh(
1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + s
inh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b) -
6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + c
osh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1)
 + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) +
6*(b*d^3*f*x + b*c*d^2*f)*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + c
osh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1)
 + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b))/(
(a^2 - b^2)*f^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)**3/(a + b*cosh(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)^3/(b*cosh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{a+b\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cosh(e + f*x)),x)
```

```
[Out] int((c + d*x)^3/(a + b*cosh(e + f*x)), x)
```

3.169 $\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=320

$$\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{2d(c+dx)\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}$$

```
[Out] (d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)-(d*x+c)^
2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)+2*d*(d*x+c)*poly
log(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-2*d*(d*x+c)*po
lylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-2*d^2*polylo
g(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f^3/(a^2-b^2)^(1/2)+2*d^2*polylog(3,
-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f^3/(a^2-b^2)^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3401, 2296, 2221, 2611, 2320, 6724}

$$\frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{2d(c+dx)\mathrm{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}+\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}}-\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}-\frac{2d^2\mathrm{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}+\frac{2d^2\mathrm{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^3\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^
2]*f) - ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[
a^2 - b^2]*f) + (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 -
 b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^2) - (2*d^2*PolyLog[3, -((b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3) + (2*d^2*PolyLo
g[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(Sqrt[a^2 - b^2]*f^3)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
```

```
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$

$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)^2}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - (2d)\int$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 2d(c$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 2d(c$$

$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + 2d(c$$

Mathematica [A]
time = 0.67, size = 247, normalized size = 0.77

$$\frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)-(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)+\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)-d\text{PolyLog}\left(3,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)\right)}{f^2}-\frac{2d\left(f(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)-d\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)\right)}{f^2}}{\sqrt{a^2-b^2}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - (c + d*x)^2*L
og[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] + (2*d*(f*(c + d*x)*PolyLog[2
, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] - d*PolyLog[3, (b*E^(e + f*x))/(-
a + Sqrt[a^2 - b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e + f*x)
)/(a + Sqrt[a^2 - b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 -
b^2]))]))/f^2)/(Sqrt[a^2 - b^2]*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{a+b\cosh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

```
[Out] int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 938 vs. 2(294) = 588.
time = 0.46, size = 938, normalized size = 2.93

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -(2*b*d^2*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + cosh(1) + sinh(1)
) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*
sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) - 2*b*d^2*sqrt((a^
2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + c
osh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1)
 + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) - 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2
- b^2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1)
+ sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh
(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2
 - b^2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1)
 + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sin
h(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*c^2*f^2 - 2*b*c*d*f*cosh(1) +
 b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*s
qrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x +
 cosh(1) + sinh(1)) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (b*c^2*f^2 - 2*b*c
*d*f*cosh(1) + b*d^2*cosh(1)^2 + b*d^2*sinh(1)^2 - 2*(b*c*d*f - b*d^2*cosh(
1))*sinh(1))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) +
2*b*sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - (b*d
```

```
^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*cosh(1) - b*d^2*cosh(1)^2 - b*d^2*si
nh(1)^2 + 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a^2 - b^2)/b^2)*log((a
*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(
f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2
)/b^2) + b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + 2*b*c*d*f*cosh(1) - b*d^2
*cosh(1)^2 - b*d^2*sinh(1)^2 + 2*(b*c*d*f - b*d^2*cosh(1))*sinh(1))*sqrt((a
^2 - b^2)/b^2)*log((a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1)
+ sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh
(1)))*sqrt((a^2 - b^2)/b^2) + b)/b))/((a^2 - b^2)*f^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cosh(f*x+e)),x)
[Out] Integral((c + d*x)**2/(a + b*cosh(e + f*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*cosh(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{a+b\cosh(e+f\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cosh(e + f*x)),x)
[Out] int((c + d*x)^2/(a + b*cosh(e + f*x)), x)
```

3.170 $\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=203

$$\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}+\frac{d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}-\frac{d\text{PolyI}}{}$$

```
[Out] (d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)-(d*x+c)*ln
(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/f/(a^2-b^2)^(1/2)+d*polylog(2,-b*exp(f
*x+e)/(a-(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)-d*polylog(2,-b*exp(f*x+e)/(a
+(a^2-b^2)^(1/2)))/f^2/(a^2-b^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3401, 2296, 2221, 2317, 2438}

$$\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f\sqrt{a^2-b^2}}-\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f\sqrt{a^2-b^2}}+\frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}-\frac{d\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

```
[Out] ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(Sqrt[a^2 - b^2]
*f) - ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(Sqrt[a^2
- b^2]*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(Sqrt[
a^2 - b^2]*f^2) - (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/
(Sqrt[a^2 - b^2]*f^2)
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = 2\int \frac{e^{e+fx}(c+dx)}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx$$
$$= \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a-2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}} - \frac{(2b)\int \frac{e^{e+fx}(c+dx)}{2a+2\sqrt{a^2-b^2}+2be^{e+fx}}\,dx}{\sqrt{a^2-b^2}}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{d\int \log}{}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{d\text{Subst}}{}$$
$$= \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{d\text{Li}_2\left(-\right.}{\sqrt{}}$$

Mathematica [A]
time = 0.63, size = 152, normalized size = 0.75

$$\frac{f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)\right)+d\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)-d\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

```
[Out] (f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E
^(e + f*x))/(a + Sqrt[a^2 - b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a + Sq
rt[a^2 - b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(
Sqrt[a^2 - b^2]*f^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(183) = 366$.
time = 1.46, size = 437, normalized size = 2.15

method	result
risch	$\frac{2c \arctan\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f\sqrt{a^2-b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)e}{f^2\sqrt{a^2-b^2}}-\frac{d\ln\left(\frac{b\,\mathrm{e}^{fx+e}+\sqrt{a}}{a+\sqrt{a}}\right.}{f\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))+1/
f*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2
)))*x+1/f^2*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2
-b^2)^(1/2)))*e-1/f*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(
a+(a^2-b^2)^(1/2)))*x-1/f^2*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^2)^(1
/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/f^2*d/(a^2-b^2)^(1/2)*dilog((-b*exp(f*x+e)+
(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/f^2*d/(a^2-b^2)^(1/2)*dilog((b*e
xp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/f^2*d*e/(-a^2+b^2)^(1/2
)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(185) = 370$.
time = 0.37, size = 559, normalized size = 2.75

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] (b*d*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh
(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x +
 cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*d*sqrt((a^2 - b^
2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + si
nh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1))
)*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (b*c*f - b*d*cosh(1) - b*d*sinh(1))*s
qrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x +
 cosh(1) + sinh(1)) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + (b*c*f - b*d*cosh(
1) - b*d*sinh(1))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + cosh(1) + sinh(1
)) + 2*b*sinh(f*x + cosh(1) + sinh(1)) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) +
 (b*d*f*x + b*d*cosh(1) + b*d*sinh(1))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*
x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + co
sh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) +
 b)/b) - (b*d*f*x + b*d*cosh(1) + b*d*sinh(1))*sqrt((a^2 - b^2)/b^2)*log((a
*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(
f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2
)/b^2) + b)/b))/((a^2 - b^2)*f^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral((c + d*x)/(a + b*cosh(e + f*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((d*x + c)/(b*cosh(f*x + e) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{a+b\cosh\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cosh(e + f*x)),x)
[Out] int((c + d*x)/(a + b*cosh(e + f*x)), x)
```

3.171 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Mathematica [A]
time = 0.67, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]
[Out] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d*x + a*c + (b*d*x + b*c)*cosh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cosh(e + fx))(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*cosh(e + f*x))*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{(a + b\cosh(e + f\,x))\,(c + d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cosh(e + f*x))*(c + d*x)),x)
[Out] int(1/((a + b*cosh(e + f*x))*(c + d*x)), x)
```

3.172 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

```
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Mathematica [A]
time = 0.79, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

```
[Out] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\,(a+b\cosh(fx+e))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

Fricas [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)
*cosh(f*x + e)), x)
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(a + b\cosh{(e + fx)})\,(c + dx)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e)),x)
```

```
[Out] Integral(1/((a + b*cosh(e + f*x))*(c + d*x)**2), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cosh\left(e+f\,x\right)\right)\,\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cosh(e + f*x))*(c + d*x)^2),x)
[Out] int(1/((a + b*cosh(e + f*x))*(c + d*x)^2), x)
```

3.173 $\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=823

$$-\frac{(c+dx)^3}{(a^2-b^2)\,f}+\frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2}+\frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f}+\frac{3d(c+dx)^2\log\left(\right.}{(a^2-\ }$$

```
[Out] -(d*x+c)^3/(a^2-b^2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))
/(a^2-b^2)/f^2+a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^2
-a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+6*d^2
*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+3*a*d*(
d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+6
*d^2*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3-3*a
*d*(d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f
^2-6*d^3*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4-6*a*d^2
*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3-6
*d^3*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+6*a*d^2*(d*
x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+6*a*d
^3*polylog(4,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^4-6*a*d^3
*polylog(4,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^4-b*(d*x+c)
^3*sinh(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Rubi [A]
time = 0.98, antiderivative size = 823, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3405, 3401, 2296, 2221, 2611, 6744, 2320, 6724, 5681}

$$-\frac{6\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)\,f^4}-\frac{6\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)\,f^4}+\frac{6a\text{Li}_4\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}\,f^4}-\frac{6a\text{Li}_4\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^3}{(a^2-b^2)^{3/2}\,f^4}+\frac{6(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^2}{(a^2-b^2)\,f^3}+\frac{6(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^2}{(a^2-b^2)\,f^3}-\frac{6a(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d^2}{(a^2-b^2)^{3/2}\,f^3}+\frac{6a(c+dx)\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d^2}{(a^2-b^2)^{3/2}\,f^3}+\frac{3(c+dx)^2\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)d}{(a^2-b^2)\,f^2}+\frac{3(c+dx)^2\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)d}{(a^2-b^2)\,f^2}+\frac{3a(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)d}{(a^2-b^2)^{3/2}\,f^2}-\frac{3a(c+dx)^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)d}{(a^2-b^2)^{3/2}\,f^2}-\frac{(c+dx)^3}{(a^2-b^2)\,f}+\frac{a(c+dx)^3\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)}{(a^2-b^2)^{3/2}\,f}-\frac{a(c+dx)^3\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)}{(a^2-b^2)^{3/2}\,f}-\frac{b(c+dx)^3\sinh(e+fx)}{(a^2-b^2)\,f\,(a+b\cosh(e+fx))}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^3/((a^2 - b^2)*f)) + (3*d*(c + d*x)^2*Log[1 + (b*E^(e + f*x))/(
a - Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^3*Log[1 + (b*E^(e +
 f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (3*d*(c + d*x)^2*Log
[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*
x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f)
 + (6*d^2*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((
a^2 - b^2)*f^3) + (3*a*d*(c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt
[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) + (6*d^2*(c + d*x)*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3) - (3*a*d*(c + d*x)^
2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
```

```
f^2) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 -
 b^2)*f^4) - (6*a*d^2*(c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2
- b^2]))])/((a^2 - b^2)^(3/2)*f^3) - (6*d^3*PolyLog[3, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^4) + (6*a*d^2*(c + d*x)*PolyLog[3, -(
(b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (6*a*d^3
*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f
^4) - (6*a*d^3*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2
- b^2)^(3/2)*f^4) - (b*(c + d*x)^3*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cos
h[e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
```

```
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rule 6744

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Dist[f*(m/(b*c*p*Log[F])), Int[(e + f*x)^
(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c,
 d, e, f, n, p}, x] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)^3\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{a\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx}{a^2-b^2} + \frac{(3bd)\int \frac{(c+dx)^2\sinh(e}{a+b\cosh(e+}}{(a^2-b^2)\,f}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} - \frac{b(c+dx)^3\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)^3}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2-b^2}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{3d(c+dx)^2\log\left(1+\right.}{(a^2-b^2)\,f}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^3\log\left(1+\right.}{(a^2-b^2)^{3/2}}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^3\log\left(1+\right.}{(a^2-b^2)^{3/2}}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^3\log\left(1+\right.}{(a^2-b^2)^{3/2}}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^3\log\left(1+\right.}{(a^2-b^2)^{3/2}}$$

$$= -\frac{(c+dx)^3}{(a^2-b^2)\,f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^3\log\left(1+\right.}{(a^2-b^2)^{3/2}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 11178 vs. $2(823) = 1646$.
time = 23.00, size = 11178, normalized size = 13.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Result too large to show
```

Maple [F]
time = 0.83, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^3}{(a+b\cosh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 11778 vs. 2(777) = 1554.
time = 0.57, size = 11778, normalized size = 14.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*(a^2*b - b^3)*c^3*f^3 - 6*(a^2*b - b^3)*c^2*d*f^2*cosh(1) + 6*(a^2*b - b
^3)*c*d^2*f*cosh(1)^2 - 2*(a^2*b - b^3)*d^3*cosh(1)^3 - 2*(a^2*b - b^3)*d^3
*sinh(1)^3 - 2*((a^2*b - b^3)*d^3*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 +
 3*(a^2*b - b^3)*c^2*d*f^3*x + 3*(a^2*b - b^3)*c^2*d*f^2*cosh(1) - 3*(a^2*b
 - b^3)*c*d^2*f*cosh(1)^2 + (a^2*b - b^3)*d^3*cosh(1)^3 + (a^2*b - b^3)*d^3
*sinh(1)^3 - 3*((a^2*b - b^3)*c*d^2*f - (a^2*b - b^3)*d^3*cosh(1))*sinh(1)^
2 + 3*((a^2*b - b^3)*c^2*d*f^2 - 2*(a^2*b - b^3)*c*d^2*f*cosh(1) + (a^2*b -
 b^3)*d^3*cosh(1)^2)*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 + 6*((a^2*b -
 b^3)*c*d^2*f - (a^2*b - b^3)*d^3*cosh(1))*sinh(1)^2 - 2*((a^2*b - b^3)*d^3
*f^3*x^3 + 3*(a^2*b - b^3)*c*d^2*f^3*x^2 + 3*(a^2*b - b^3)*c^2*d*f^3*x + 3*
(a^2*b - b^3)*c^2*d*f^2*cosh(1) - 3*(a^2*b - b^3)*c*d^2*f*cosh(1)^2 + (a^2*
b - b^3)*d^3*cosh(1)^3 + (a^2*b - b^3)*d^3*sinh(1)^3 - 3*((a^2*b - b^3)*c*d
```

```
^2*f - (a^2*b - b^3)*d^3*cosh(1))*sinh(1)^2 + 3*((a^2*b - b^3)*c^2*d*f^2 -
2*(a^2*b - b^3)*c*d^2*f*cosh(1) + (a^2*b - b^3)*d^3*cosh(1)^2)*sinh(1))*sin
h(f*x + cosh(1) + sinh(1))^2 + 6*(a*b^2*d^3*cosh(f*x + cosh(1) + sinh(1))^2
 + a*b^2*d^3*sinh(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d^3*cosh(f*x + cosh(
1) + sinh(1)) + a*b^2*d^3 + 2*(a*b^2*d^3*cosh(f*x + cosh(1) + sinh(1)) + a^
2*b*d^3)*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*polylog(4, -(
a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh
(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^
2)/b^2))/b) - 6*(a*b^2*d^3*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d^3*sinh
(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d^3*cosh(f*x + cosh(1) + sinh(1)) + a
*b^2*d^3 + 2*(a*b^2*d^3*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d^3)*sinh(f*x
 + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + cosh
(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) +
sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) - 2*(
(a^3 - a*b^2)*d^3*f^3*x^3 + 3*(a^3 - a*b^2)*c*d^2*f^3*x^2 + 3*(a^3 - a*b^2)
*c^2*d*f^3*x - (a^3 - a*b^2)*c^3*f^3 + 6*(a^3 - a*b^2)*c^2*d*f^2*cosh(1) -
6*(a^3 - a*b^2)*c*d^2*f*cosh(1)^2 + 2*(a^3 - a*b^2)*d^3*cosh(1)^3 + 2*(a^3
- a*b^2)*d^3*sinh(1)^3 - 6*((a^3 - a*b^2)*c*d^2*f - (a^3 - a*b^2)*d^3*cosh(
1))*sinh(1)^2 + 6*((a^3 - a*b^2)*c^2*d*f^2 - 2*(a^3 - a*b^2)*c*d^2*f*cosh(1
) + (a^3 - a*b^2)*d^3*cosh(1)^2)*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 3
*(2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f + 2*((a^2*b - b^3)*d^3*
f*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1))^2 + 2*((a^2*b -
b^3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + cosh(1) + sinh(1))^2 + 4*(
(a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)
) + 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f
*x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(
1) + sinh(1)) + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2
+ (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + co
sh(1) + sinh(1))^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d
*f^2)*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^
2*f^2*x + a^2*b*c^2*d*f^2)*cosh(f*x + cosh(1) + sinh(1)) + 2*(a^2*b*d^3*f^2
*x^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2
*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + c
osh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x + cosh(1) + si
nh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1))
 + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 3*(
2*(a^2*b - b^3)*d^3*f*x + 2*(a^2*b - b^3)*c*d^2*f + 2*((a^2*b - b^3)*d^3*f*
x + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1))^2 + 2*((a^2*b - b^
3)*d^3*f*x + (a^2*b - b^3)*c*d^2*f)*sinh(f*x + cosh(1) + sinh(1))^2 + 4*((a
^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1))
+ 4*((a^3 - a*b^2)*d^3*f*x + (a^3 - a*b^2)*c*d^2*f + ((a^2*b - b^3)*d^3*f*x
 + (a^2*b - b^3)*c*d^2*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1)
 + sinh(1)) - (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2 +
(a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + cosh
(1) + sinh(1))^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c*d^2*f^2*x + a*b^2*c^2*d*f
```

```
^2)*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^2*b*d^3*f^2*x^2 + 2*a^2*b*c*d^2*
f^2*x + a^2*b*c^2*d*f^2)*cosh(f*x + cosh(1) + sinh(1)) + 2*(a^2*b*d^3*f^2*x
^2 + 2*a^2*b*c*d^2*f^2*x + a^2*b*c^2*d*f^2 + (a*b^2*d^3*f^2*x^2 + 2*a*b^2*c
*d^2*f^2*x + a*b^2*c^2*d*f^2)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cos
h(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))*dilog(-(a*cosh(f*x + cosh(1) + sinh
(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) +
 b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (3*(a
^2*b - b^3)*c^2*d*f^2 - 6*(a^2*b - b^3)*c*d^2*f*cosh(1) + 3*(a^2*b - b^3)*d
^3*cosh(1)^2 + 3*(a^2*b - b^3)*d^3*sinh(1)^2 + 3*((a^2*b - b^3)*c^2*d*f^2 -
 2*(a^2*b - b^3)*c*d^2*f*cosh(1) + (a^2*b - b^3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**3/(a+b*cosh(f*x+e))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^3/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^3}{(a+b\cosh\left(e+f\,x\right))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^3/(a + b*cosh(e + f*x))^2,x)
[Out] int((c + d*x)^3/(a + b*cosh(e + f*x))^2, x)
```

3.174 $\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=593

$$-\frac{(c+dx)^2}{(a^2-b^2)f}+\frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2}+\frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}+\frac{2d(c+dx)\log\left(1+\right.}{(a^2-b^2}$$

```
[Out] -(d*x+c)^2/(a^2-b^2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(
a^2-b^2)/f^2+a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(
3/2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^2-a*(
d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+2*d^2*pol
ylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+2*a*d*(d*x+c)*polyl
og(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+2*d^2*polylog(2
,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3-2*a*d*(d*x+c)*polylog(2,-
b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2-2*a*d^2*polylog(3,-b*
exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+2*a*d^2*polylog(3,-b*ex
p(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3-b*(d*x+c)^2*sinh(f*x+e)/(
a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Rubi [A]

time = 0.73, antiderivative size = 593, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 10, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3405, 3401, 2296, 2221, 2611, 2320, 6724, 5681, 2317, 2438}

$$\frac{2ad(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}-\frac{2ad(c+dx)\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}+\frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f^2(a^2-b^2)}+\frac{2d(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f^2(a^2-b^2)}+\frac{a(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f(a^2-b^2)^{3/2}}-\frac{a(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f(a^2-b^2)^{3/2}}-\frac{b(c+dx)^2\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}-\frac{(c+dx)^2}{f(a^2-b^2)}+\frac{2d^2\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)}+\frac{2d^2\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)}-\frac{2ad^2\text{Li}_3\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)^{3/2}}+\frac{2ad^2\text{Li}_3\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^3(a^2-b^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -((c + d*x)^2/((a^2 - b^2)*f)) + (2*d*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a
- Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) + (a*(c + d*x)^2*Log[1 + (b*E^(e + f
*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2*d*(c + d*x)*Log[1 +
 (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)*f^2) - (a*(c + d*x)^2
*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^2 - b^2)^(3/2)*f) + (2
*d^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)*f^3
) + (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/
((a^2 - b^2)^(3/2)*f^2) + (2*d^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2
 - b^2]))])/((a^2 - b^2)*f^3) - (2*a*d*(c + d*x)*PolyLog[2, -((b*E^(e + f*x
))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2) - (2*a*d^2*PolyLog[3, -
((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^3) + (2*a*d^
2*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*
f^3) - (b*(c + d*x)^2*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*x]))
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2320

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Dist[v/D[v, x]
, Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Functi
onOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ[
{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))*
(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2611

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Dist[g*(m/(b*c*n*Log[F])), Int[(f + g*x)^(m
 - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e,
 f, g, n}, x] && GtQ[m, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
```

```
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rule 5681

```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Rule 6724

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Rubi steps

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)^2\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{a\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx}{a^2-b^2} + \frac{(2bd)\int \frac{(c+dx)\sinh(\cdots}{a+b\cosh(e\cdots}}{(a^2-b^2)\,f\cdots}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} - \frac{b(c+dx)^2\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)^2}{b+2ae^{e+fx}+be^{2(e+fx)}}\,d\cdots}{a^2-b^2}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{2d(c+dx)\log\left(1+\cdots\right.}{(a^2-b^2)\,f\cdots}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\cdots\right.}{(a^2-b^2)^{3/2}\cdots}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\cdots\right.}{(a^2-b^2)^{3/2}\cdots}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\cdots\right.}{(a^2-b^2)^{3/2}\cdots}$$

$$= -\frac{(c+dx)^2}{(a^2-b^2)\,f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)\,f^2} + \frac{a(c+dx)^2\log\left(1+\cdots\right.}{(a^2-b^2)^{3/2}\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2854 vs. $2(593) = 1186$.
time = 18.62, size = 2854, normalized size = 4.81

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] -((4*(a^2 - b^2)^2*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f^2*x + 2*(a^2 - b^2)^2*
d^2*((a^2 - b^2)*E^(2*e))^(3/2)*f^2*x^2 + 4*a^3*Sqrt[a^2 - b^2]*Sqrt[-(a^2
- b^2)^2]*c*d*Sqrt[(a^2 - b^2)*E^(2*e)]*f*ArcTan[(a + b*E^(e + f*x))/Sqrt[-
a^2 + b^2]] - 4*a*b^2*Sqrt[a^2 - b^2]*Sqrt[-(a^2 - b^2)^2]*c*d*Sqrt[(a^2 -
b^2)*E^(2*e)]*f*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] + (4*a*b^2*(a^
2 - b^2)^(3/2)*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*ArcTan[(a + b*E^(e + f*x))
```

```
/Sqrt[-a^2 + b^2]])/Sqrt[-(a^2 - b^2)^2] + (4*a^3*Sqrt[-(a^2 - b^2)^2]*c*d*
((a^2 - b^2)*E^(2*e))^(3/2)*f*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]])
/Sqrt[a^2 - b^2] - 4*a*(a^2 - b^2)^(5/2)*c*d*Sqrt[(a^2 - b^2)*E^(2*e)]*f*Ar
cTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 4*a*(a^2 - b^2)^(3/2)*c*d*((a^
2 - b^2)*E^(2*e))^(3/2)*f*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] + 2*
a*(a^2 - b^2)^(5/2)*c^2*Sqrt[(a^2 - b^2)*E^(2*e)]*f^2*ArcTanh[(a + b*E^(e +
 f*x))/Sqrt[a^2 - b^2]] + 2*a*(a^2 - b^2)^(3/2)*c^2*((a^2 - b^2)*E^(2*e))^(
3/2)*f^2*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*(a^2 - b^2)^3*c*d
*Sqrt[(a^2 - b^2)*E^(2*e)]*f*Log[b + 2*a*E^(e + f*x) + b*E^(2*(e + f*x))] -
 2*(a^2 - b^2)^2*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*Log[b + 2*a*E^(e + f*x)
+ b*E^(2*(e + f*x))] - 2*(a^2 - b^2)^3*d^2*Sqrt[(a^2 - b^2)*E^(2*e)]*f*x*Lo
g[1 + (b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)])] - 2*(a^2 - b^2
)^2*d^2*((a^2 - b^2)*E^(2*e))^(3/2)*f*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e -
Sqrt[(a^2 - b^2)*E^(2*e)])] - 2*a*(a^2 - b^2)^3*c*d*E^e*f^2*x*Log[1 + (b*E^
(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)])] - 2*a*(a^2 - b^2)^3*c*d*E
^(3*e)*f^2*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)])]
 - a*(a^2 - b^2)^3*d^2*E^e*f^2*x^2*Log[1 + (b*E^(2*e + f*x))/(a*E^e - Sqrt[
(a^2 - b^2)*E^(2*e)])] - a*(a^2 - b^2)^3*d^2*E^(3*e)*f^2*x^2*Log[1 + (b*E^(
2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)])] - 2*(a^2 - b^2)^3*d^2*Sqrt
[(a^2 - b^2)*E^(2*e)]*f*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^
2)*E^(2*e)])] - 2*(a^2 - b^2)^2*d^2*((a^2 - b^2)*E^(2*e))^(3/2)*f*x*Log[1 +
 (b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)])] + 2*a*(a^2 - b^2)^3
*c*d*E^e*f^2*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)]
)] + 2*a*(a^2 - b^2)^3*c*d*E^(3*e)*f^2*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e +
 Sqrt[(a^2 - b^2)*E^(2*e)])] + a*(a^2 - b^2)^3*d^2*E^e*f^2*x^2*Log[1 + (b*E
^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)])] + a*(a^2 - b^2)^3*d^2*E^
(3*e)*f^2*x^2*Log[1 + (b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)])
] - 2*(a^2 - b^2)^3*d^2*Sqrt[(a^2 - b^2)*E^(2*e)]*PolyLog[2, -((b*E^(2*e +
f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] - 2*(a^2 - b^2)^2*d^2*((a^2 - b
^2)*E^(2*e))^(3/2)*PolyLog[2, -((b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)
*E^(2*e)]))] - 2*a*(a^2 - b^2)^3*c*d*E^e*f*PolyLog[2, -((b*E^(2*e + f*x))/(
a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] - 2*a*(a^2 - b^2)^3*c*d*E^(3*e)*f*Poly
Log[2, -((b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] - 2*a*(a^2
 - b^2)^3*d^2*E^e*f*x*PolyLog[2, -((b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b
^2)*E^(2*e)]))] - 2*a*(a^2 - b^2)^3*d^2*E^(3*e)*f*x*PolyLog[2, -((b*E^(2*e
+ f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] - 2*(a^2 - b^2)^3*d^2*Sqrt[(a
^2 - b^2)*E^(2*e)]*PolyLog[2, -((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)
*E^(2*e)]))] - 2*(a^2 - b^2)^2*d^2*((a^2 - b^2)*E^(2*e))^(3/2)*PolyLog[2, -
((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)]))] + 2*a*(a^2 - b^2)^
3*c*d*E^e*f*PolyLog[2, -((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e
)]))] + 2*a*(a^2 - b^2)^3*c*d*E^(3*e)*f*PolyLog[2, -((b*E^(2*e + f*x))/(a*E
^e + Sqrt[(a^2 - b^2)*E^(2*e)]))] + 2*a*(a^2 - b^2)^3*d^2*E^e*f*x*PolyLog[2
, -((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^(2*e)]))] + 2*a*(a^2 - b^
2)^3*d^2*E^(3*e)*f*x*PolyLog[2, -((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^
2)*E^(2*e)]))] + 2*a*(a^2 - b^2)^3*d^2*E^e*PolyLog[3, -((b*E^(2*e + f*x))/(
```

```
a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] + 2*a*(a^2 - b^2)^3*d^2*E^(3*e)*PolyLo
g[3, -((b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)]))] - 2*a*(a^2 -
 b^2)^3*d^2*E^e*PolyLog[3, -((b*E^(2*e + f*x))/(a*E^e + Sqrt[(a^2 - b^2)*E^
(2*e)]))] - 2*a*(a^2 - b^2)^3*d^2*E^(3*e)*PolyLog[3, -((b*E^(2*e + f*x))/(a
*E^e + Sqrt[(a^2 - b^2)*E^(2*e)]))])/((a^2 - b^2)^4*Sqrt[(a^2 - b^2)*E^(2*e
)]*(1 + E^(2*e))*f^3)) + (Sech[e]*(a*c^2*Sinh[e] + 2*a*c*d*x*Sinh[e] + a*d^
2*x^2*Sinh[e] - b*c^2*Sinh[f*x] - 2*b*c*d*x*Sinh[f*x] - b*d^2*x^2*Sinh[f*x]
))/((a - b)*(a + b)*f*(a + b*Cosh[e + f*x]))
```

Maple [F]
time = 0.85, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^2}{(a+b\cosh(fx+e))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6143 vs. 2(559) = 1118.
time = 0.68, size = 6143, normalized size = 10.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*(a^2*b - b^3)*c^2*f^2 - 4*(a^2*b - b^3)*c*d*f*cosh(1) + 2*(a^2*b - b^3)*
d^2*cosh(1)^2 + 2*(a^2*b - b^3)*d^2*sinh(1)^2 - 2*((a^2*b - b^3)*d^2*f^2*x^
2 + 2*(a^2*b - b^3)*c*d*f^2*x + 2*(a^2*b - b^3)*c*d*f*cosh(1) - (a^2*b - b^
3)*d^2*cosh(1)^2 - (a^2*b - b^3)*d^2*sinh(1)^2 + 2*((a^2*b - b^3)*c*d*f - (
```

a^2*b - b^3)*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 - 2*((a^
2*b - b^3)*d^2*f^2*x^2 + 2*(a^2*b - b^3)*c*d*f^2*x + 2*(a^2*b - b^3)*c*d*f*
cosh(1) - (a^2*b - b^3)*d^2*cosh(1)^2 - (a^2*b - b^3)*d^2*sinh(1)^2 + 2*((a
^2*b - b^3)*c*d*f - (a^2*b - b^3)*d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1)
+ sinh(1))^2 - 2*(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d^2*sin
h(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d^2*cosh(f*x + cosh(1) + sinh(1)) +
a*b^2*d^2 + 2*(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d^2)*sinh(f*
x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + cos
h(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(1) +
sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) + 2*
(a*b^2*d^2*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d^2*sinh(f*x + cosh(1) +
sinh(1))^2 + 2*a^2*b*d^2*cosh(f*x + cosh(1) + sinh(1)) + a*b^2*d^2 + 2*(a*
b^2*d^2*cosh(f*x + cosh(1) + sinh(1)) + a^2*b*d^2)*sinh(f*x + cosh(1) + sin
h(1)))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + cosh(1) + sinh(1)) +
a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sin
h(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))/b) - 2*((a^3 - a*b^2)*d^
2*f^2*x^2 + 2*(a^3 - a*b^2)*c*d*f^2*x - (a^3 - a*b^2)*c^2*f^2 + 4*(a^3 - a*
b^2)*c*d*f*cosh(1) - 2*(a^3 - a*b^2)*d^2*cosh(1)^2 - 2*(a^3 - a*b^2)*d^2*si
nh(1)^2 + 4*((a^3 - a*b^2)*c*d*f - (a^3 - a*b^2)*d^2*cosh(1))*sinh(1))*cosh
(f*x + cosh(1) + sinh(1)) + 2*((a^2*b - b^3)*d^2*cosh(f*x + cosh(1) + sinh(
1))^2 + (a^2*b - b^3)*d^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 - a*b^2)
*d^2*cosh(f*x + cosh(1) + sinh(1)) + (a^2*b - b^3)*d^2 + 2*((a^2*b - b^3)*d
^2*cosh(f*x + cosh(1) + sinh(1)) + (a^3 - a*b^2)*d^2)*sinh(f*x + cosh(1) +
sinh(1)) + (a*b^2*d^2*f*x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*cos
h(f*x + cosh(1) + sinh(1))^2 + (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(f*x + cos
h(1) + sinh(1))^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(f*x + cosh(1) + si
nh(1)) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c*d*f)*cos
h(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)
/b^2))*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sin
h(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))
sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2((a^2*b - b^3)*d^2*cosh(f*x + cosh(1
) + sinh(1))^2 + (a^2*b - b^3)*d^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3
- a*b^2)*d^2*cosh(f*x + cosh(1) + sinh(1)) + (a^2*b - b^3)*d^2 + 2*((a^2*b
- b^3)*d^2*cosh(f*x + cosh(1) + sinh(1)) + (a^3 - a*b^2)*d^2)*sinh(f*x + c
osh(1) + sinh(1)) - (a*b^2*d^2*f*x + a*b^2*c*d*f + (a*b^2*d^2*f*x + a*b^2*c
*d*f)*cosh(f*x + cosh(1) + sinh(1))^2 + (a*b^2*d^2*f*x + a*b^2*c*d*f)*sinh(
f*x + cosh(1) + sinh(1))^2 + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f)*cosh(f*x + cos
h(1) + sinh(1)) + 2*(a^2*b*d^2*f*x + a^2*b*c*d*f + (a*b^2*d^2*f*x + a*b^2*c
*d*f)*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a
^2 - b^2)/b^2))*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh
(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) +
sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (2*(a^2*b - b^3)*c*d*f - 2*(a
^2*b - b^3)*d^2*cosh(1) - 2*(a^2*b - b^3)*d^2*sinh(1) + 2*((a^2*b - b^3)*c*
d*f - (a^2*b - b^3)*d^2*cosh(1) - (a^2*b - b^3)*d^2*sinh(1))*cosh(f*x + cos
h(1) + sinh(1))^2 + 2*((a^2*b - b^3)*c*d*f - (a^2*b - b^3)*d^2*cosh(1) - (a

```
^2*b - b^3)*d^2*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 + 4*((a^3 - a*b^2)
*c*d*f - (a^3 - a*b^2)*d^2*cosh(1) - (a^3 - a*b^2)*d^2*sinh(1))*cosh(f*x +
cosh(1) + sinh(1)) + 4*((a^3 - a*b^2)*c*d*f - (a^3 - a*b^2)*d^2*cosh(1) - (
a^3 - a*b^2)*d^2*sinh(1) + ((a^2*b - b^3)*c*d*f - (a^2*b - b^3)*d^2*cosh(1)
 - (a^2*b - b^3)*d^2*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cos
h(1) + sinh(1)) - (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*d^2*cosh(1
)^2 + a*b^2*d^2*sinh(1)^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*cosh(1) + a*b^2*
d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*d^2*cosh(1))*s
inh(1))*cosh(f*x + cosh(1) + sinh(1))^2 + (a*b^2*c^2*f^2 - 2*a*b^2*c*d*f*co
sh(1) + a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d*f - a*b^2*
d^2*cosh(1))*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^2*b*c^2*f^2 -
2*a^2*b*c*d*f*cosh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 - 2*(a^2*
b*c*d*f - a^2*b*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a*
b^2*c*d*f - a*b^2*d^2*cosh(1))*sinh(1) + 2*(a^2*b*c^2*f^2 - 2*a^2*b*c*d*f*c
osh(1) + a^2*b*d^2*cosh(1)^2 + a^2*b*d^2*sinh(1)^2 + (a*b^2*c^2*f^2 - 2*a*b
^2*c*d*f*cosh(1) + a*b^2*d^2*cosh(1)^2 + a*b^2*d^2*sinh(1)^2 - 2*(a*b^2*c*d
*f - a*b^2*d^2*cosh(1))*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) - 2*(a^2*b*c
*d*f - a^2*b*d^2*cosh(1))*sinh(1))*sinh(f*x + c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^2/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(c+d\,x)^2}{(a+b\cosh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^2/(a + b*cosh(e + f*x))^2,x)
[Out] int((c + d*x)^2/(a + b*cosh(e + f*x))^2, x)
```

3.175 $\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=274

$$\frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}-\frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f}+\frac{d\log(a+b\cosh(e+fx))}{(a^2-b^2)f^2}+\frac{ad\text{PolyL}}{}$$

```
[Out] d*ln(a+b*cosh(f*x+e))/(a^2-b^2)/f^2+a*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2
)^(1/2)))/(a^2-b^2)^(3/2)/f-a*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2))
)/(a^2-b^2)^(3/2)/f+a*d*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b
^2)^(3/2)/f^2-a*d*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3
/2)/f^2-b*(d*x+c)*sinh(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Rubi [A]
time = 0.33, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 8, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3405, 3401, 2296, 2221, 2317, 2438, 2747, 31}

$$\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{f(a^2-b^2)^{3/2}}-\frac{a(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{f(a^2-b^2)^{3/2}}-\frac{b(c+dx)\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}+\frac{ad\text{Li}_2\left(-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}-\frac{ad\text{Li}_2\left(-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2(a^2-b^2)^{3/2}}+\frac{d\log(a+b\cosh(e+fx))}{f^2(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/((a^2 - b^2)^(
3/2)*f) - (a*(c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/((a^
2 - b^2)^(3/2)*f) + (d*Log[a + b*Cosh[e + f*x]])/((a^2 - b^2)*f^2) + (a*d*P
olyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/((a^2 - b^2)^(3/2)*f^2
) - (a*d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/((a^2 - b^2)
^(3/2)*f^2) - (b*(c + d*x)*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e + f*
x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 2221

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Di
st[d*(m/(b*f*g*n*Log[F])), Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x)
))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

Rule 2296

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/q), Int[
(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Dist[2*(c/q), Int[(f + g*x)^m
*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[v,
 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

Rule 2317

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Dist[1/(d*e*n*Log[F]), Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x))
)^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

Rule 2438

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

Rule 2747

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Dist[1/(b^p*f), Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
- 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 3401

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Dist[2, Int[((c + d*x)^m*(E^((-I)*e +
 f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)*
e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c,
 d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3405

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Dist[a/(a^2 - b^2), Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x],
 x] - Dist[b*d*(m/(f*(a^2 - b^2))), Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(a
+ b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 - b^
2, 0] && IGtQ[m, 0]
```

Rubi steps

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = -\frac{b(c+dx)\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{a\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx}{a^2-b^2} + \frac{(bd)\int \frac{\sinh(e+fx)}{a+b\cosh(e+fx)}}{(a^2-b^2)\,f}$$

$$= -\frac{b(c+dx)\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} + \frac{(2a)\int \frac{e^{e+fx}(c+dx)}{b+2ae^{e+fx}+be^{2(e+fx)}}\,dx}{a^2-b^2} +$$

$$= \frac{d\log(a+b\cosh(e+fx))}{(a^2-b^2)\,f^2} - \frac{b(c+dx)\sinh(e+fx)}{(a^2-b^2)\,f(a+b\cosh(e+fx))} +$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} +$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} +$$

$$= \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}\,f} +$$

Mathematica [**A**]
time = 3.31, size = 509, normalized size = 1.86

$$\frac{\frac{(a^2-b^2)\left(\sqrt{-(a^2-b^2)^2}\,d(e+fx)-2a\sqrt{a^2-b^2}\,d\,\text{ArcTan}\left(\frac{a+be^{e+fx}}{\sqrt{-a^2+b^2}}\right)-2a\sqrt{-a^2+b^2}\,d\tanh^{-1}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right)-2a\sqrt{-a^2+b^2}\,de\tanh^{-1}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right)+2a\sqrt{-a^2+b^2}\,cf\tanh^{-1}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right)-a\sqrt{-a^2+b^2}\,d(e+fx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)+a\sqrt{-a^2+b^2}\,d(e+fx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)-\sqrt{-(a^2-b^2)^2}\,d\log\left(b+2ae^{e+fx}+be^{2(e+fx)}\right)-a\sqrt{-a^2+b^2}\,d\,\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)+a\sqrt{-a^2+b^2}\,d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)\right)}{\left(-(a^2-b^2)^2\right)^{3/2}}-\frac{bf(c+dx)\sinh(e+fx)}{(a-b)(a+b)(a+b\cosh(e+fx))}}{f^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (((a^2 - b^2)*(Sqrt[-(a^2 - b^2)^2]*d*(e + f*x) - 2*a*Sqrt[a^2 - b^2]*d*Arc
Tan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*ArcTanh[
(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*a*Sqrt[-a^2 + b^2]*d*e*ArcTanh[(a
+ b*E^(e + f*x))/Sqrt[a^2 - b^2]] + 2*a*Sqrt[-a^2 + b^2]*c*f*ArcTanh[(a + b
*E^(e + f*x))/Sqrt[a^2 - b^2]] - a*Sqrt[-a^2 + b^2]*d*(e + f*x)*Log[1 + (b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2])] + a*Sqrt[-a^2 + b^2]*d*(e + f*x)*Log[1
+ (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] - Sqrt[-(a^2 - b^2)^2]*d*Log[b + 2
*a*E^(e + f*x) + b*E^(2*(e + f*x))] - a*Sqrt[-a^2 + b^2]*d*PolyLog[2, (b*E^
(e + f*x))/(-a + Sqrt[a^2 - b^2])] + a*Sqrt[-a^2 + b^2]*d*PolyLog[2, -((b*E
^(e + f*x))/(a + Sqrt[a^2 - b^2]))]))/(-(a^2 - b^2)^2)^(3/2) - (b*f*(c + d*
x)*Sinh[e + f*x])/((a - b)*(a + b)*(a + b*Cosh[e + f*x])))/f^2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(254) = 508$.
time = 2.04, size = 585, normalized size = 2.14

method	result
risch	$\frac{2(dx+c)\left(a\,\mathrm{e}^{fx+e}+b\right)}{f(a^2-b^2)\left(b\,\mathrm{e}^{2fx+2e}+2a\,\mathrm{e}^{fx+e}+b\right)}+\frac{2ac\arctan\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{ad\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f(a^2-b^2)^{\frac{3}{2}}}+\frac{ad\ln\left(\frac{-b\,\mathrm{e}^{fx\cdots}}{-\cdots}\right.}{f\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)/(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 2*(d*x+c)*(a*exp(f*x+e)+b)/f/(a^2-b^2)/(b*exp(2*f*x+2*e)+2*a*exp(f*x+e)+b)+
2/f/(a^2-b^2)*a*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^
2)^(1/2))+1/f/(a^2-b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+
(a^2-b^2)^(1/2)))*x+1/f^2/(a^2-b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2-b^2)^(
1/2)-a)/(-a+(a^2-b^2)^(1/2)))*e-1/f/(a^2-b^2)^(3/2)*a*d*ln((b*exp(f*x+e)+(a
^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-1/f^2/(a^2-b^2)^(3/2)*a*d*ln((b*exp
(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/f^2/(a^2-b^2)^(3/2)*a*d
*dilog((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/f^2/(a^2-b
^2)^(3/2)*a*d*dilog((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2
/f^2/(a^2-b^2)*d*ln(exp(f*x+e))+1/f^2/(a^2-b^2)*d*ln(b*exp(2*f*x+2*e)+2*a*e
xp(f*x+e)+b)-2/f^2/(a^2-b^2)*a*d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x
+e)+2*a)/(-a^2+b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2297 vs. $2(259) = 518$.
time = 0.41, size = 2297, normalized size = 8.38

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] (2*(a^2*b - b^3)*c*f - 2*(a^2*b - b^3)*d*cosh(1) - 2*((a^2*b - b^3)*d*f*x +
 (a^2*b - b^3)*d*cosh(1) + (a^2*b - b^3)*d*sinh(1))*cosh(f*x + cosh(1) + si
```

```
nh(1))^2 - 2*(a^2*b - b^3)*d*sinh(1) - 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^
3)*d*cosh(1) + (a^2*b - b^3)*d*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 + (
a*b^2*d*cosh(f*x + cosh(1) + sinh(1))^2 + a*b^2*d*sinh(f*x + cosh(1) + sinh
(1))^2 + 2*a^2*b*d*cosh(f*x + cosh(1) + sinh(1)) + a*b^2*d + 2*(a*b^2*d*cos
h(f*x + cosh(1) + sinh(1)) + a^2*b*d)*sinh(f*x + cosh(1) + sinh(1)))*sqrt((
a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh
(1) + sinh(1)) + (b*cosh(f*x + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) +
sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (a*b^2*d*cosh(f*x + cosh(1) +
 sinh(1))^2 + a*b^2*d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*a^2*b*d*cosh(f*x
+ cosh(1) + sinh(1)) + a*b^2*d + 2*(a*b^2*d*cosh(f*x + cosh(1) + sinh(1)) +
 a^2*b*d)*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*co
sh(f*x + cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x
 + cosh(1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b
^2) + b)/b + 1) + (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1) + (a*b^2
*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2
 + (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + s
inh(1))^2 + 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1))*cosh(f*x +
cosh(1) + sinh(1)) + 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1) + (
a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(
1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x +
 cosh(1) + sinh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) + (b*cosh(f*x + cosh(
1) + sinh(1)) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)
/b) - (a*b^2*d*f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1) + (a*b^2*d*f*x + a*b
^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 + (a*b^2*d*
f*x + a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + sinh(1))^2 +
2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1))*cosh(f*x + cosh(1) + si
nh(1)) + 2*(a^2*b*d*f*x + a^2*b*d*cosh(1) + a^2*b*d*sinh(1) + (a*b^2*d*f*x
+ a*b^2*d*cosh(1) + a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(f*
x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + cosh(1) + s
inh(1)) + a*sinh(f*x + cosh(1) + sinh(1)) - (b*cosh(f*x + cosh(1) + sinh(1)
) + b*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*((a^
3 - a*b^2)*d*f*x - (a^3 - a*b^2)*c*f + 2*(a^3 - a*b^2)*d*cosh(1) + 2*(a^3 -
 a*b^2)*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + ((a^2*b - b^3)*d*cosh(f*
x + cosh(1) + sinh(1))^2 + (a^2*b - b^3)*d*sinh(f*x + cosh(1) + sinh(1))^2
+ 2*(a^3 - a*b^2)*d*cosh(f*x + cosh(1) + sinh(1)) + (a^2*b - b^3)*d + 2*((a
^2*b - b^3)*d*cosh(f*x + cosh(1) + sinh(1)) + (a^3 - a*b^2)*d)*sinh(f*x + c
osh(1) + sinh(1)) - (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1) + (a*b^2
*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1))^2 +
 (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*sinh(f*x + cosh(1) + sinh(
1))^2 + 2*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1))*cosh(f*x + cosh(1
) + sinh(1)) + 2*(a^2*b*c*f - a^2*b*d*cosh(1) - a^2*b*d*sinh(1) + (a*b^2*c*
f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)))*sinh(
f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2)/b^2))*log(2*b*cosh(f*x + cosh(1)
 + sinh(1)) + 2*b*sinh(f*x + cosh(1) + sinh(1)) + 2*b*sqrt((a^2 - b^2)/b^2)
 + 2*a) + ((a^2*b - b^3)*d*cosh(f*x + cosh(1) + sinh(1))^2 + (a^2*b - b^3)*
```

```
d*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^3 - a*b^2)*d*cosh(f*x + cosh(1) +
sinh(1)) + (a^2*b - b^3)*d + 2*((a^2*b - b^3)*d*cosh(f*x + cosh(1) + sinh(1
)) + (a^3 - a*b^2)*d)*sinh(f*x + cosh(1) + sinh(1)) + (a*b^2*c*f - a*b^2*d*
cosh(1) - a*b^2*d*sinh(1) + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))
*cosh(f*x + cosh(1) + sinh(1))^2 + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*s
inh(1))*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^2*b*c*f - a^2*b*d*cosh(1) -
a^2*b*d*sinh(1))*cosh(f*x + cosh(1) + sinh(1)) + 2*(a^2*b*c*f - a^2*b*d*cos
h(1) - a^2*b*d*sinh(1) + (a*b^2*c*f - a*b^2*d*cosh(1) - a*b^2*d*sinh(1))*co
sh(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))*sqrt((a^2 - b^2
)/b^2))*log(2*b*cosh(f*x + cosh(1) + sinh(1)) + 2*b*sinh(f*x + cosh(1) + si
nh(1)) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*((a^3 - a*b^2)*d*f*x - (a^3 -
 a*b^2)*c*f + 2*(a^3 - a*b^2)*d*cosh(1) + 2*(a^3 - a*b^2)*d*sinh(1) + 2*((a
^2*b - b^3)*d*f*x + (a^2*b - b^3)*d*cosh(1) + (a^2*b - b^3)*d*sinh(1))*cosh
(f*x + cosh(1) + sinh(1)))*sinh(f*x + cosh(1) + sinh(1)))/((a^4*b - 2*a^2*b
^3 + b^5)*f^2*cosh(f*x + cosh(1) + sinh(1))^2 + (a^4*b - 2*a^2*b^3 + b^5)*f
^2*sinh(f*x + cosh(1) + sinh(1))^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*f^2*cosh(f
*x + cosh(1) + sinh(1)) + (a^4*b - 2*a^2*b^3 + b^5)*f^2 + 2*((a^4*b - 2*a^2
*b^3 + b^5)*f^2*cosh(f*x + cosh(1) + sinh(1)) + (a^5 - 2*a^3*b^2 + a*b^4)*f
^2)*sinh(f*x + cosh(1) + sinh(1)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))**2,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{c+d\,x}{{\left(a+b\,\mathrm{cosh}\left(e+f\,x\right)\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)/(a + b*cosh(e + f*x))^2,x)
[Out] int((c + d*x)/(a + b*cosh(e + f*x))^2, x)
```

3.176 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Mathematica [A]
time = 33.69, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(a*e^(f*x + e) + b)/(a^2*b*c*f - b^3*c*f + (a^2*b*d*f - b^3*d*f)*x + ((a^
2*b*d*f - b^3*d*f)*x*e^(2*e) + (a^2*b*c*f - b^3*c*f)*e^(2*e))*e^(2*f*x) + 2
*((a^3*d*f - a*b^2*d*f)*x*e^e + (a^3*c*f - a*b^2*c*f)*e^e)*e^(f*x)) + integ
rate(2*(b*d + (a*d*f*x*e^e + (c*f + d)*a*e^e)*e^(f*x))/(a^2*b*c^2*f - b^3*c
^2*f + (a^2*b*d^2*f - b^3*d^2*f)*x^2 + 2*(a^2*b*c*d*f - b^3*c*d*f)*x + ((a^
2*b*d^2*f - b^3*d^2*f)*x^2*e^(2*e) + 2*(a^2*b*c*d*f - b^3*c*d*f)*x*e^(2*e)
+ (a^2*b*c^2*f - b^3*c^2*f)*e^(2*e))*e^(2*f*x) + 2*((a^3*d^2*f - a*b^2*d^2*
f)*x^2*e^e + 2*(a^3*c*d*f - a*b^2*c*d*f)*x*e^e + (a^3*c^2*f - a*b^2*c^2*f)*
e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cosh(f*x + e)^2 + 2*(a*b*d*
x + a*b*c)*cosh(f*x + e)), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a + b\cosh\left(e + fx\right)\right)^{2}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Integral(1/((a + b*cosh(e + f*x))**2*(c + d*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)^2), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{\left(a+b\cosh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)),x)
```

```
[Out] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)), x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
[Out] Defer[Int][1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Rubi steps

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Mathematica [A]
time = 36.26, size = 0, normalized size = 0.00

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
[Out] Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(dx+c)^2\left(a+b\cosh\left(fx+e\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] 2*(a*e^(f*x + e) + b)/(a^2*b*c^2*f - b^3*c^2*f + (a^2*b*d^2*f - b^3*d^2*f)*
x^2 + 2*(a^2*b*c*d*f - b^3*c*d*f)*x + ((a^2*b*d^2*f - b^3*d^2*f)*x^2*e^(2*e
) + 2*(a^2*b*c*d*f - b^3*c*d*f)*x*e^(2*e) + (a^2*b*c^2*f - b^3*c^2*f)*e^(2*
e))*e^(2*f*x) + 2*((a^3*d^2*f - a*b^2*d^2*f)*x^2*e^e + 2*(a^3*c*d*f - a*b^2
*c*d*f)*x*e^e + (a^3*c^2*f - a*b^2*c^2*f)*e^e)*e^(f*x)) + integrate(2*(2*b*
d + (a*d*f*x*e^e + (c*f + 2*d)*a*e^e)*e^(f*x))/(a^2*b*c^3*f - b^3*c^3*f + (
a^2*b*d^3*f - b^3*d^3*f)*x^3 + 3*(a^2*b*c*d^2*f - b^3*c*d^2*f)*x^2 + 3*(a^2
*b*c^2*d*f - b^3*c^2*d*f)*x + ((a^2*b*d^3*f - b^3*d^3*f)*x^3*e^(2*e) + 3*(a
^2*b*c*d^2*f - b^3*c*d^2*f)*x^2*e^(2*e) + 3*(a^2*b*c^2*d*f - b^3*c^2*d*f)*x
*e^(2*e) + (a^2*b*c^3*f - b^3*c^3*f)*e^(2*e))*e^(2*f*x) + 2*((a^3*d^3*f - a
*b^2*d^3*f)*x^3*e^e + 3*(a^3*c*d^2*f - a*b^2*c*d^2*f)*x^2*e^e + 3*(a^3*c^2*
d*f - a*b^2*c^2*d*f)*x*e^e + (a^3*c^3*f - a*b^2*c^3*f)*e^e)*e^(f*x)), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d*
x + b^2*c^2)*cosh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*cosh
(f*x + e)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)^2), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{1}{{\left(a+b\,\cosh\left(e+f\,x\right)\right)}^2\,{\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2),x)
```

```
[Out] int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2), x)
```

3.178 $\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$

Optimal. Leaf size=23

$$\operatorname{Int}((c+dx)^m(a+b\cosh(e+fx))^n,x)$$

```
[Out] Unintegrable((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

Rubi [A]
time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

```
[Out] Defer[Int][(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Mathematica [A]
time = 3.02, size = 0, normalized size = 0.00

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

```
[Out] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+b\cosh\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="maxima")
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="fricas")
[Out] integral((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**n,x)
[Out] Timed out
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="giac")
[Out] integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int (a+b\cosh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^n*(c + d*x)^m,x)
[Out] int((a + b*cosh(e + f*x))^n*(c + d*x)^m, x)
```

3.179 $\int (c+dx)^m (a+b\cosh(e+fx))^3 \, dx$

Optimal. Leaf size=543

$$\frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3\,2^{-3-m}a}{}$$

```
[Out] a^3*(d*x+c)^(1+m)/d/(1+m)+3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*b^3*
exp(3*e-3*c*f/d)*(d*x+c)^m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3
*2^(-3-m)*a*b^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f
*(d*x+c)/d)^m)+3/2*a^2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/(
(-f*(d*x+c)/d)^m)+3/8*b^3*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/
((-f*(d*x+c)/d)^m)-3/2*a^2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)-3/8*b^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a*b^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m
,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-1/8*3^(-1-m)*b^3*exp(-3*e+3*c*f/d)*(d*x
+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.54, antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3398, 3388, 2212, 3393}

$$\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{3ab^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{3ab^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{b^33^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}-\frac{b^33^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}+\frac{a^3(c+dx)^{m+1}}{d(m+1)}+\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1 +
 m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(
c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e - (
2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))
/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))
/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (3*b^3*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (3*a^2*b*E^(-
e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))
/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])
/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 -
m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(8
*f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
```

```
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx = \int \left(a^3(c+dx)^m+3a^2b(c+dx)^m\cosh(e+fx)+3ab^2(c+dx)^m\cos\right.$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\left(3a^2b\right)\int (c+dx)^m\cosh(e+fx)\,dx+\left(3ab^2\right)\int (c$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{1}{2}\left(3a^2b\right)\int e^{-i(ie+ifx)}(c+dx)^m\,dx+\frac{1}{2}\left(3a^2b\right)\int e$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}+\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}+\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right.}{2f}$$

$$= \frac{a^3(c+dx)^{1+m}}{d(1+m)}+\frac{3ab^2(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}b^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\right.}{8}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2639 vs. $2(543) = 1086$.

time = 9.71, size = 2639, normalized size = 4.86

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```

```
[Out] (a^3*(c + d*x)^m*(c*f + d*f*x))/(d*f*(1 + m)) + (3*a^2*b*((f*Cosh[((-c + (d
*e)/f)*f)/d]*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f
+ (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d*(e + f
*x))/f))/d)]) - (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f +
(d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e + f*x))/
f))/d]))/(2*d) + (f*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (
d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d
*(e + f*x))/f))/d)]) + (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*
e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d])*Sinh[((-c + (d*e)/f)*f)/d])/(2*d)))/f + (3*a*b^2*((f*Cosh[((
-c + (d*e)/f)*f)/d]^2*((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)/(2*(1 + m))
+ (-(2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f
+ (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c - (d*e)/f + (d*(e +
f*x))/f))/d]) - 2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c -
 (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (2*f*(c - (d*e)/f + (
d*(e + f*x))/f))/d])/4))/d + (f*Cosh[((-c + (d*e)/f)*f)/d]*(-(2^(-1 - m)*(c
 - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f)
)/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c - (d*e)/f + (d*(e + f*x))/f))/d]) + 2^
(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (2*f*(c - (d*e)/f + (d*(e + f*x))/f))/d
])*Sinh[((-c + (d*e)/f)*f)/d])/(2*d) + (f*(-1/2*(c - (d*e)/f + (d*(e + f*x)
)/f)^(1 + m)/(1 + m) + (-(2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m
)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-2*f*(c
 - (d*e)/f + (d*(e + f*x))/f))/d]) - 2^(-1 - m)*(c - (d*e)/f + (d*(e + f*x)
)/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m,
(2*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])/4)*Sinh[((-c + (d*e)/f)*f)/d]^2)/
d))/f + (b^3*((f*Cosh[((-c + (d*e)/f)*f)/d]^3*((3*(-((c - (d*e)/f + (d*(e +
 f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma
[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)]) - (c - (d*e)/f + (d*(e +
 f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1
+ m, (f*(c - (d*e)/f + (d*(e + f*x))/f))/d]))/8 + (-(3^(-1 - m)*(c - (d*e)/
f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (d*(e + f*x))/f))/d))^(-1
 - m)*Gamma[1 + m, (-3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d]) - 3^(-1 - m)*
(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f)
)/d)^(-1 - m)*Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])/8))/d
+ (f*((-3*(-((c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c - (d*e)/f + (
d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x)
)/f))/d)]) + (c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*((f*(c - (d*e)/f + (d*
```

(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e + f*x))/f))
/d]))/8 + (-(3^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*(-((f*(c -
(d*e)/f + (d*(e + f*x))/f))/d))^(-1 - m)*Gamma[1 + m, (-3*f*(c - (d*e)/f +
(d*(e + f*x))/f))/d]) + 3^(-1 - m)*(c - (d*e)/f + (d*(e + f*x))/f)^(1 + m)*
((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(-1 - m)*Gamma[1 + m, (3*f*(c - (d*
e)/f + (d*(e + f*x))/f))/d])/8)*Sinh[((-c + (d*e)/f)*f)/d]^3)/d + ((c - (d*
e)/f + (d*(e + f*x))/f)^m*Cosh[e - (c*f)/d]^2*((-((f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d))^m*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(2*m)*Gamma[1 + m,
 (-3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d] + (-((f^2*(c - (d*e)/f + (d*(e +
 f*x))/f)^2)/d^2))^m*(3^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^m*G
amma[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)] + (-((f*(c - (d*e)/f
+ (d*(e + f*x))/f))/d))^m*(3^(1 + m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e +
 f*x))/f))/d] + Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])))*Si
nh[e - (c*f)/d])/(8*3^m*(-((f^2*(c - (d*e)/f + (d*(e + f*x))/f)^2)/d^2))^(2
*m)) + ((c - (d*e)/f + (d*(e + f*x))/f)^m*((-((f*(c - (d*e)/f + (d*(e + f*x
))/f))/d))^m*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^(2*m)*Gamma[1 + m, (-3
f(c - (d*e)/f + (d*(e + f*x))/f))/d] - (-((f^2*(c - (d*e)/f + (d*(e + f*x
))/f)^2)/d^2))^m*(3^(1 + m)*((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)^m*Gamma
[1 + m, -((f*(c - (d*e)/f + (d*(e + f*x))/f))/d)] + (-((f*(c - (d*e)/f + (d
*(e + f*x))/f))/d))^m*(-(3^(1 + m)*Gamma[1 + m, (f*(c - (d*e)/f + (d*(e + f
*x))/f))/d]) + Gamma[1 + m, (3*f*(c - (d*e)/f + (d*(e + f*x))/f))/d])))*Sin
h[e - (c*f)/d]*Sinh[2*(e - (c*f)/d)])/(16*3^m*(-((f^2*(c - (d*e)/f + (d*(e
+ f*x))/f)^2)/d^2))^(2*m))))/f

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m} \left(a+b\cosh \left(fx+e\right)\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)

[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)

Maxima [A]
time = 0.13, size = 383, normalized size = 0.71

$$-\frac{3}{2} \left(\frac{(dx+c)^{m+1} e^{\left(\frac{cf}{d}-e\right)} E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(-\frac{cf}{d}+e\right)} E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right) a^2 b - \frac{3}{4} \left(\frac{(dx+c)^{m+1} e^{\left(\frac{2\,cf}{d}-2\,e\right)} E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(-\frac{2\,cf}{d}+2\,e\right)} E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d} - \frac{2\,(dx+c)^{m+1}}{d(m+1)}\right) a b^2 - \frac{1}{8} \left(\frac{(dx+c)^{m+1} e^{\left(\frac{3\,cf}{d}-3\,e\right)} E_{-m}\left(\frac{3\,(dx+c)f}{d}\right)}{d} + \frac{3\,(dx+c)^{m+1} e^{\left(\frac{cf}{d}-e\right)} E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{3\,(dx+c)^{m+1} e^{\left(-\frac{cf}{d}+e\right)} E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1} e^{\left(-\frac{3\,cf}{d}+3\,e\right)} E_{-m}\left(-\frac{3\,(dx+c)f}{d}\right)}{d}\right) b^3 + \frac{(dx+c)^{m+1} a^3}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="maxima")

[Out] -3/2*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d +
 (d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2
b - 3/4((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c

```
)*f/d)/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x
+ c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a*b^2 - 1/8*((d*x + c)^(m +
1)*e^(3*c*f/d - 3*e)*exp_integral_e(-m, 3*(d*x + c)*f/d)/d + 3*(d*x + c)^(m
 + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d + 3*(d*x + c)^(m +
1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*
e^(-3*c*f/d + 3*e)*exp_integral_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (d*x + c)^
(m + 1)*a^3/(d*(m + 1))
```

Fricas [A]
time = 0.13, size = 883, normalized size = 1.63

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="fricas")
```

```
[Out] -1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) - 3*c*f + 3*d*cosh(1) + 3*d*s
inh(1))/d)*gamma(m + 1, 3*(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*cosh((
d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(1))/d)*gamma(m + 1, 2*(d*f*
x + c*f)/d) + 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*cosh((d*m*log(f/d
) - c*f + d*cosh(1) + d*sinh(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) - 9*((4*a
^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*cosh((d*m*log(-f/d) + c*f - d*cosh(1)
- d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*co
sh((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d)*gamma(m + 1, -2
*(d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) + 3*c*f - 3*d*c
osh(1) - 3*d*sinh(1))/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) - (b^3*d*m + b^3*
d)*gamma(m + 1, 3*(d*f*x + c*f)/d)*sinh((d*m*log(3*f/d) - 3*c*f + 3*d*cosh(
1) + 3*d*sinh(1))/d) - 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)
/d)*sinh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*sinh(1))/d) - 9*((4*a^
2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m
*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d) + 9*((4*a^2*b + b^3)*d*m + (4*a
^2*b + b^3)*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d
*cosh(1) - d*sinh(1))/d) + 9*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, -2*(d*f*x +
 c*f)/d)*sinh((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) + (b
^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log(-3*f/d) + 3*
c*f - 3*d*cosh(1) - 3*d*sinh(1))/d) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3
+ 3*a*b^2)*c*f)*cosh(m*log(d*x + c)) - 12*((2*a^3 + 3*a*b^2)*d*f*x + (2*a^3
 + 3*a*b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**3,x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="giac")
[Out] integrate((b*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cosh(e+f\,x))^3\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^3*(c + d*x)^m,x)
[Out] int((a + b*cosh(e + f*x))^3*(c + d*x)^m, x)
```

3.180 $\int (c+dx)^m (a+b\cosh(e+fx))^2 \, dx$

Optimal. Leaf size=282

$$\frac{a^2(c+dx)^{1+m}}{d(1+m)}+\frac{b^2(c+dx)^{1+m}}{2d(1+m)}+\frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{abe^{e-\frac{cf}{d}}(c+$$

```
[Out] a^2*(d*x+c)^(1+m)/d/(1+m)+1/2*b^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*b^2*exp(2*
e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*exp
(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a*b*exp(-e
+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*b^2*e
xp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.28, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3398, 3388, 2212, 3393}

$$\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}-\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^22^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^22^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{a^2(c+dx)^{m+1}}{d(m+1)}+\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] (a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m))
 + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c +
d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*Gam
ma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (a*b*E^(-e + (c
*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) -
 (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*
x))/d])/(f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3393

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx = \int \left(a^2(c+dx)^m + 2ab(c+dx)^m\cosh(e+fx) + b^2(c+dx)^m\cosh^2(\right.$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (2ab)\int (c+dx)^m\cosh(e+fx)\,dx + b^2\int (c+dx$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + (ab)\int e^{-i(ie+ifx)}(c+dx)^m\,dx + (ab)\int e^{i(ie+ifx)}($$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}$$

$$= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+}{d}\right.}{f}$$

Mathematica [A]
time = 5.83, size = 241, normalized size = 0.85

$$\frac{(c+dx)^m\left(\frac{8a^2f(c+dx)}{d(1+m)}+\frac{4b^2f(c+dx)}{d(1+m)}-8abe^{-e+\frac{cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,f\left(\frac{c}{d}+x\right)\right)+2^{-m}b^2e^{2e-\frac{2cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)+8abe^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)-b^2e^{-2e+\frac{2cf}{d}}\left(\frac{2cf}{d}+2fx\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] ((c + d*x)^m*((8*a^2*f*(c + d*x))/(d*(1 + m)) + (4*b^2*f*(c + d*x))/(d*(1 +
 m)) - (8*a*b*E^(-e + (c*f)/d)*Gamma[1 + m, f*(c/d + x)])/(f*(c/d + x))^m +
 (b^2*E^(2*e - (2*c*f)/d)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c +
```

```
d*x))/d))^m) + (8*a*b*E^(e - (c*f)/d)*Gamma[1 + m, -((f*(c + d*x))/d)])/(-
((f*(c + d*x))/d))^m - (b^2*E^(-2*e + (2*c*f)/d)*Gamma[1 + m, (2*f*(c + d*x
))/d])/((2*c*f)/d + 2*f*x)^m))/(8*f)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m} \left(a+b\cosh\left(fx+e\right)\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

```
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.08, size = 212, normalized size = 0.75

$$-\left(\frac{\left(dx+c\right)^{m+1} e^{\left(\frac{cf}{d}-e\right)} E_{-m}\left(\frac{\left(dx+c\right)f}{d}\right)}{d}+\frac{\left(dx+c\right)^{m+1} e^{\left(-\frac{cf}{d}+e\right)} E_{-m}\left(-\frac{\left(dx+c\right)f}{d}\right)}{d}\right)ab-\frac{1}{4}\left(\frac{\left(dx+c\right)^{m+1} e^{\left(\frac{2cf}{d}-2e\right)} E_{-m}\left(\frac{2\left(dx+c\right)f}{d}\right)}{d}+\frac{\left(dx+c\right)^{m+1} e^{\left(-\frac{2cf}{d}+2e\right)} E_{-m}\left(-\frac{2\left(dx+c\right)f}{d}\right)}{d}-\frac{2\left(dx+c\right)^{m+1}}{d(m+1)}\right)b^{2}+\frac{\left(dx+c\right)^{m+1}a^{2}}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

```
[Out] -((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*
x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b - 1
/4*((d*x + c)^(m + 1)*e^(2*c*f/d - 2*e)*exp_integral_e(-m, 2*(d*x + c)*f/d)
/d + (d*x + c)^(m + 1)*e^(-2*c*f/d + 2*e)*exp_integral_e(-m, -2*(d*x + c)*f
/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(d*(m
+ 1))
```

Fricas [A]
time = 0.15, size = 555, normalized size = 1.97

$$-\frac{\left(b^{2}dm+b^{2}d\right)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh\left(1\right)+2d\sinh\left(1\right)}{d}\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)+8\left(abdm+abd\right)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh\left(1\right)+d\sinh\left(1\right)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-8\left(abdm+abd\right)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh\left(1\right)-d\sinh\left(1\right)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-\left(b^{2}dm+b^{2}d\right)\cosh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh\left(1\right)-2d\sinh\left(1\right)}{d}\right)\Gamma\left(m+1,-\frac{2\left(dfx+cf\right)}{d}\right)-\left(b^{2}dm+b^{2}d\right)\Gamma\left(m+1,\frac{2\left(dfx+cf\right)}{d}\right)\sinh\left(\frac{dm\log\left(\frac{2f}{d}\right)-2cf+2d\cosh\left(1\right)+2d\sinh\left(1\right)}{d}\right)-8\left(abdm+abd\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh\left(1\right)+d\sinh\left(1\right)}{d}\right)+8\left(abdm+abd\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh\left(1\right)-d\sinh\left(1\right)}{d}\right)+\left(b^{2}dm+b^{2}d\right)\Gamma\left(m+1,-\frac{2\left(dfx+cf\right)}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{2f}{d}\right)+2cf-2d\cosh\left(1\right)-2d\sinh\left(1\right)}{d}\right)-4\left(\left(2a^{2}+b^{2}\right)dfx+\left(2a^{2}+b^{2}\right)cf\right)\cosh\left(m\log\left(dx+c\right)\right)-4\left(\left(2a^{2}+b^{2}\right)dfx+\left(2a^{2}+b^{2}\right)cf\right)\sinh\left(m\log\left(dx+c\right)\right)}{8\left(dfm+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) - 2*c*f + 2*d*cosh(1) + 2*d*si
nh(1))/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 8*(a*b*d*m + a*b*d)*cosh((d*m*l
og(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8
*(a*b*d*m + a*b*d)*cosh((d*m*log(-f/d) + c*f - d*cosh(1) - d*sinh(1))/d)*ga
mma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cosh((d*m*log(-2*f/d) + 2*
c*f - 2*d*cosh(1) - 2*d*sinh(1))/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) - (b^2
*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) - 2*c*f
+ 2*d*cosh(1) + 2*d*sinh(1))/d) - 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x +
```

```
 c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d) + 8*(a*b*d*m
+ a*b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f - d*cosh(
1) - d*sinh(1))/d) + (b^2*d*m + b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sin
h((d*m*log(-2*f/d) + 2*c*f - 2*d*cosh(1) - 2*d*sinh(1))/d) - 4*((2*a^2 + b^
2)*d*f*x + (2*a^2 + b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 + b^2)*d*f*x
 + (2*a^2 + b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e))**2,x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

```
[Out] integrate((b*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (a+b\cosh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))^2*(c + d*x)^m,x)
```

```
[Out] int((a + b*cosh(e + f*x))^2*(c + d*x)^m, x)
```

3.181 $\int (c+dx)^m(a+b\cosh(e+fx))\,dx$

Optimal. Leaf size=131

$$\frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\Big(1+m}{2f}$$

```
[Out] a*(d*x+c)^(1+m)/d/(1+m)+1/2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d
)/f/((-f*(d*x+c)/d)^m)-1/2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)
/f/((f*(d*x+c)/d)^m)
```

Rubi [A]
time = 0.10, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3398, 3388, 2212}

$$\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\text{Gamma}\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
```

```
[Out] (a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
+ m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)/d
)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Rule 2212

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

Rule 3388

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Dist[I/2, Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Dist[
I/2, Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e,
f, m}, x] && IntegerQ[2*k]
```

Rule 3398

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
```

```
m, 0] || NeQ[a^2 - b^2, 0])
```

Rubi steps

$$\begin{aligned}\int (c+dx)^m(a+b\cosh(e+fx))\,dx &= \int \left(a(c+dx)^m+b(c+dx)^m\cosh(e+fx)\right)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+b\int (c+dx)^m\cosh(e+fx)\,dx\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{1}{2}b\int e^{-i(ie+ifx)}(c+dx)^m\,dx+\frac{1}{2}b\int e^{i(ie+ifx)}(c+d\\ &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}\end{aligned}$$

Mathematica [A]
time = 0.97, size = 202, normalized size = 1.54

$$\frac{e^{-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(\cosh\left(\frac{3cf}{d}\right)+\sinh\left(\frac{3cf}{d}\right)\right)\left(2af(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m-bd(1+m)\left(-\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,f\left(\frac{c}{d}+x\right)\right)(\cosh(e)-\sinh(e))\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)+bd(1+m)\left(f\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{cf}{d}\right)+\sinh\left(e-\frac{cf}{d}\right)\right)\right)}{2df(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
[Out] ((c + d*x)^m*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(2*a*f*(c + d*x)*(-((f^2*(
c + d*x)^2)/d^2))^m - b*d*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, f*(c/
d + x)]*(Cosh[e] - Sinh[e])*(Cosh[(c*f)/d] + Sinh[(c*f)/d]) + b*d*(1 + m)*(
f*(c/d + x))^m*Gamma[1 + m, -((f*(c + d*x))/d)]*(Cosh[e - (c*f)/d] + Sinh[e
 - (c*f)/d])))/(2*d*E^((3*c*f)/d)*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Maple [F]
time = 180.00, size = 0, normalized size = 0.00

$$\int \left(dx+c\right)^{m}\left(a+b\cosh\left(fx+e\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
[Out] int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.06, size = 102, normalized size = 0.78

$$-\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(\frac{cf}{d}-e\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(-\frac{cf}{d}+e\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)b+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

```
[Out] -1/2*((d*x + c)^(m + 1)*e^(c*f/d - e)*exp_integral_e(-m, (d*x + c)*f/d)/d +
 (d*x + c)^(m + 1)*e^(-c*f/d + e)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*b +
 (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(127) = 254$.
time = 0.09, size = 271, normalized size = 2.07

$$-\frac{(bdm+bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-(bdm+bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)-(bdm+bd)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(\frac{f}{d}\right)-cf+d\cosh(1)+d\sinh(1)}{d}\right)+(bdm+bd)\Gamma\left(m+1,-\frac{dfx+cf}{d}\right)\sinh\left(\frac{dm\log\left(-\frac{f}{d}\right)+cf-d\cosh(1)-d\sinh(1)}{d}\right)-2\left(adfx+acf\right)\cosh\left(m\log\left(dx+c\right)\right)-2\left(adfx+acf\right)\sinh\left(m\log\left(dx+c\right)\right)}{2\left(dfm+df\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

```
[Out] -1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/d)*ga
mma(m + 1, (d*f*x + c*f)/d) - (b*d*m + b*d)*cosh((d*m*log(-f/d) + c*f - d*c
osh(1) - d*sinh(1))/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma
(m + 1, (d*f*x + c*f)/d)*sinh((d*m*log(f/d) - c*f + d*cosh(1) + d*sinh(1))/
d) + (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) + c*f
 - d*cosh(1) - d*sinh(1))/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x + c)) - 2
*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m*(a+b*cosh(f*x+e)),x)
```

```
[Out] Exception raised: TypeError >> cannot determine truth value of Relational
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

```
[Out] integrate((b*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (a+b\cosh(e+f\,x))\,\,{(c+d\,x)}^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*cosh(e + f*x))*(c + d*x)^m,x)
[Out] int((a + b*cosh(e + f*x))*(c + d*x)^m, x)
```

3.182 $\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{a+b\cosh(e+fx)},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Mathematica [A]
time = 0.88, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{a+b\cosh(fx+e)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
[Out] int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="fricas")
[Out] integral((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c + dx)^m}{a + b\cosh{(e + fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*cosh(f*x+e)),x)
[Out] Integral((c + d*x)**m/(a + b*cosh(e + f*x)), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="giac")
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{{\left(c+d\,x\right)}^m}{a+b\,\mathrm{cosh}\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*cosh(e + f*x)),x)
[Out] int((c + d*x)^m/(a + b*cosh(e + f*x)), x)
```

3.183 $\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} dx$

Optimal. Leaf size=23

$$\text{Int}\left(\frac{(c+dx)^m}{(a+b\cosh(e+fx))^2},x\right)$$

```
[Out] Unintegrable((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

Rubi [A]
time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} dx$$

Verification is not applicable to the result.

```
[In] Int[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Defer[Int][(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Rubi steps

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} dx = \int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} dx$$

Mathematica [A]
time = 3.94, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

```
[Out] Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Maple [A]
time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{(dx+c)^m}{(a+b\cosh(fx+e))^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
[Out] int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
[Out] integral((d*x + c)^m/(b^2*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e) + a^2), x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)**m/(a+b*cosh(f*x+e))**2,x)
[Out] Integral((c + d*x)**m/(a + b*cosh(e + f*x))**2, x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
[Out] integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{(c+d\,x)^m}{(a+b\cosh(e+f\,x))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x)^m/(a + b*cosh(e + f*x))^2,x)
[Out] int((c + d*x)^m/(a + b*cosh(e + f*x))^2, x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```